A Max K-Cut implementation for QAOA in the measurement based quantum computing formalism

S. Corli
Polimi, CNR

D. Dragoni
Leonardo S.p.A.

M. Proietti
Leonardo S.p.A.

M. Dispenza
Leonardo S.p.A.

C. Cavazzoni
Leonardo S.p.A.

E. Prati
Unimi, CNR

Quantum Team in Milan (Italy)

the Quantum Intelligence Lab - Milan

Università Statale Milano

Istituto di Fotonica e Nanotecnnologie

Politecnico Milano

CNR-IFN Milano

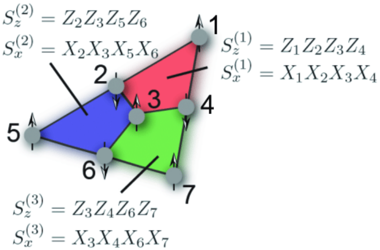
Quantum Team

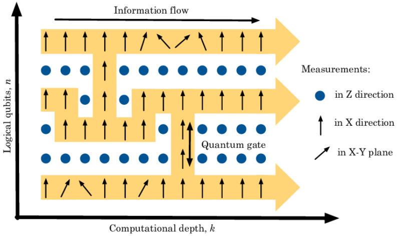
- Enrico Prati (Head, UNIMI)
- Paolo Zentilini (UNIMI)
- Luca Nigro (UNIMI)
- Francesco Monzani (UNIMI)
- Lorenzo Moro (POLIMI)
- Gabriele Agliardi (IBM POLIMI)
- Sebastiano Corli (POLIMI)
- Marco Maronese (UNIBO IIT)
- Lorenzo Rocutto (UNIBO)
- Andrea Zanetti (POLIMI)
- Davide Noè (Tohoku University)
- Matteo Di Giancamillo (POLIMI)
- Manuel Peracci (POLIMI)
- Rebecca Casati (UNIMI)
- Stefano Bruni (POLIMI)

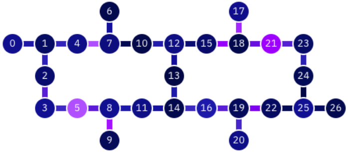
Graph Theory in Quantum Computing

Fields of interest

1. Error correction codes
2. Qubit connectivity
3. One-Way Quantum Computing

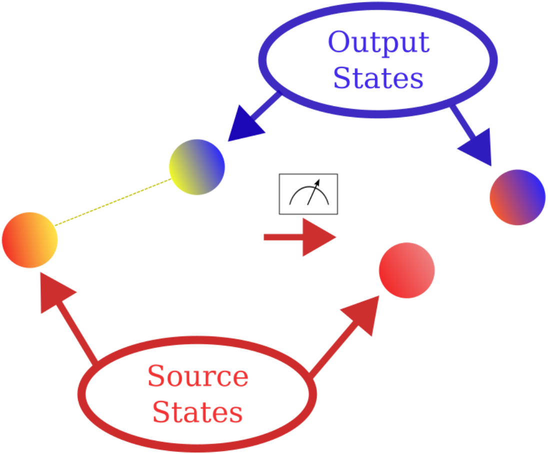






A. Bermudez et al. "Assessing the progress of trapped-ion processors towards fault-tolerant quantum computation." Physical Review X 7.4 (2017): 041061.

Measurement Based Quantum Computing (MBQC)



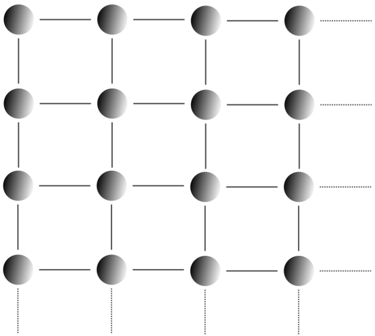
MBQC States

1. Source states
   - inputs
   - ancillae
2. Output states

MBQC Operators

- Entanglement
- Measurements
- Corrections

Graph States



Representation

- Vertices:
  $V = \{1, 2, ..., n\}$
- Edges:
  $E = \{(1,2), (2,3), ..., (n-1, n)\}$
- Neighborhood of a:
  $N_a = \{b \in V | (a, b) \in E\}$

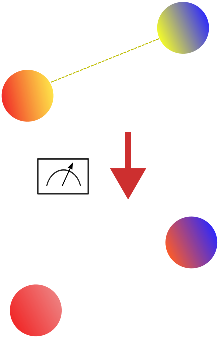
Encoding

- Vertices: qubits
- Edges: entanglements

! Caveat !

- Increasing number of logical qubits

Measurement-Based Quantum Computing



MBQC Pipeline

1. Input state: $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$
2. Projection: $|\theta\rangle = |0\rangle + e^{i\theta}\,|1\rangle$
3. Entanglement: $\hat{C}Z_{12}\,|\psi\rangle_1\,|+\rangle_2 = |\Psi\rangle$

$$|\Psi\rangle = a\,|0\rangle_1\,|+\rangle_2 + b\,|1\rangle_1\,|-\rangle_2$$

4. Measurement: $|\theta\rangle_1\,\langle\theta|\Psi\rangle = |\theta\rangle_1\,|\psi'\rangle$
5. Output: $|\psi'\rangle = a\,|+\rangle + b\,|-\rangle$

The Max k-Cut Problem

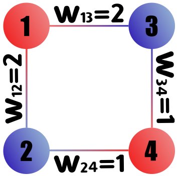
Features

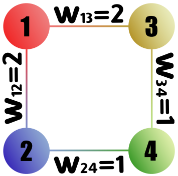
- Combinatorial optimization
- Encoding: Ising Hamiltonian

$$w(S, \bar{S}) = \frac{1}{2} \sum_{i \in S, j \in \bar{S}} w_{ij}$$

$$\max \frac{1}{2} \sum_{1 \leq i < j \leq n} w_{ij}(1 - z_i z_j)$$

$$\hat{H}_C = \sum_{ij} \frac{w_{ij}}{2}(I - \hat{Z}_i \hat{Z}_j)$$





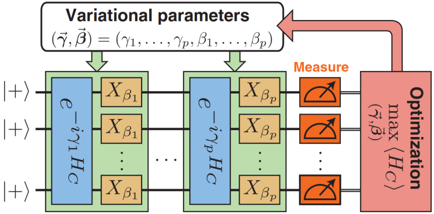
QAOA algorithm

Features

- Combinatorial optimization
- Variational algorithm: parameters $\{\beta_i, \gamma_i\}_{i=1}^{p}$
- Hybrid algorithm
  1. Backpropagation: classical
  2. State evolution: quantum

Quantum features

- Mixing Hamiltonian: $\hat{H}_M \left|+\right\rangle^{\otimes n} = \left|+\right\rangle^{\otimes n}$
- Cost Hamiltonian: $\hat{H}_C \left|\psi\right\rangle = E \left|\psi\right\rangle$
- Goal state: $\hat{H}_C \left|\tilde{\psi}\right\rangle = E_{max} \left|\tilde{\psi}\right\rangle$
- Approx. ratio: $r = E / E_{max}$



L. Zhou et al., *Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices*, PHYSICAL REVIEW X, 2020

Derivation from Adiabatic Theorem

Adiabatic Theorem

- Adiabatic evolution, $t \in [0, T]$

$$\hat{H}(t) = \left[1 - \frac{t}{T}\right] \hat{H}_M - \frac{t}{T} \hat{H}_C$$

- Time evolution

$$\hat{U}(t) = e^{-i \int_0^T \mathrm{d}t \hat{H}(t)}$$

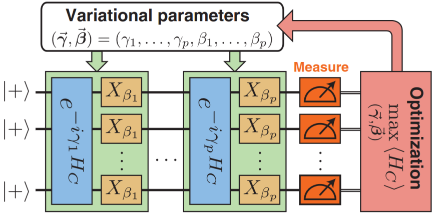
QAOA algorithm

- QAOA approximation on $\hat{H}(t)$

$$\prod_{i=1}^{p} e^{-i\gamma_i \hat{H}_C} e^{-i\beta_i \hat{H}_M}$$

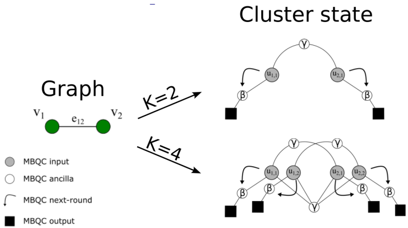
- Backpropagation on angles

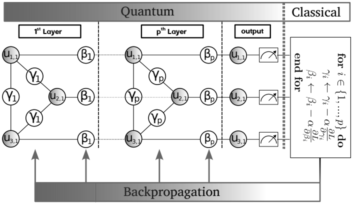
$$\beta_i \to \beta_i - \nabla_{\beta_i} E$$



L. Zhou et al., *Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices*, PHYSICAL REVIEW X, 2020

QAOA in the MBQC Frame





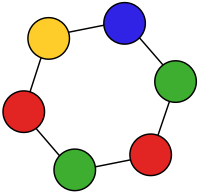
Algorithm MBQC-QAOA

Set graph $G[V, E]$
Set K, p
for i in p **do**:
  Install graph state
  for $|+\rangle_j \in \textcircled{\gamma}$ **do**:
    $\langle \gamma_i | + \rangle_j$
  end for
  for $|+\rangle_j \in \textcircled{\beta}$ **do**:
    $\langle \beta_i | u_j \rangle$
    $\langle + | + \rangle_j$
  end for
end for

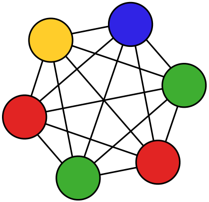
M. Proietti, F. Cerocchi, M. Dispenza, *Native measurement-based quantum approximate optimization algorithm applied to the Max K-Cut problem*, PHYSICAL REVIEW A, 2022

Environment Settings

Cyclic Graphs



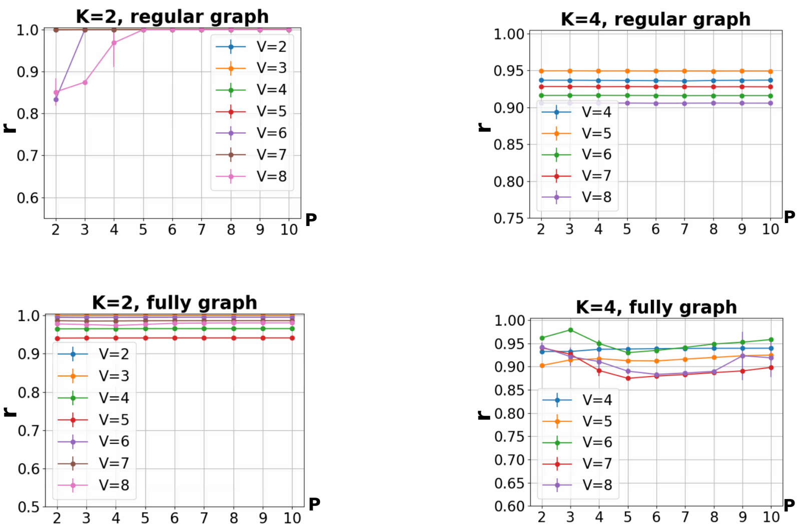
Fully Connected Graphs



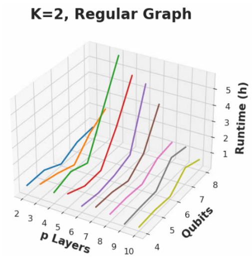
Computational frame

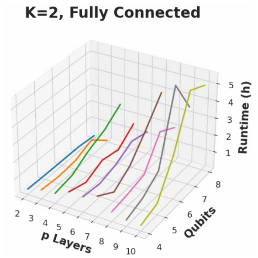
1. Software: python libraries
   - paddle (Baidu)
   - paddle-quantum
2. Hardware: HPC system davinci-1 (Leonardo Company)
   - AMD Epyc CPUs (512GB)
   - A100 GPUs (40GB)

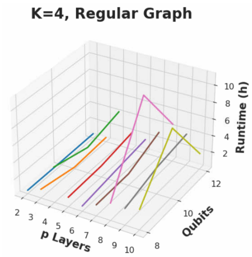
Results: Approximation Ratio $r = E/E_{max}$

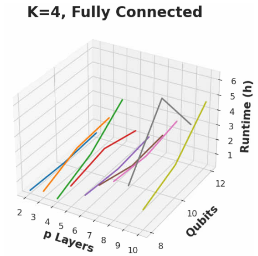


Results: Global Runtime for Simulations









Acknowledgements and Perspectives

- Politecnico di Milano: actual institution
- Leonardo company: developers team & davinci-1 hardware provider
- On road: implementation on QuiX Quantum photonic hardware

Max k-Cut Hamiltonian

$$\hat{H}_C = \sum_{ij} \frac{w_{ij}}{2^q} \sum_{n=1}^{q} \left(C_n I - \hat{C}_n[\{\hat{Z}_{il}\hat{Z}_{jl}\}_{l=1}^{q}] \right)$$

$$q = \log_2(k)$$

- $q = 1$
- $C_1 = 1$

$$\hat{H}_C = \frac{1}{2}(w_{12} + w_{13} + w_{23} + -w_{12}\hat{Z}_1\hat{Z}_2 - w_{13}\hat{Z}_1\hat{Z}_3 + -w_{23}\hat{Z}_2\hat{Z}_3)$$