Targeting quantum many-body scars with shallow variational quantum circuits

GABRIELE CENEDESE

Università degli Studi dell'Insubria – INFN Milano

In collaboration with:
M. Bondani, A. Andreanov, M. Carrega, G. Benenti and D. Rosa

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Introduction

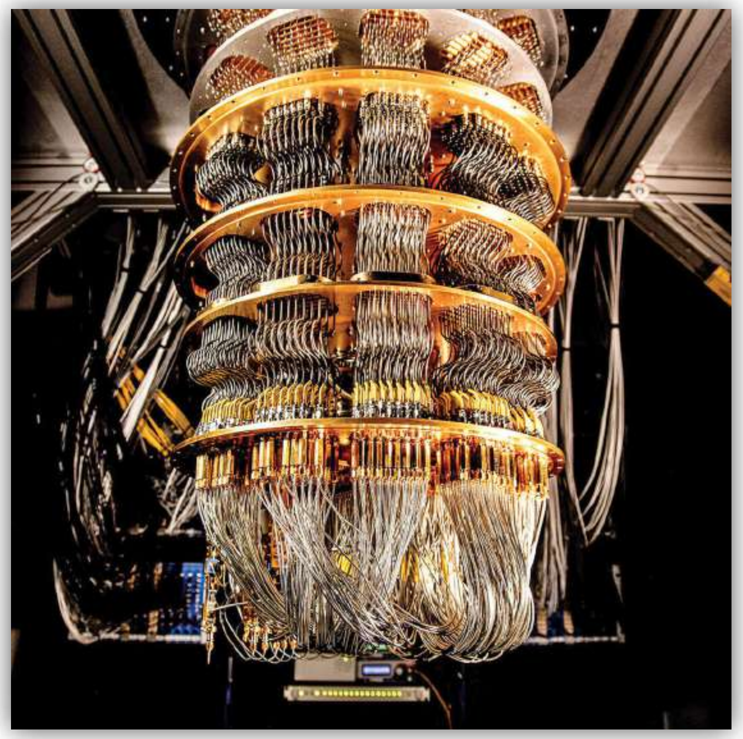
G. Cenedese, et al. Generation of Pseudo-Random Quantum States on Actual Quantum Processors. *Entropy 25 (4), 607* (2023).

G. Cenedese, et al. Correcting Coherent Errors by Random Operators on Actual Quantum Hardware. *Entropy 25 (2), 324* (2023).

Quantum computers are prone to **noise** and **decoherence:**

- **Coherent errors**
  - **Gate imperfections, drift in control parameters**
  - **Crosstalk**
- **Incoherent errors**

  Due to the interaction with the enviroment
  - **Bit-flip, phase-flip**
  - **Amplitude damping**
  - **Phase damping**
  - **Depolarizig noise**
  - **Readout errors**



Credits: Erik Lucero/Google Quantum AI

Introduction

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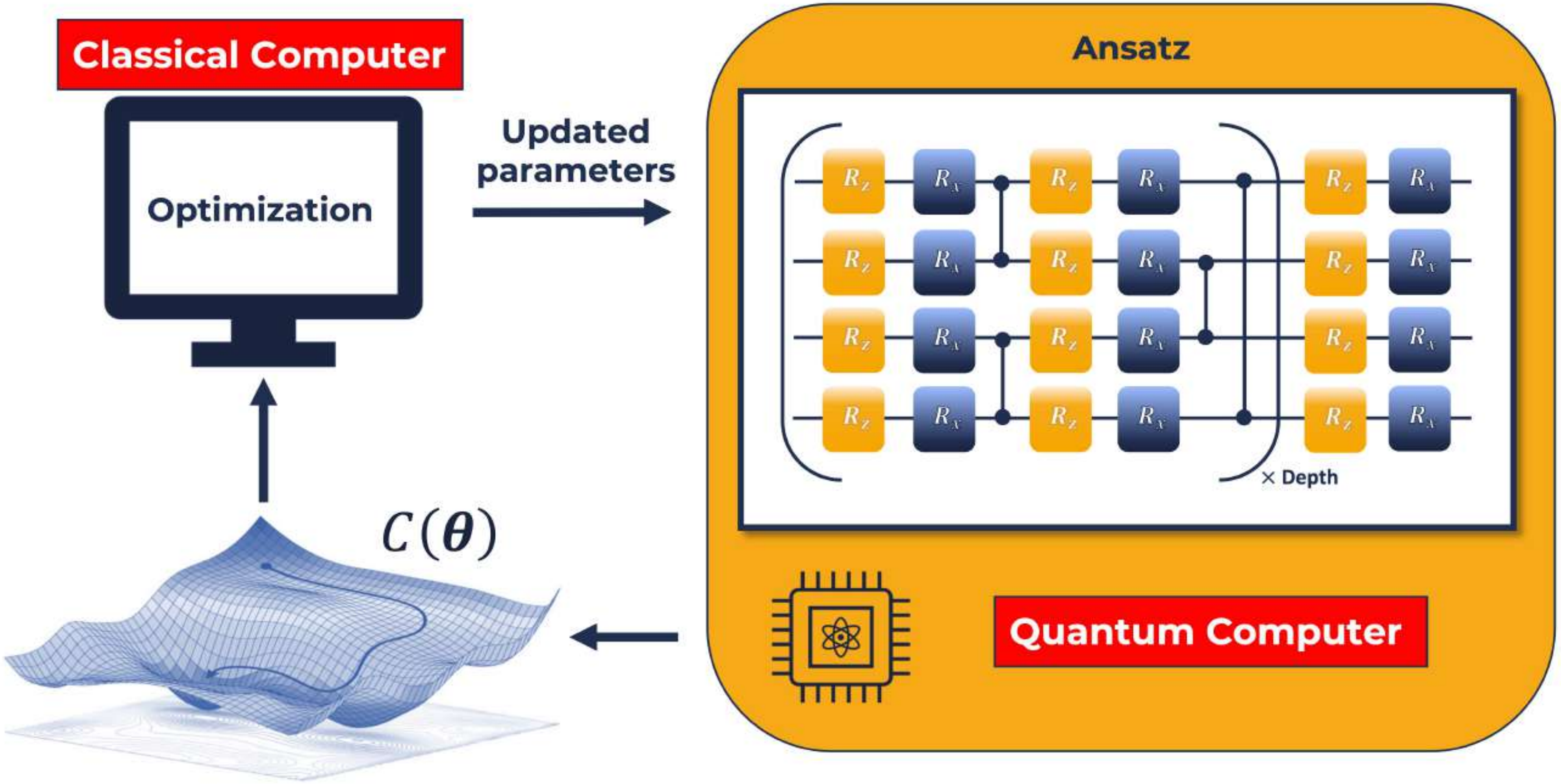
→

- **Error mitigation**
- **Variational Quantum Algorithms (VQAs)**

VQA

A **variational quantum algorithm** combines **quantum circuits** with **classical optimization techniques**. These algorithms are designed to solve optimization problems and are particularly well-suited for **near-term quantum computers**, which are limited by factors such as qubit noise and gate errors



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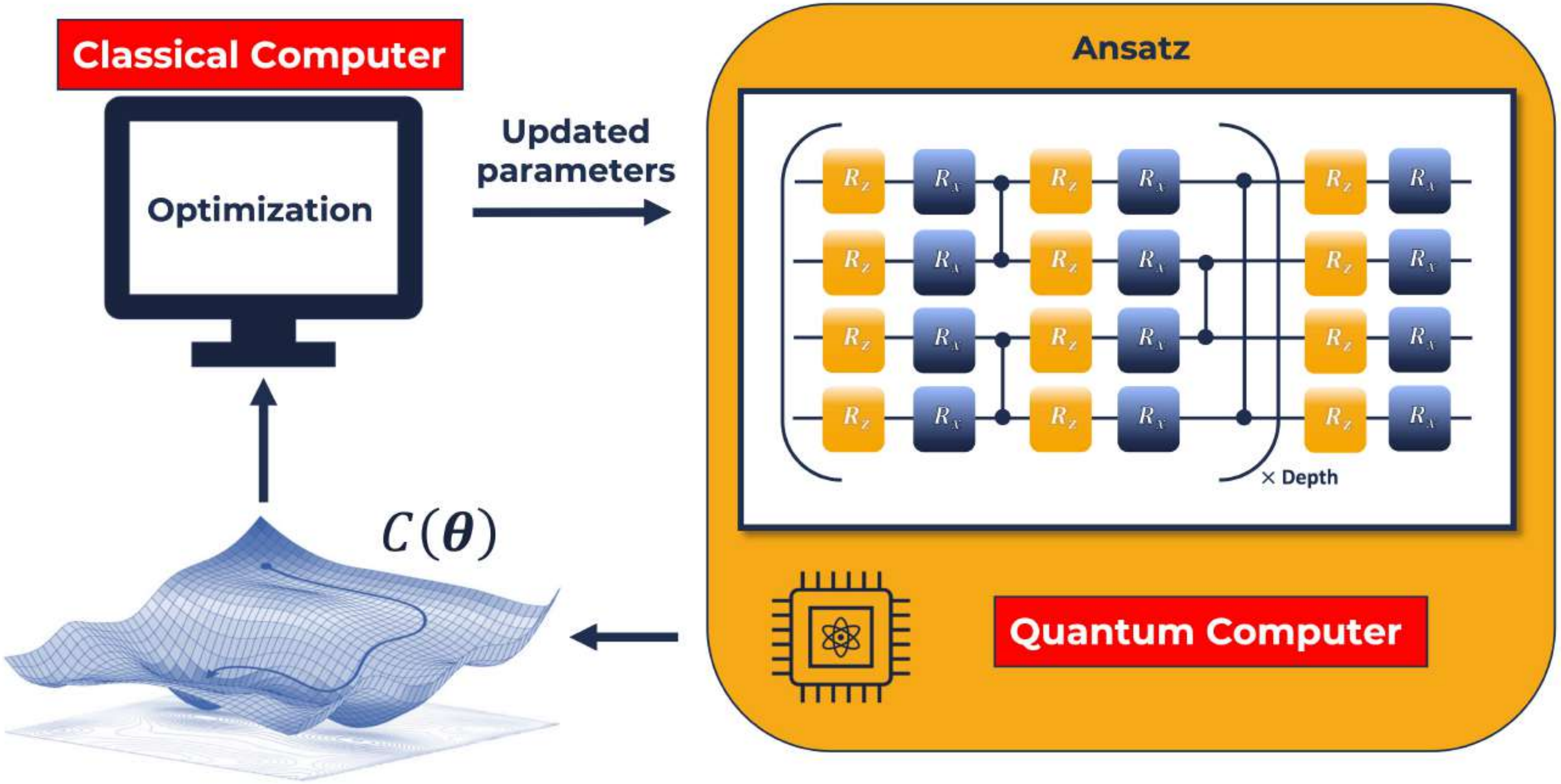
- **Cost function $C(\theta)$:** encodes the solution of the problem, estimated by a quantum computer (or its gradient)



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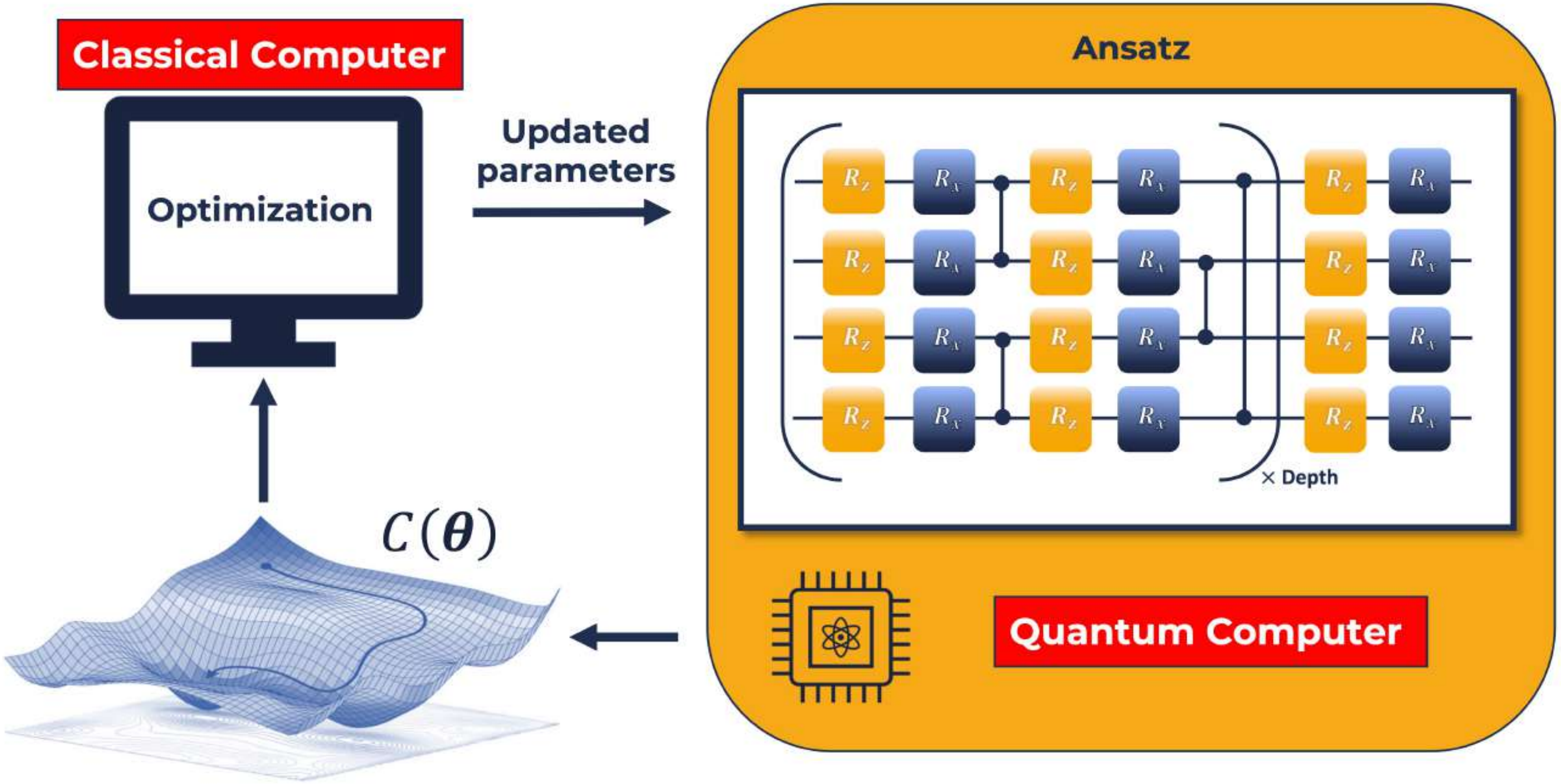
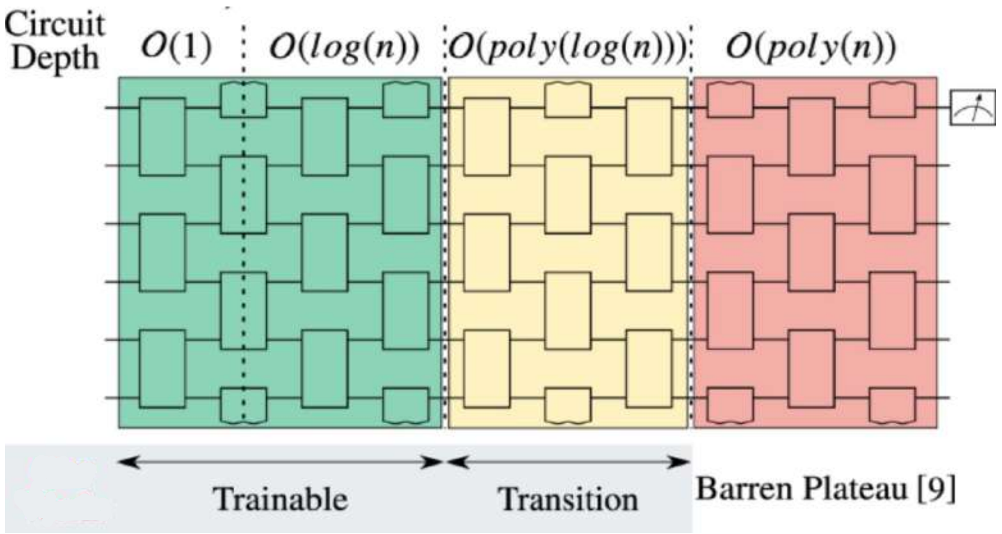
- **Cost function $C(\boldsymbol{\theta})$:** encodes the solution of the problem, estimated by a quantum computer (or its gradient)

- **Ansatz:** parametric quantum circuit $U(\boldsymbol{\theta})$, it can be problem-inspired or problem-agnostic, the circuit depth and ancilla requirements must be kept small

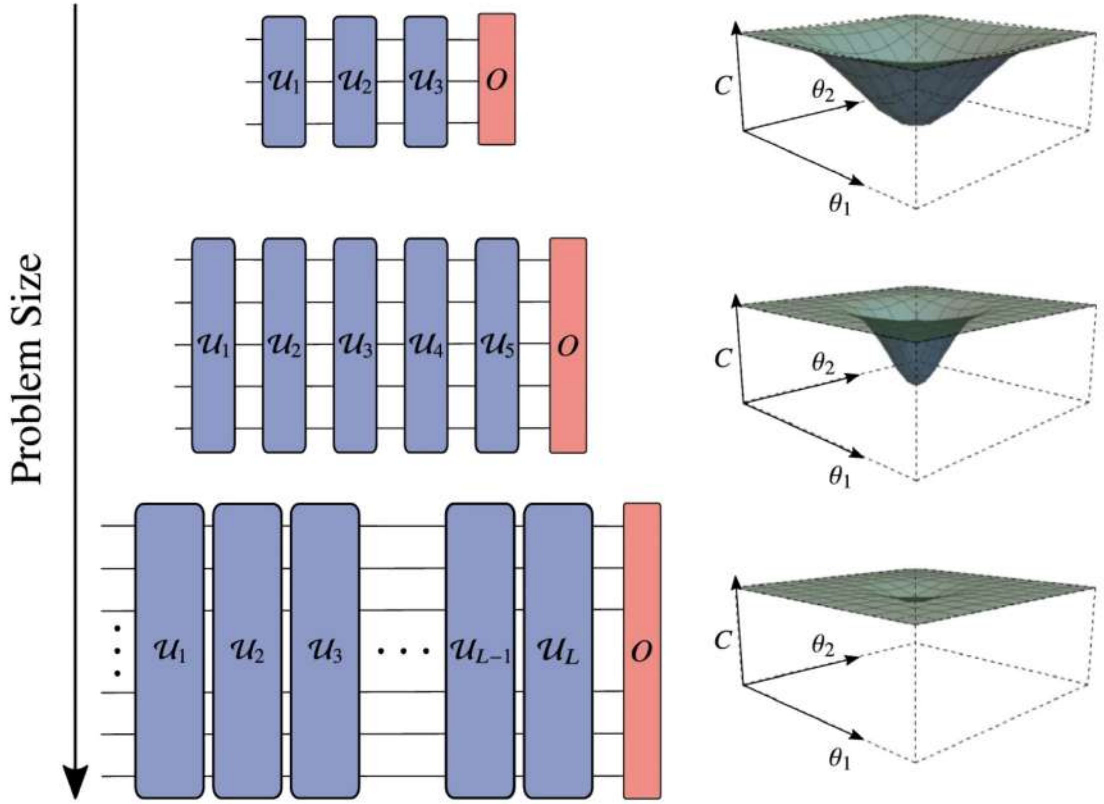


VQAs: Barren Plateaus

Noisy and **deep** ansatzes induce **Barren plateau**, vanishing gradient:



M. Cerezo, et al. Cost function dependent barren plateaus in shallow parametrized quantum circuits. *Nature communications* 12.1: 1791 (2021).



➔ Ansatz depth must be kept **shallow**

J. R. McClean, et al. Barren plateaus in quantum neural network training landscapes. *Nature communications* 9.1: 4812 (2018).

S. Wang, et al. Noise-induced barren plateaus in variational quantum algorithms. *Nature communications* 12.1: 6961 (2021).

Quantum Many-Body Scars

Eigenstates of **non-integrable** many-body systems (in a lattice) which present peculiar characteristics such as **area-law entanglement** entropy and breaking of **eigenstate thermalization hypothesis (ETH)**

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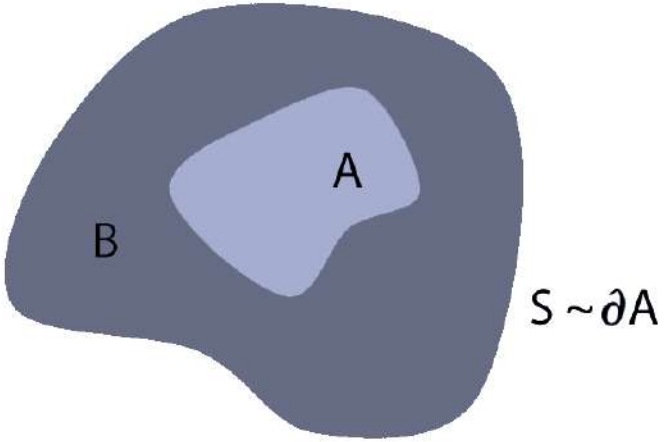
Area-law →



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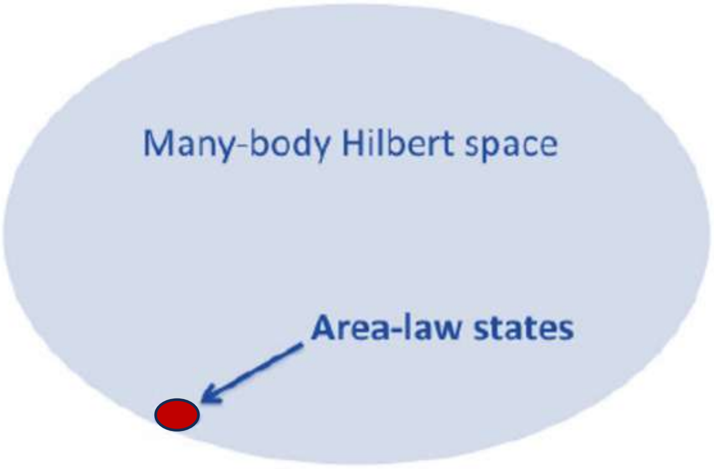

J. Eisert, et al. Colloquium: Area laws for the entanglement entropy. *Reviews of modern physics* 82.1:277 (2010).

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ETH ➔

$$\mathcal{H}|E_\alpha\rangle = E_\alpha|E_\alpha\rangle$$

$$|\psi(0)\rangle = \sum_\alpha C_\alpha |E_\alpha\rangle$$

Narrow in energy
➔ $\Delta\mathcal{H} \ll \langle\mathcal{H}\rangle$

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$$A_{\alpha\beta} = \langle E_\alpha | A | E_\beta \rangle$$

Infinite time average of an observable

$$\bar{A} \equiv \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau dt \, \langle A \rangle_t = \sum_\alpha |C_\alpha|^2 A_{\alpha\alpha}$$

Depends on the initial conditions

ETH →

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→ $\Delta\mathcal{H} \ll \langle\mathcal{H}\rangle$

Quantum statistical average

$$\langle A \rangle_{mc}(\langle\mathcal{H}\rangle) = \frac{1}{N_{\langle\mathcal{H}\rangle,\Delta\mathcal{H}}} \sum_{\alpha:E_\alpha\in I} A_{\alpha\alpha}$$

$I = [\langle\mathcal{H}\rangle - \Delta\mathcal{H}, \langle\mathcal{H}\rangle + \Delta\mathcal{H}]$

Depends on $\langle\mathcal{H}\rangle = \sum_\alpha |C_\alpha|^2 E_\alpha$

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Depends on the initial conditions

ETH →

Each eigenstate of the Hamiltonian implicitly contains a thermal state

$\mathcal{H}|E_\alpha\rangle = E_\alpha |E_\alpha\rangle$

$|\psi(0)\rangle = \sum_\alpha C_\alpha |E_\alpha\rangle$

Narrow in energy
➔ $\Delta\mathcal{H} \ll \langle\mathcal{H}\rangle$

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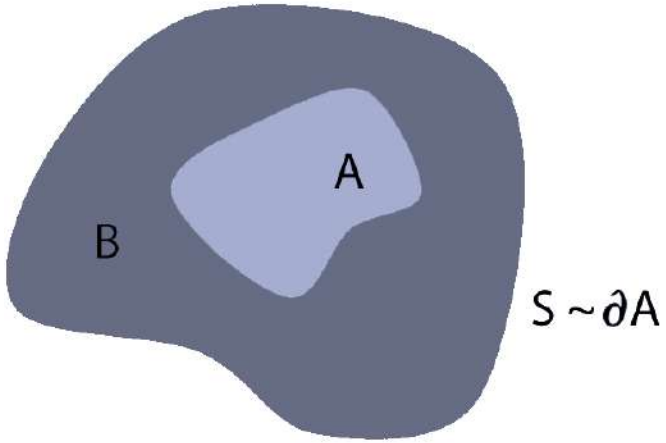

ETH

Ergodic dynamics of the observables, thermal behaviour
➔ loss of the information about the initial state
➔ no dynamical revivals

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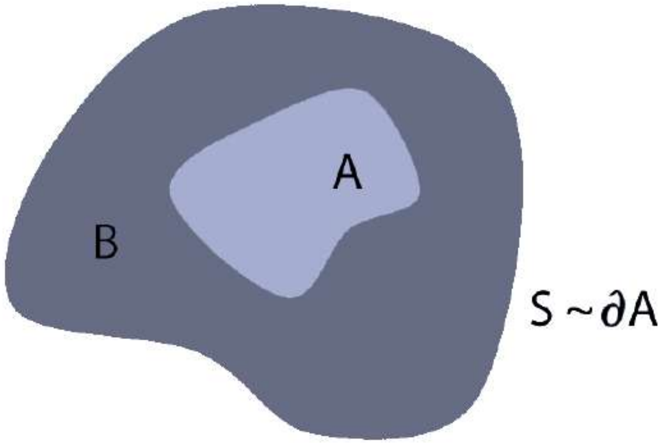
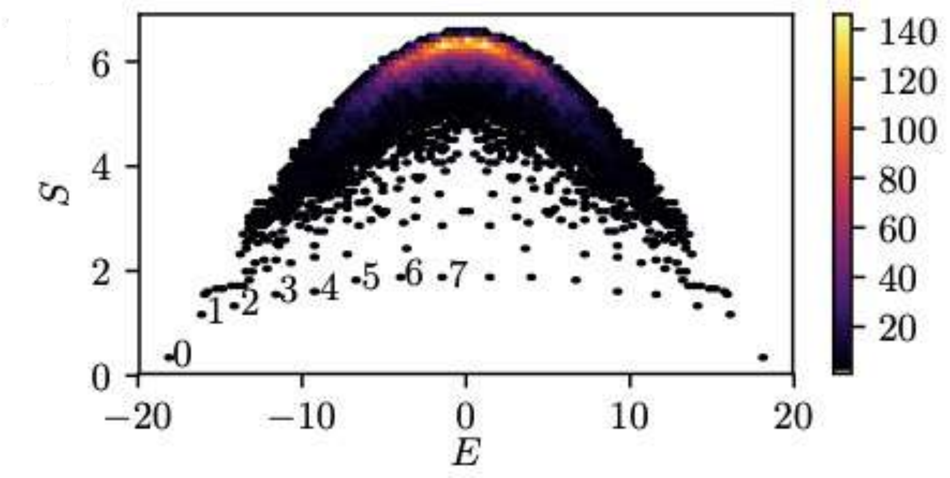


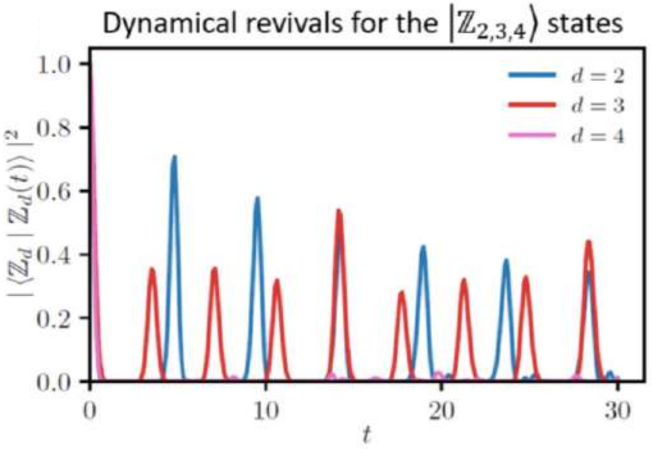
ETH

Ergodic dynamics of the ob… …bles, thermal behaviour
➔ loss of the informatio… …bou… …e initial state
➔ no dynamical revival…

States with **overlap** with scars exhibit **long-lived oscillations** (revivals) and thus **non-thermal behaviour**

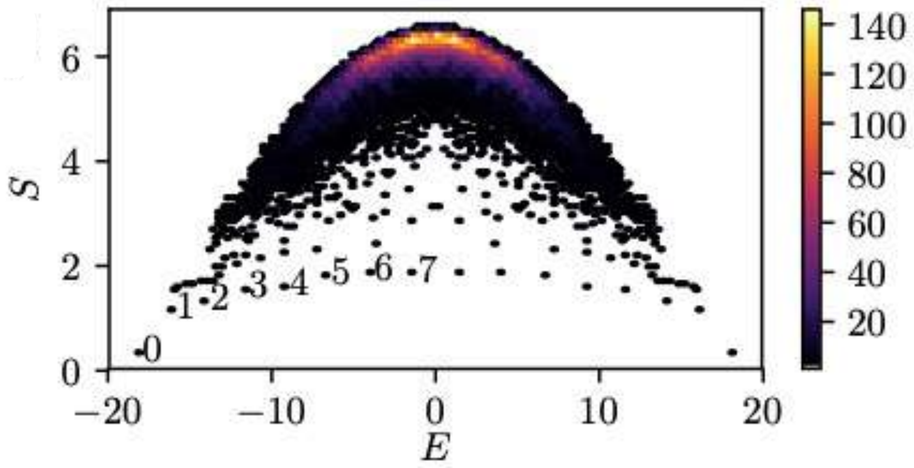
Quantum Many-Body Scars

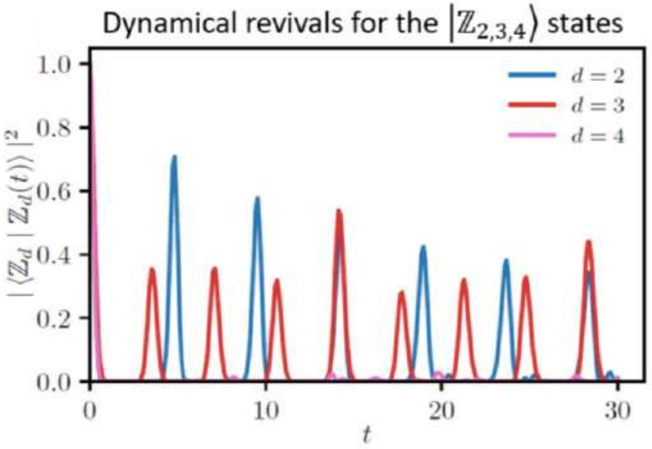


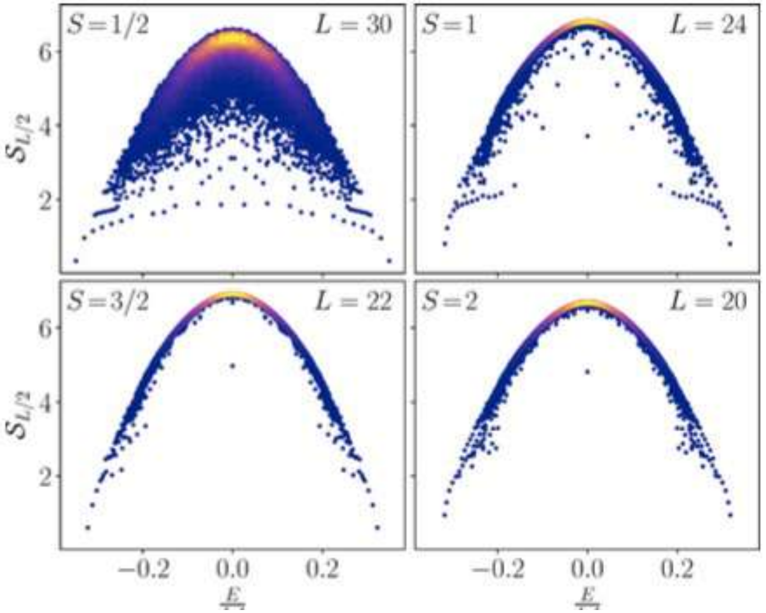


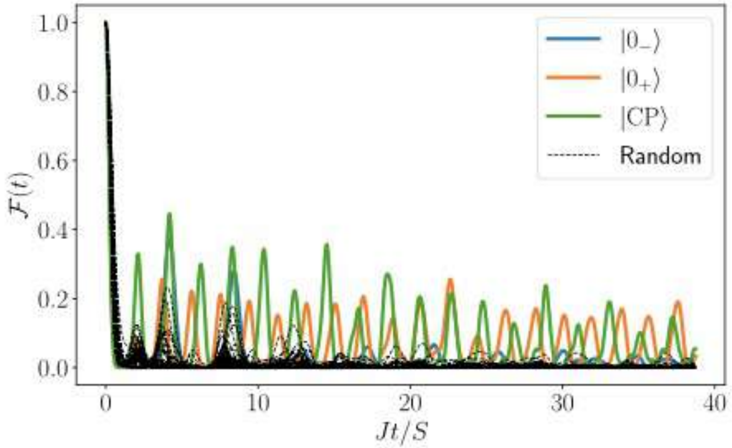
C. J. Turner, et al. Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations. *PRB* 98.15: 155134 (2018).

Quantum Many-Body Scars









J.-Y. Desaules, et al. Prominent quantum many-body scars in a truncated Schwinger model. *PRB* 107.20: 205112 (2023).

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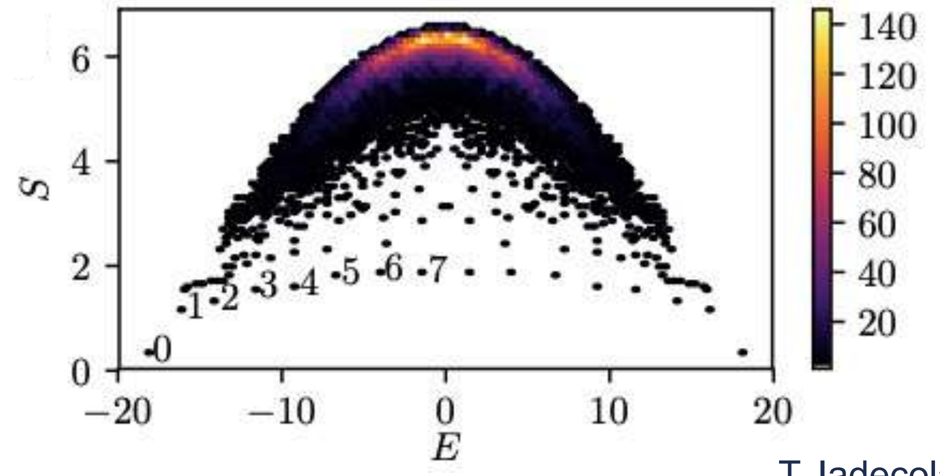
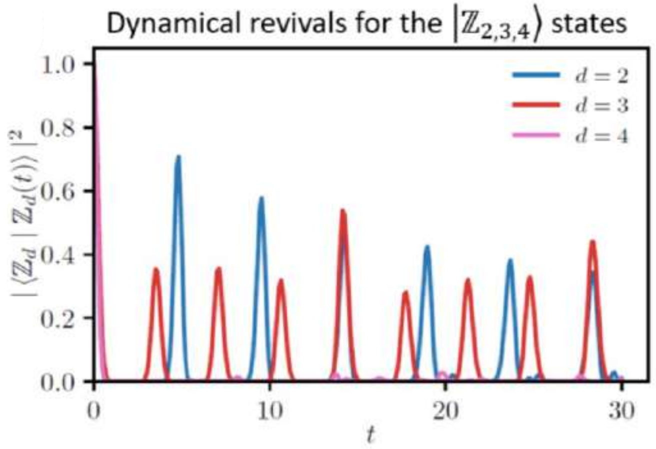
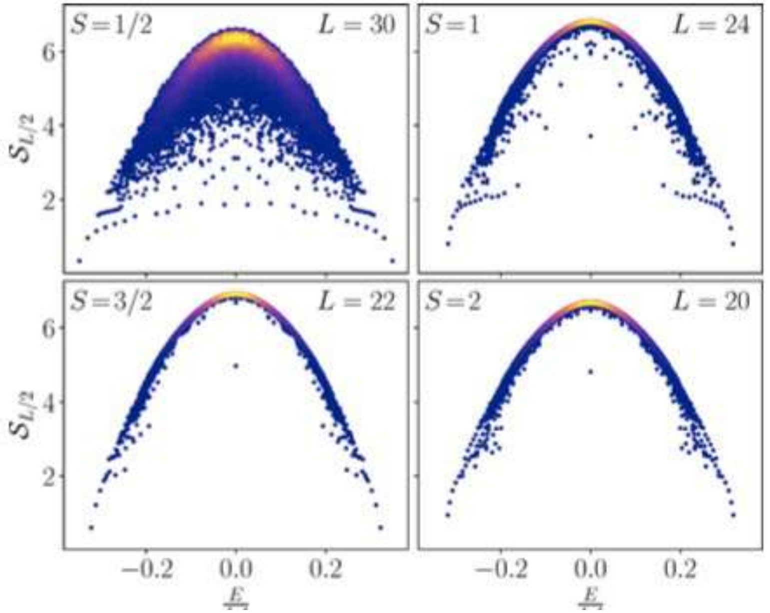
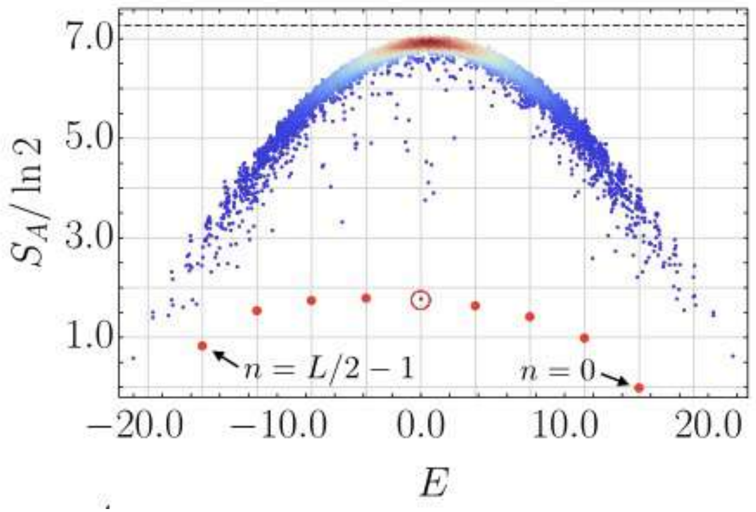
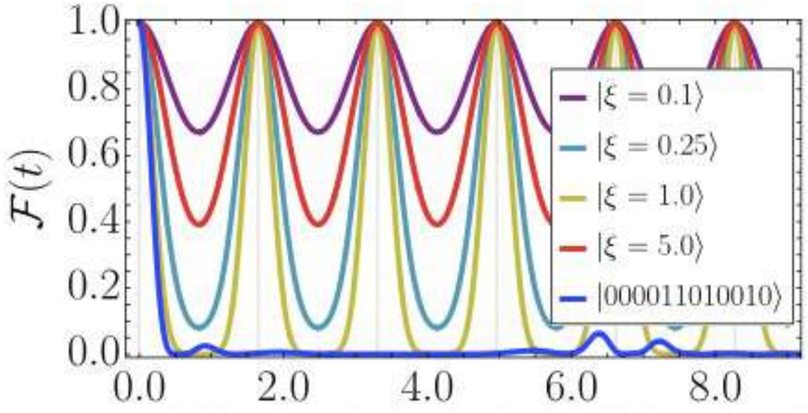


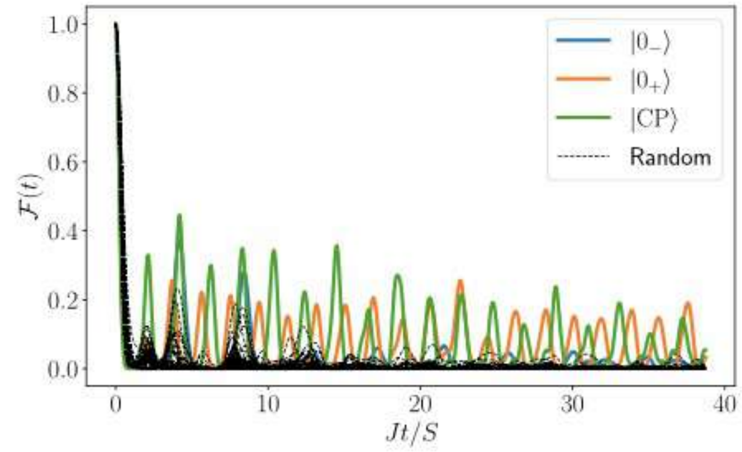




T. Iadecola and M. Schecter. Quantum many-body scar states with emergent kinetic constraints and finite-entanglement revivals. *PRB* 101.2: 024306 (2020).







VQE-Scars

Cost function:

$$C(\boldsymbol{\theta}) = a\,\langle (H-E)^2\rangle + b(\langle H^2\rangle - \langle H\rangle^2) + c f_{symm}$$

$a, b, c \in [0,1] : \; a + b + c = 1$

$$\boldsymbol{\theta}_{opt} = \mathrm{argmin}_{\theta} C(\boldsymbol{\theta})$$

$$|\psi_{vqe}\rangle = U(\boldsymbol{\theta}_{opt})|0\rangle$$

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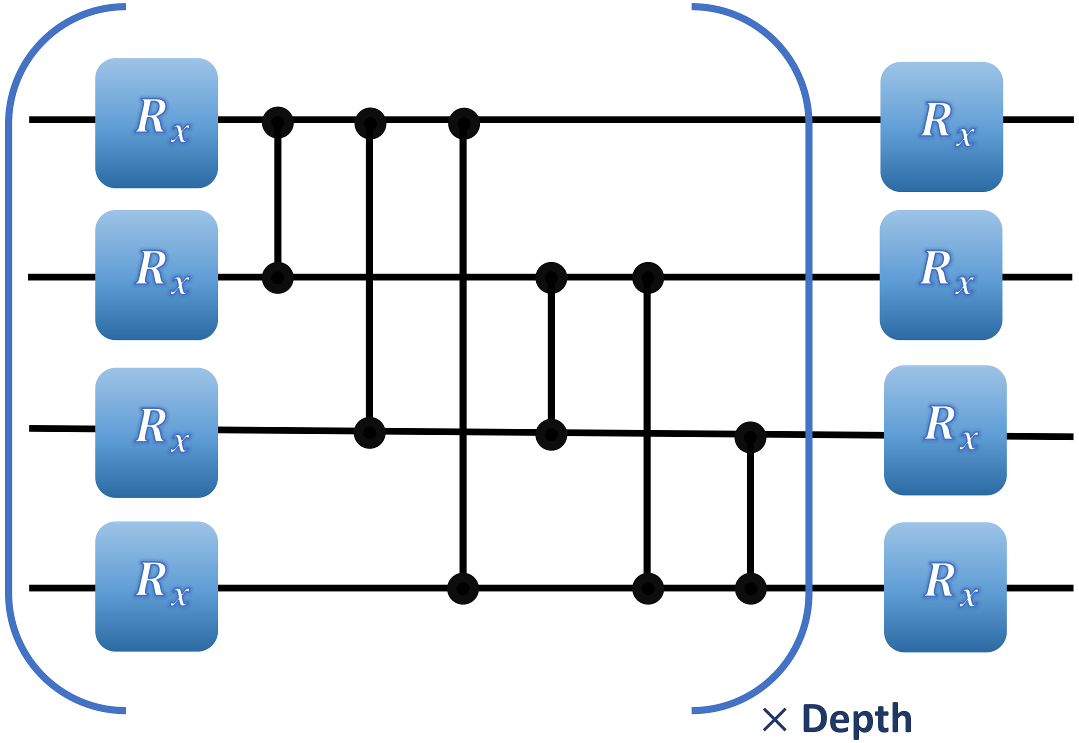
All-to-all ansatz:



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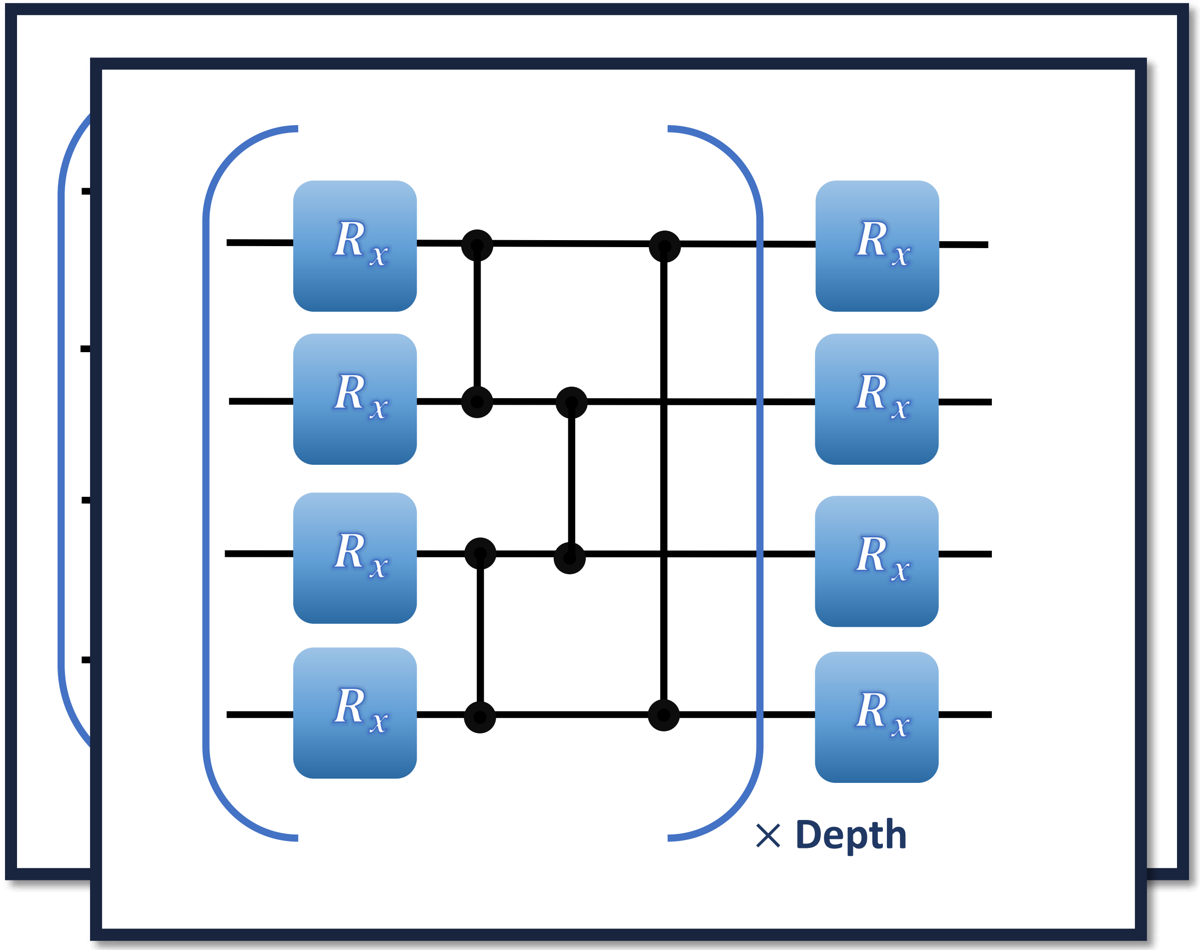
Nearest-neighbour ansatz:



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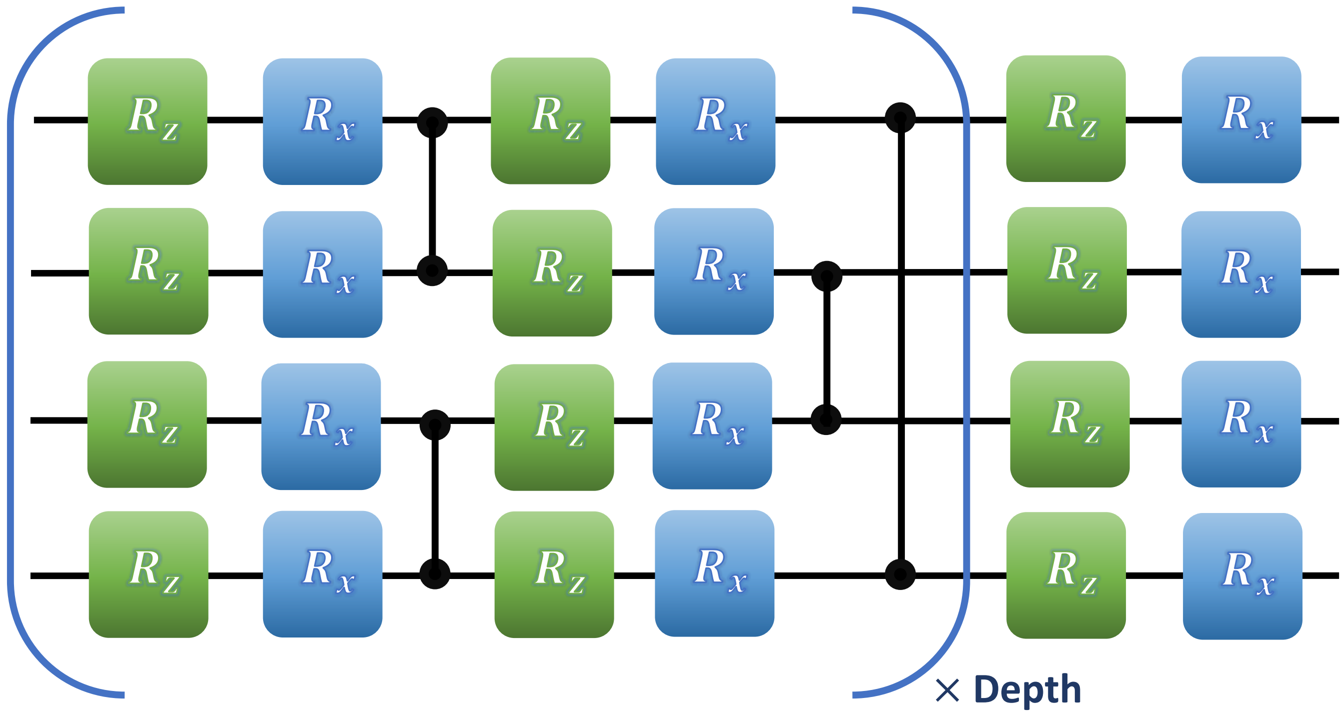
Hardware-efficient ansatz:



VQE-scars: model 1

1D Hamiltonian of N **hardcore bosons** placed in a circular lattice:

$$H = \sum_{i \neq j} G_{ij}^{A}\, d_i^{\dagger} d_j + \sum_{i \neq j} G_{ij}^{B}\, n_i n_j + \sum_{i \neq j \neq l} G_{ijl}^{C}\, d_i^{\dagger} d_l n_j + \sum_{i} G_i^{D}\, n_i + G^{E}$$

d_i and $d_i^{\dagger}$ are the annihilation and creation operators, respectively and $n_i = d_i^{\dagger} d_i$

$$N_b = \sum_{i=1}^{N} n_i \qquad [N_b, H] = 0$$

$$\boxed{f_{symm} = \langle (N_b - n_b)^2 \rangle}$$

N.S. Srivatsa, et al. Quantum many-body scars with chiral topological order in two dimensions and critical properties in one dimension. *Physical Review B* 102.23:235106 (2020).

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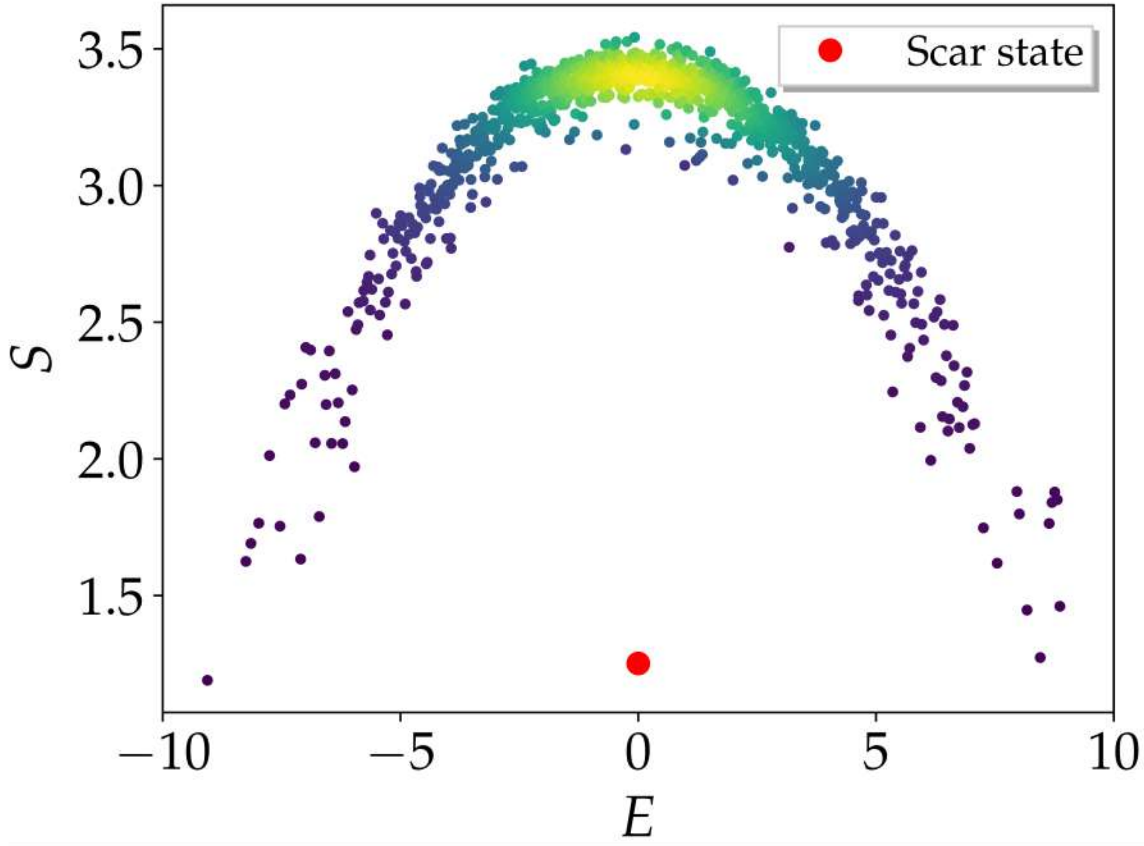
$N = 12, n_b = 6$



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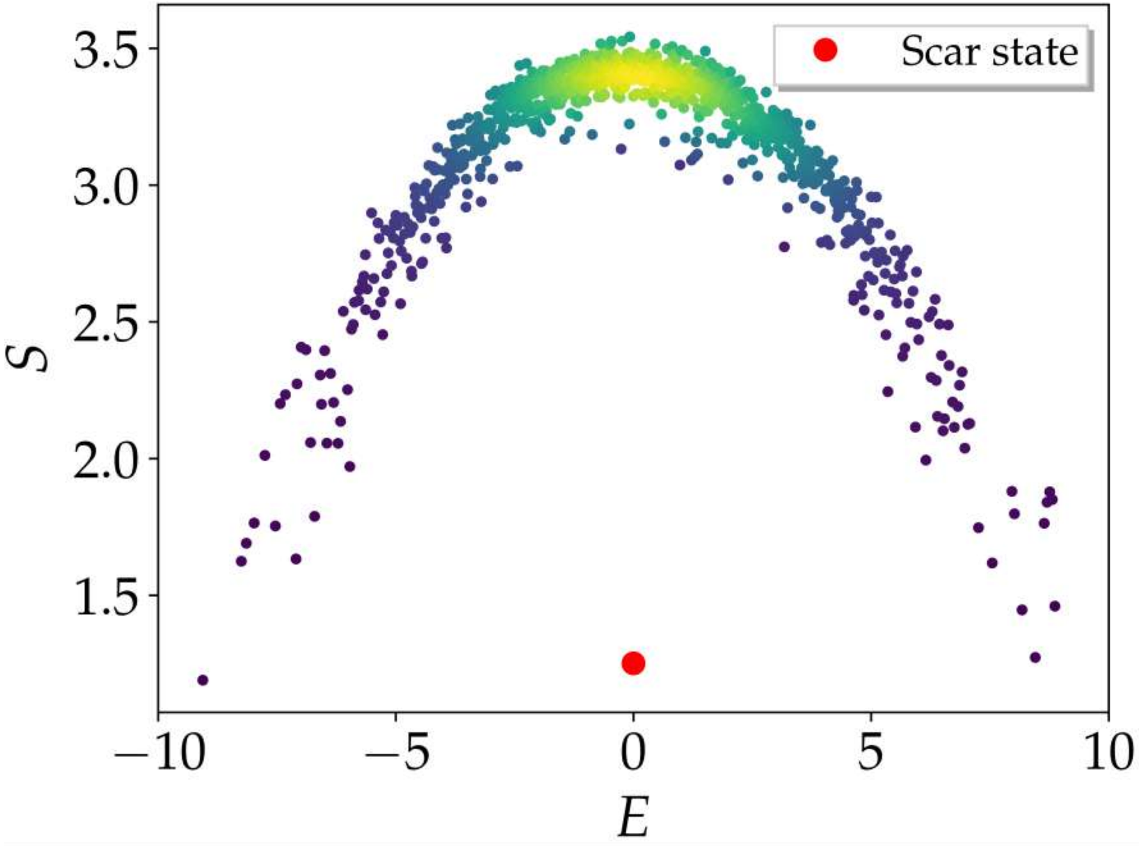

Why this Hamiltonian?

- ❖ The **location** of the scar **can be adjusted** by tuning a parameter of the Hamiltonian
- ❖ The algebra of hardcore bosons can be easily mapped into Pauli algebra su(2)

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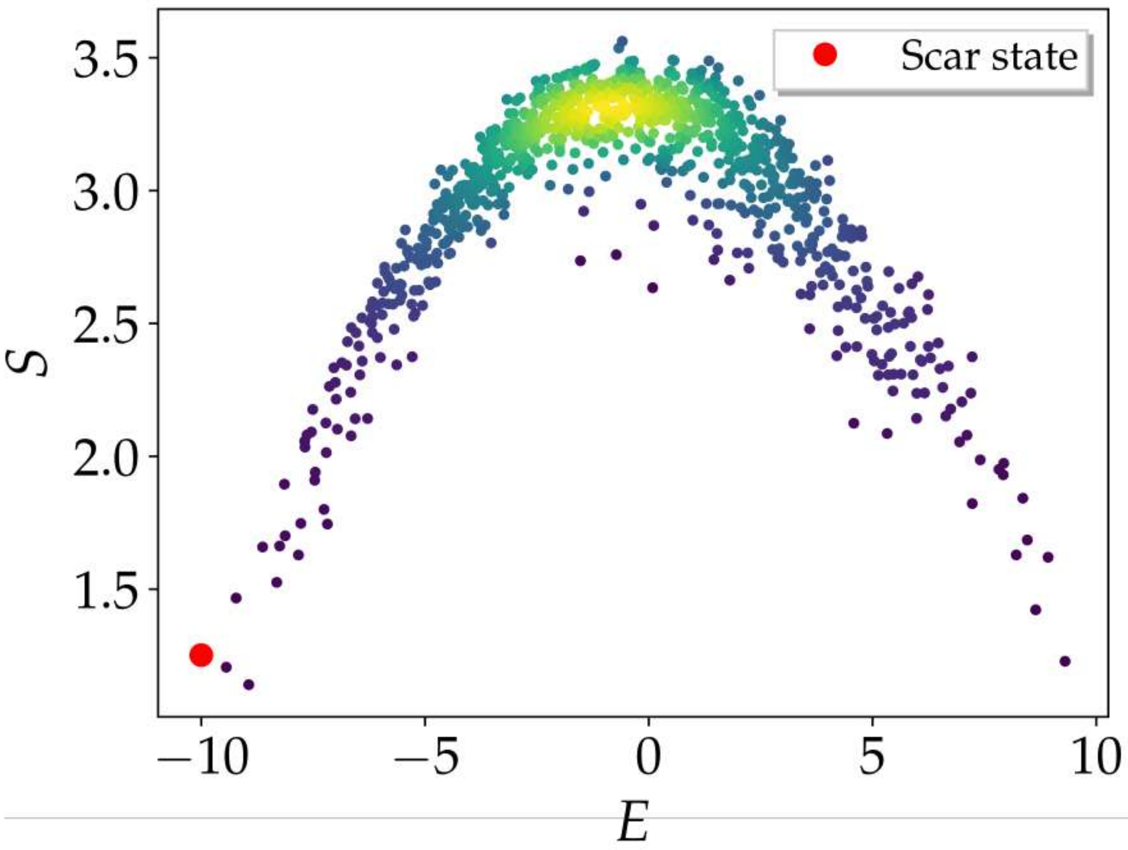

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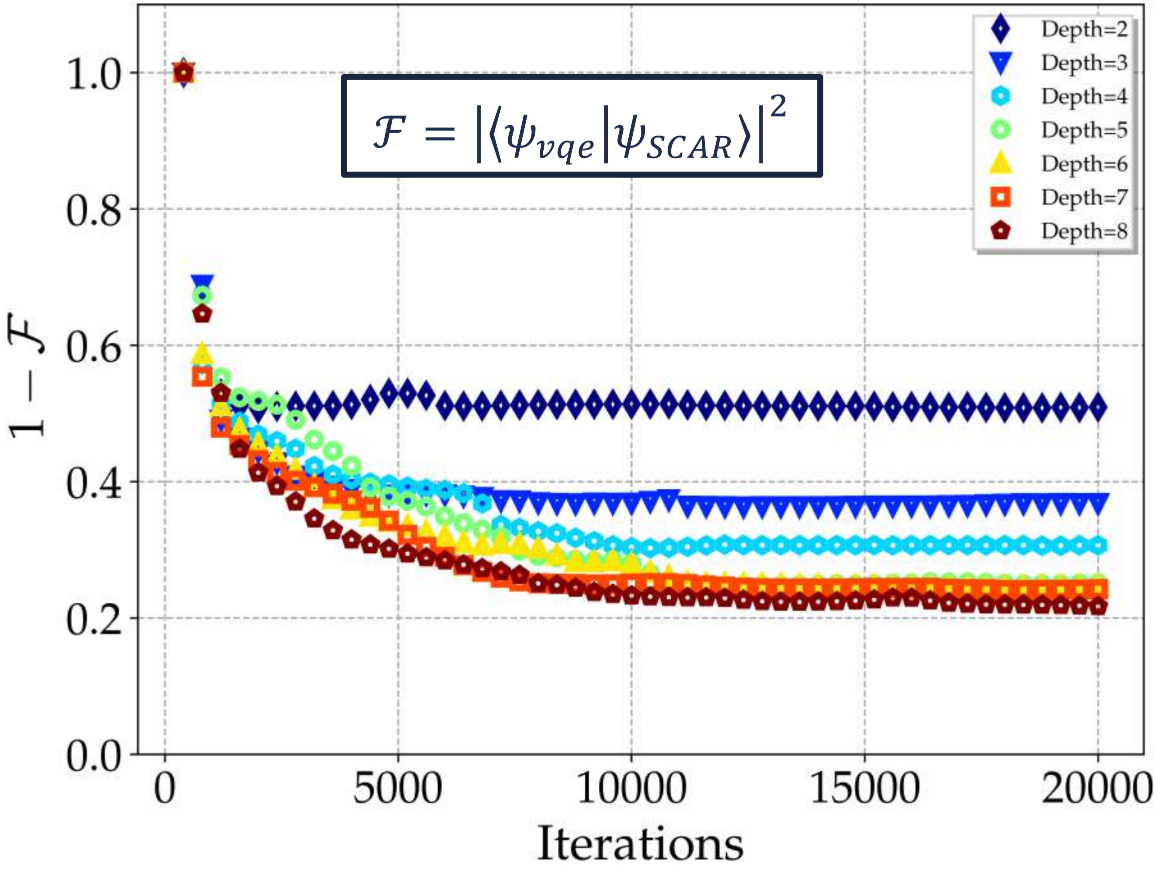
VQE-scars: model 1

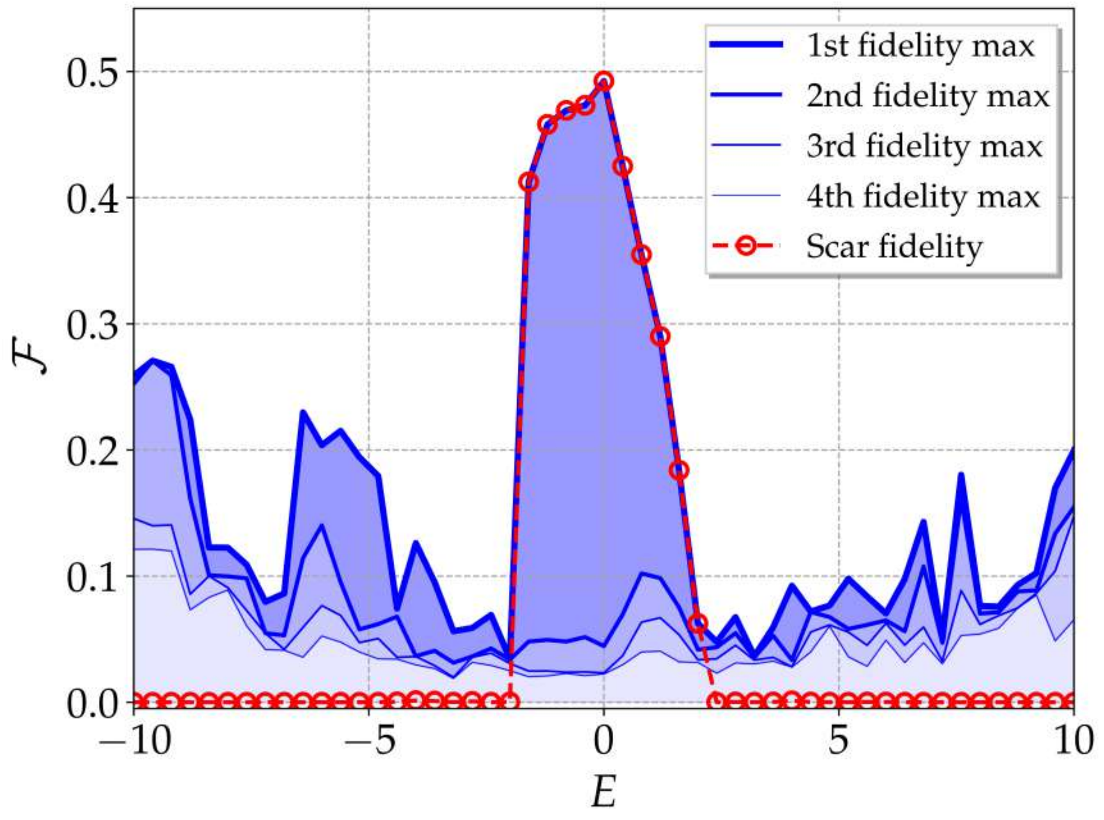
Some **preliminary results** using the HE ansatz:



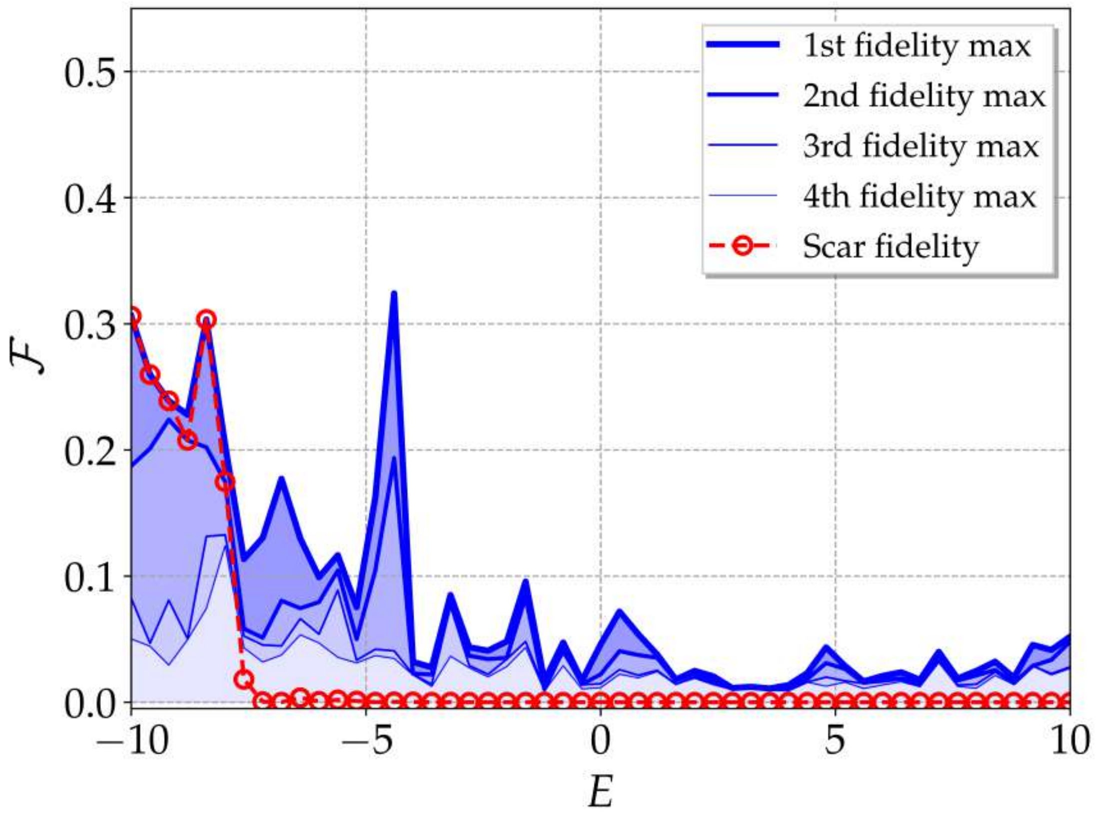
VQE-scars: model 1

$N = 12, Depth = 2,$
$a = 0.05, b = 0.25, c = 0.70,$
Iterations=1000

Energy sweep $E \in [-10,10]$, to discriminate the presence of a scar depending on the convergence of the algorithm



Scar



No-scar

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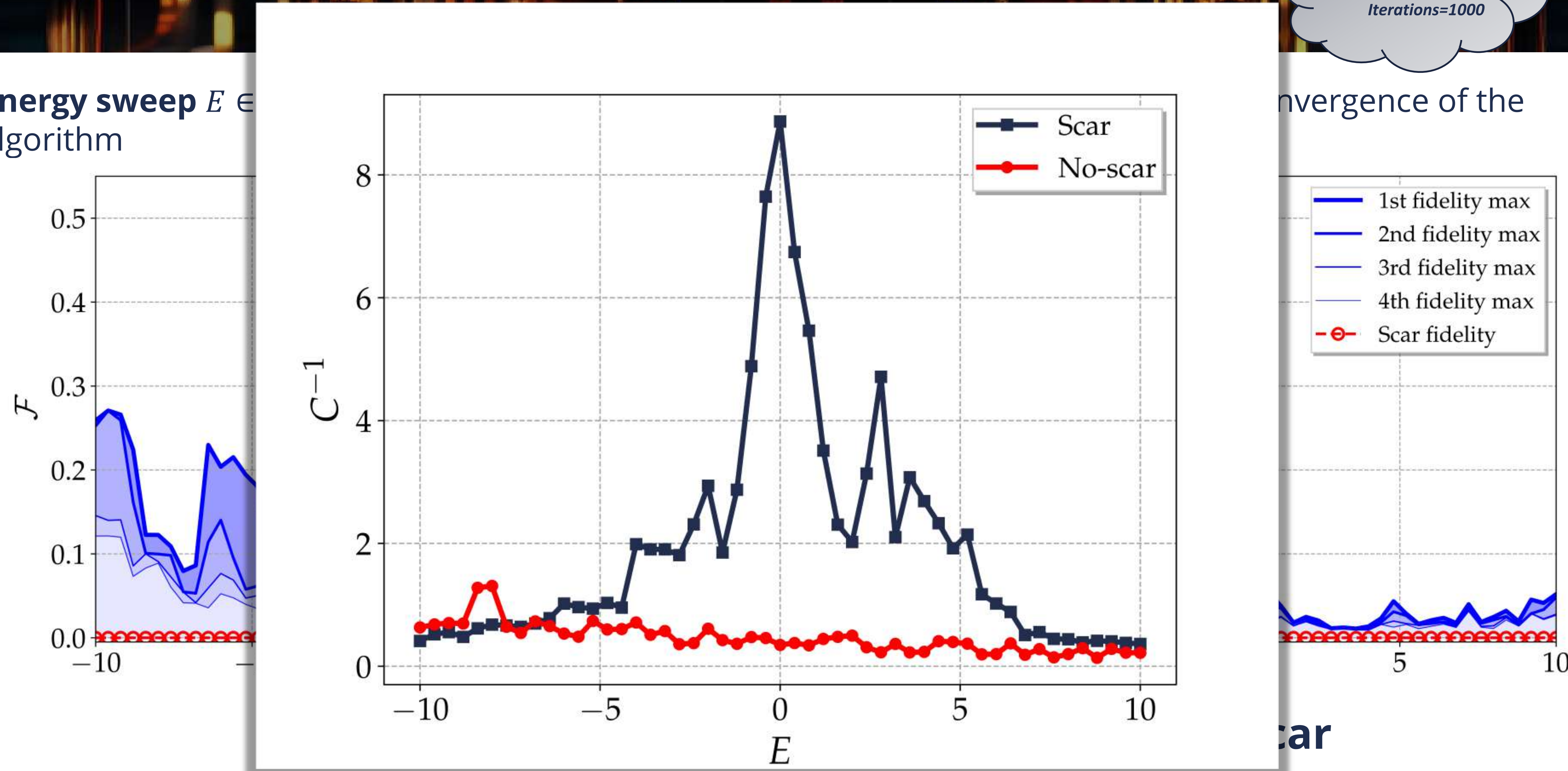


car

VQE-scars: model 2

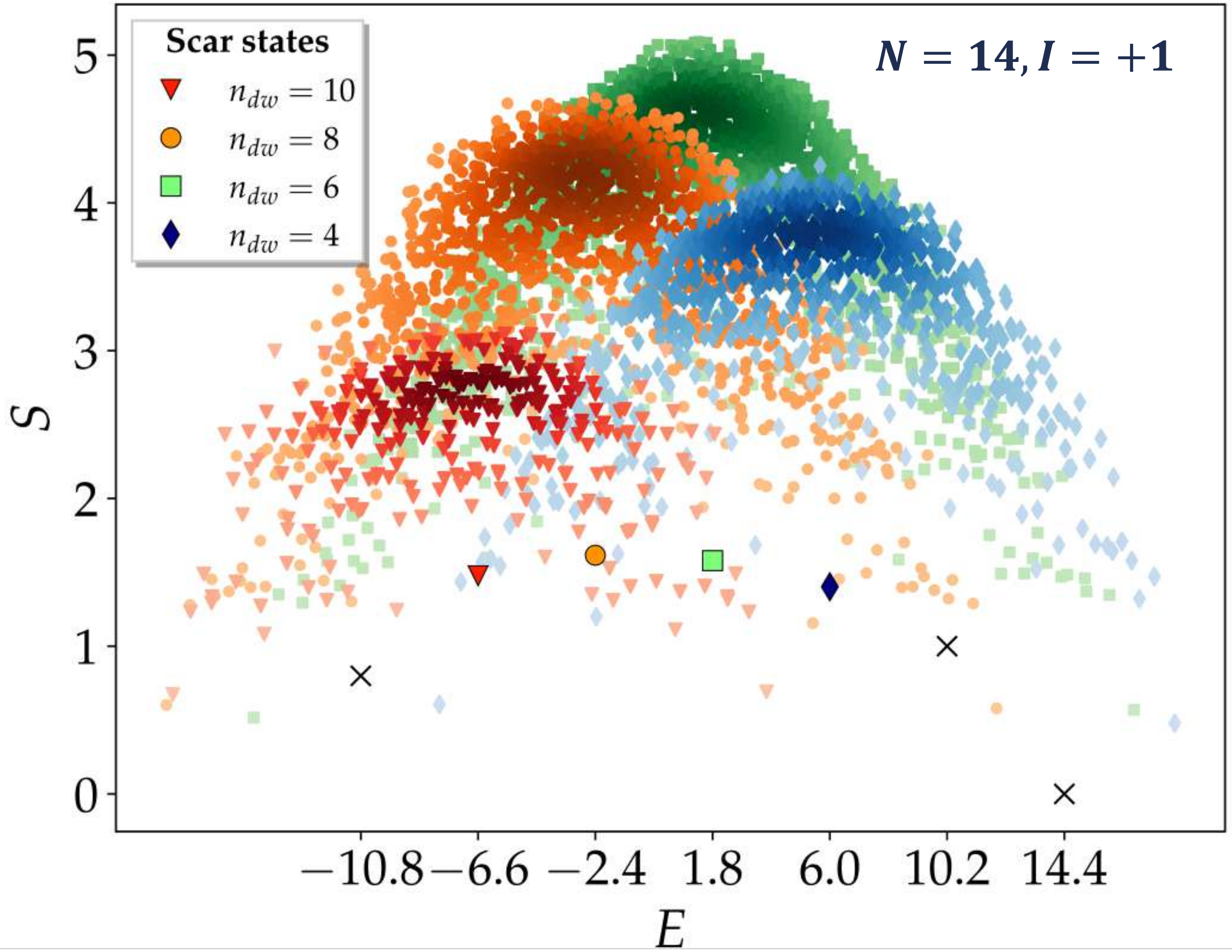
1D Hamiltonian of **spin-1/2 model** on a *N*-length chain:

$$H = \lambda \sum_{i=2}^{N-1} (\sigma_i^x - \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z) + \Delta \sum_{i=1}^{N} \sigma_i^z + J \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z$$

$$N_{dw} = \sum_{i=1}^{N-1} (1 - \sigma_i^z \sigma_{i+1}^z)/2 \qquad [N_{dw}, H] = 0$$

$$I = \prod_{i=1}^{N} \sigma_i^x \qquad [I, H] = 0$$

$$f_{symm} = \langle (N_{dw} - n_{dw})^2 \rangle$$



T. Iadecola, et al. Quantum many-body scar states with emergent kinetic constraints and finite-entanglement revivals. *Physical Review B* 101.2:024306 (2020).

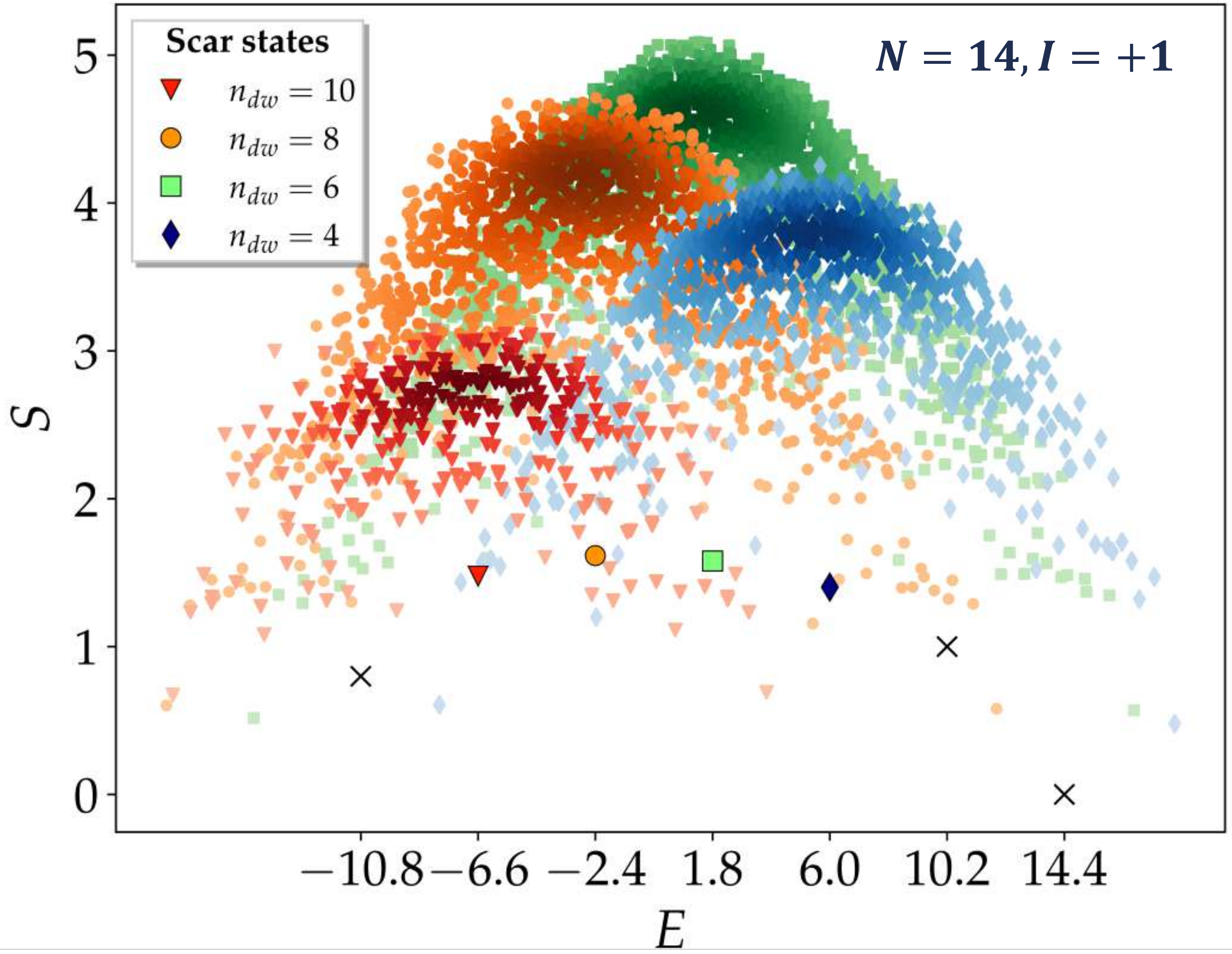
VQE-scars: model 2

1D Hamiltonian of **spin-1/2 model** on a *N*-length chain:

$$H = \lambda \sum_{i=2}^{N-1} (\sigma_i^x - \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z) + \Delta \sum_{i=1}^{N} \sigma_i^z + \mathrm{J} \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z$$

Why this Hamiltonian?

- two **towers of scar eigenstates** (for each inversion symmetry sector)
- each scar is associated with a **number of domain walls**

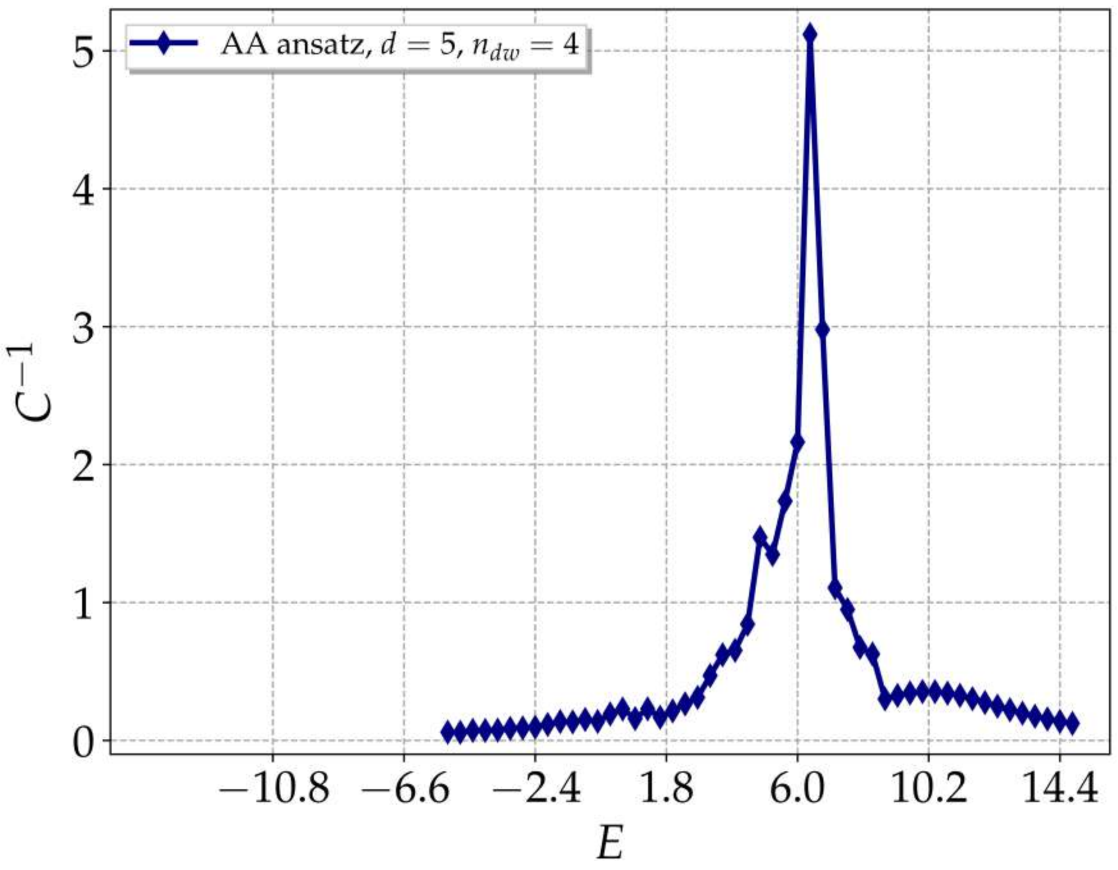


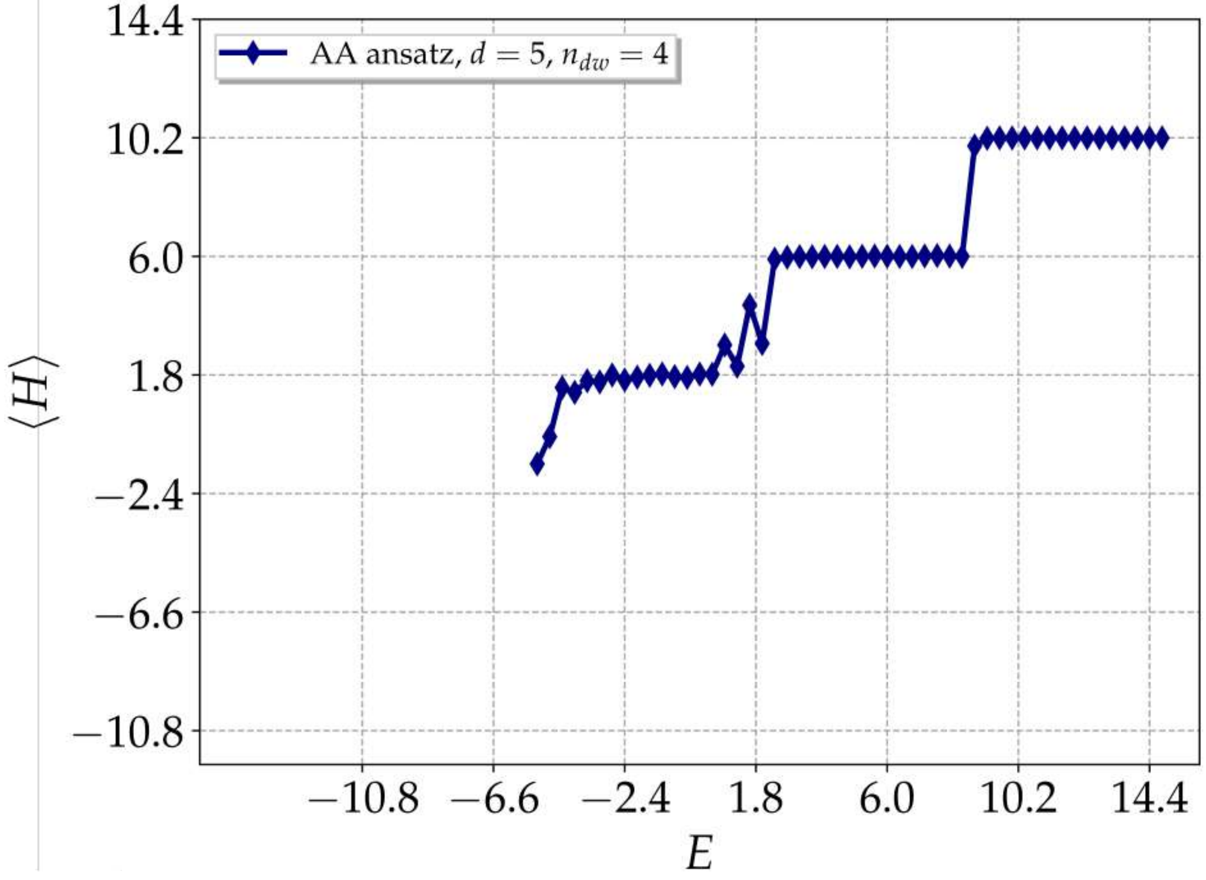
T. Iadecola, et al. Quantum many-body scar states with emergent kinetic constraints and finite-entanglement revivals. *Physical Review B* 101.2:024306 (2020).

VQE-scars: model 2

$a = 0.05, b = 0.25, c = 0.70,$
Iterations=1000

Energy sweep to discriminate the presence of a scar depending on the convergence of the algorithm





VQE-scars: model 2

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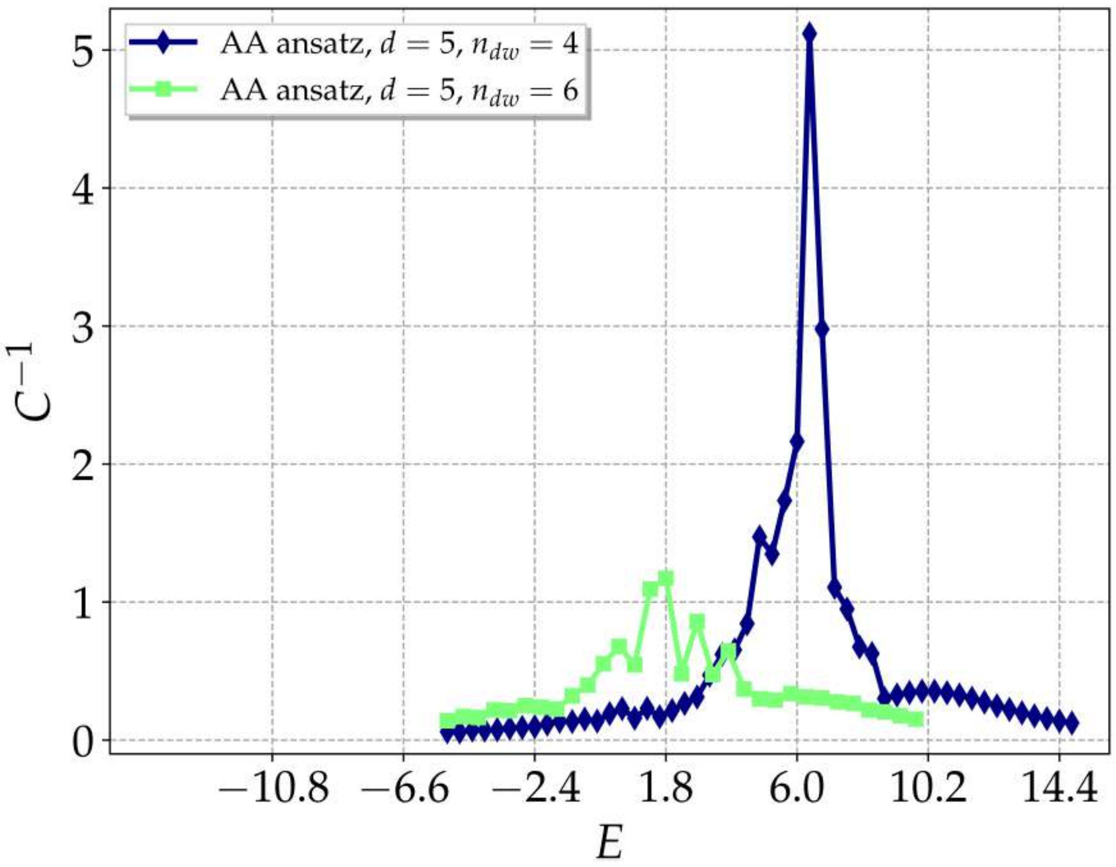

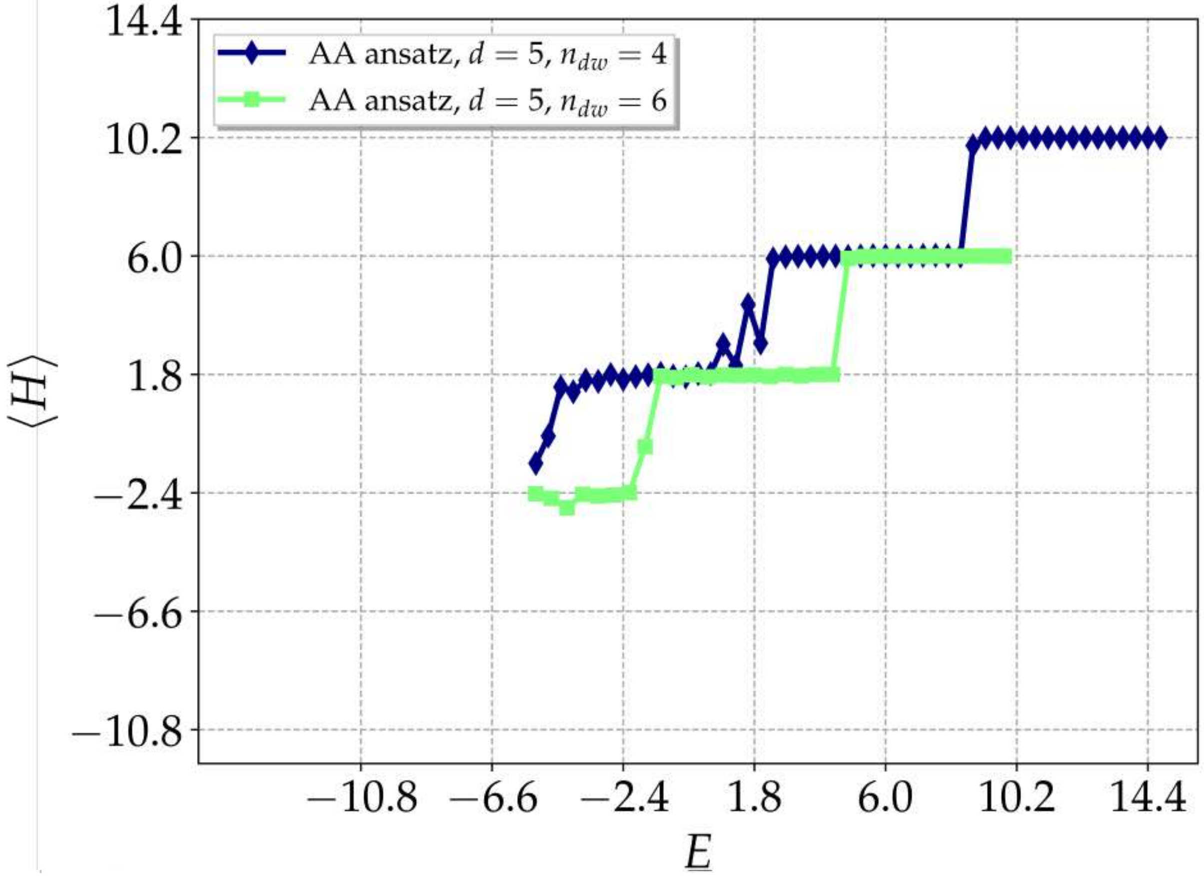

VQE-scars: model 2

$a = 0.05, b = 0.25, c = 0.70,$
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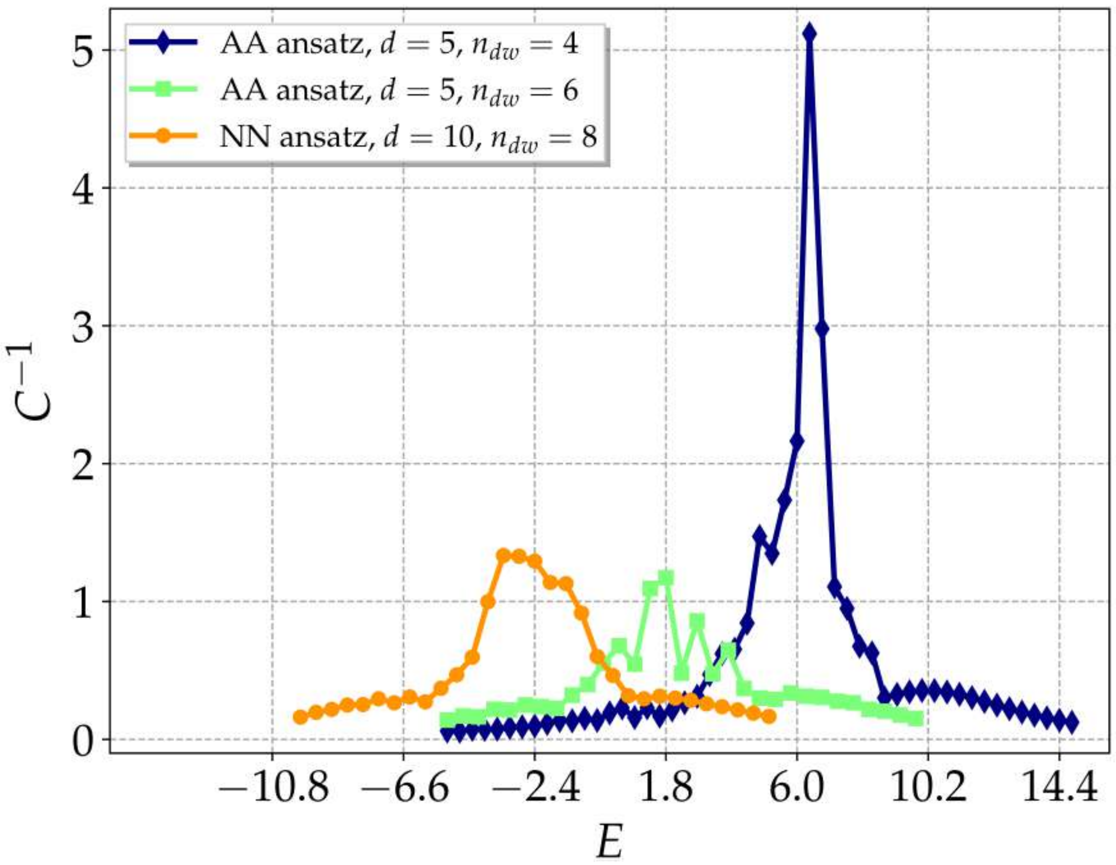
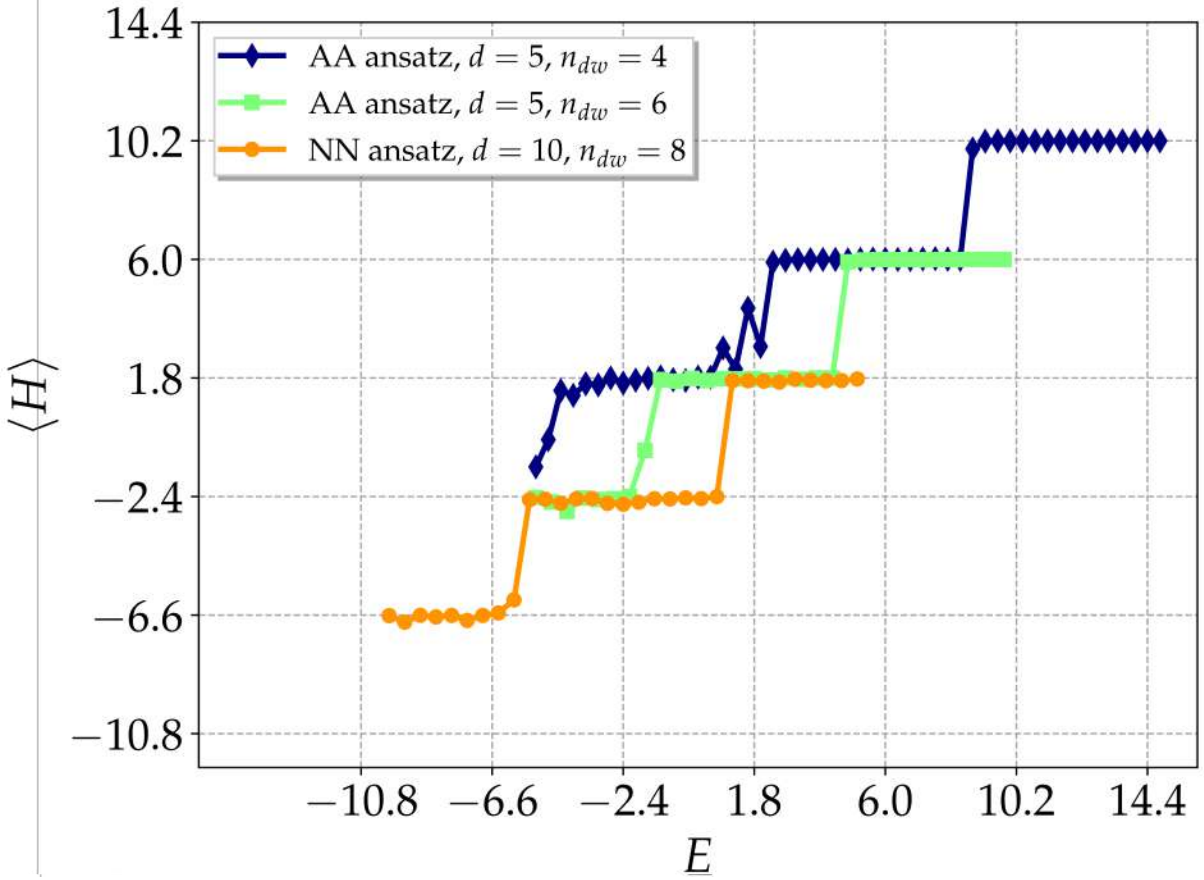
Energy sweep to discriminate the presence of a scar depending on the convergence of the algorithm

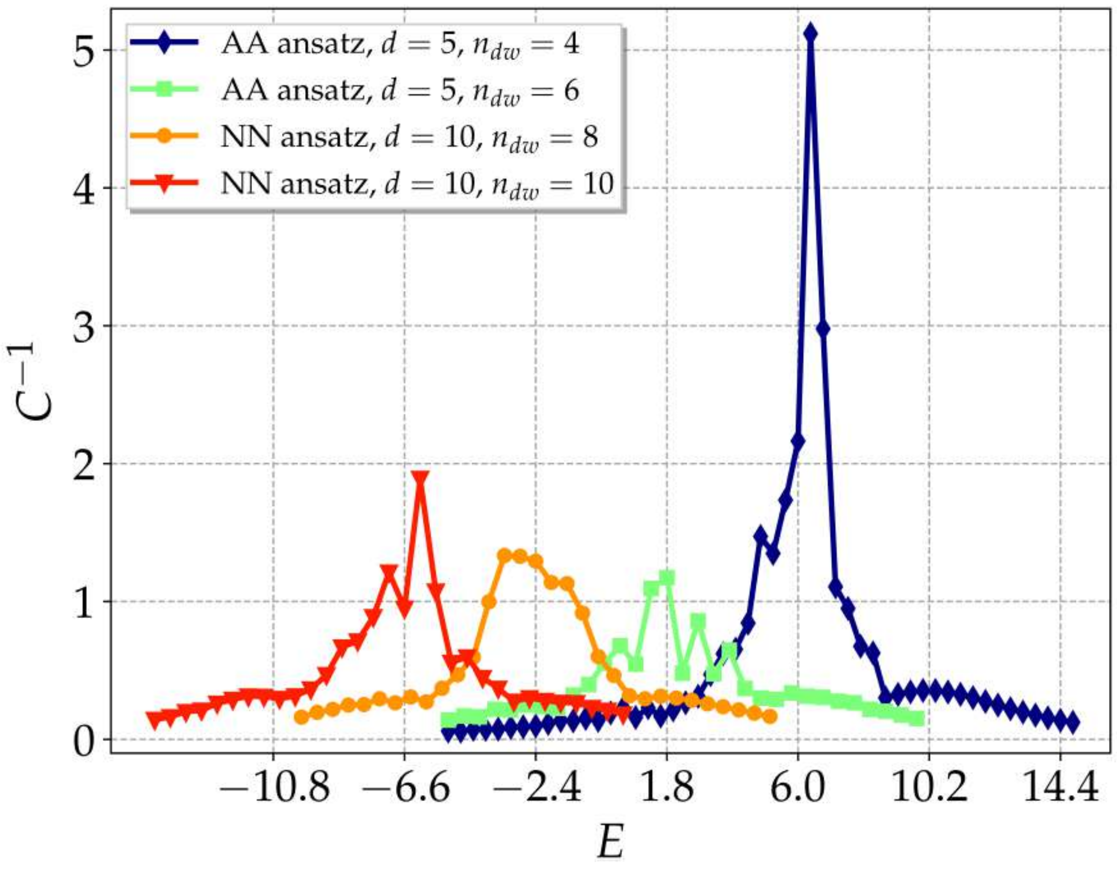


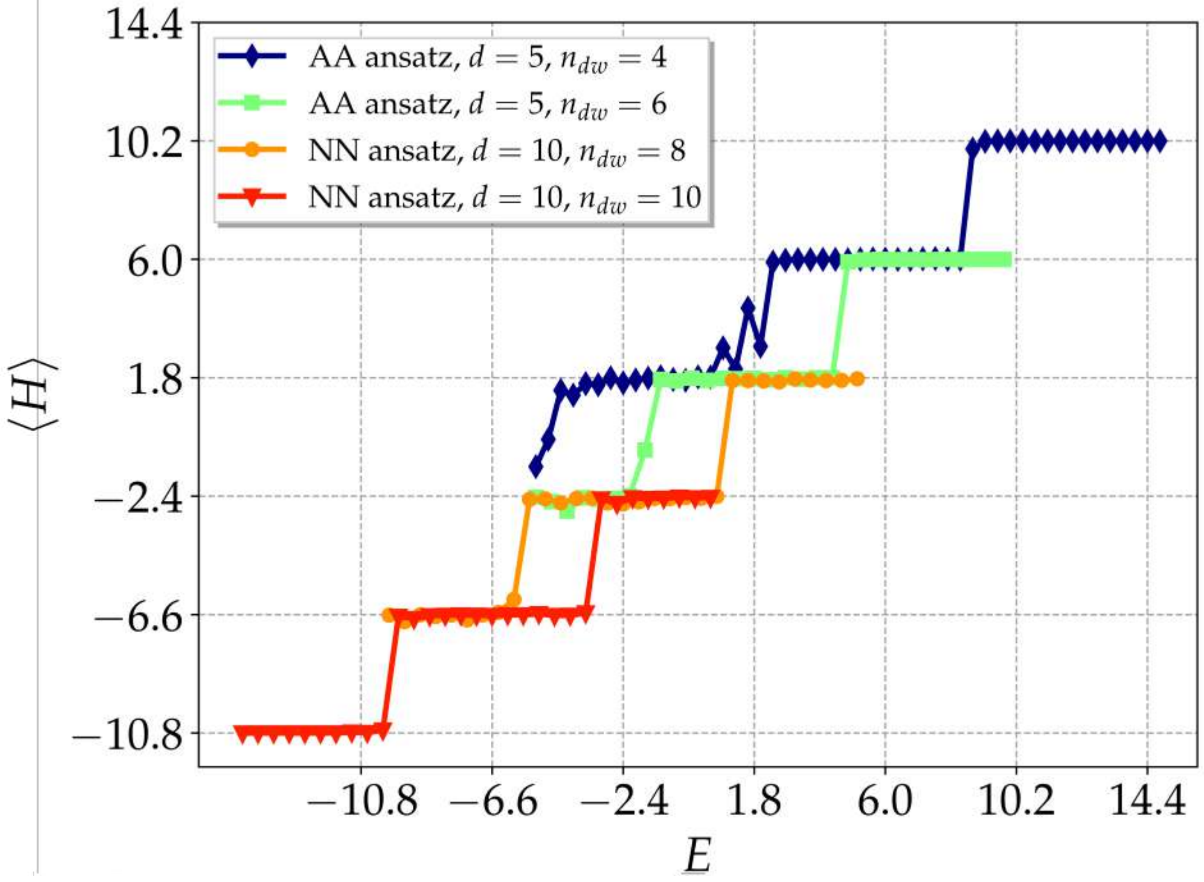


VQE-scars: model 2

$a = 0.05, b = 0.25, c = 0.70,$
Iterations=1000

Energy sweep to discriminate the presence of a scar depending on the convergence of the algorithm



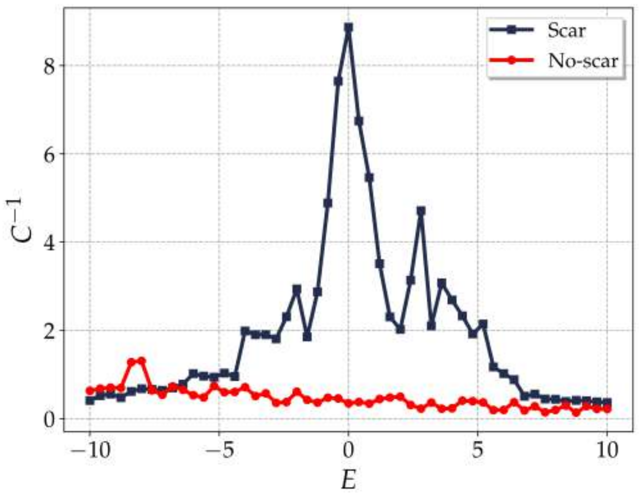


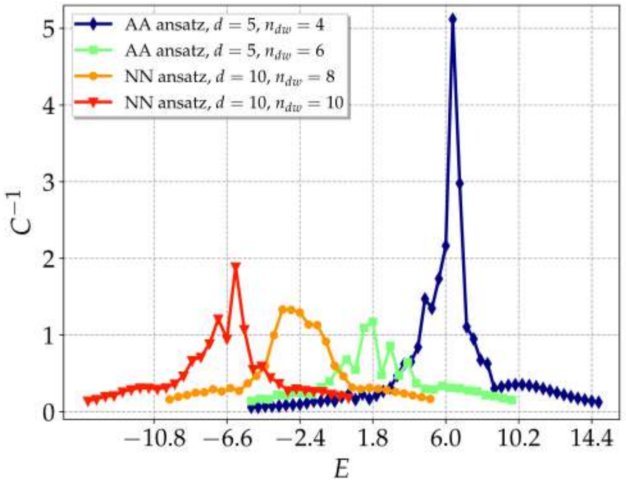
Conclusions

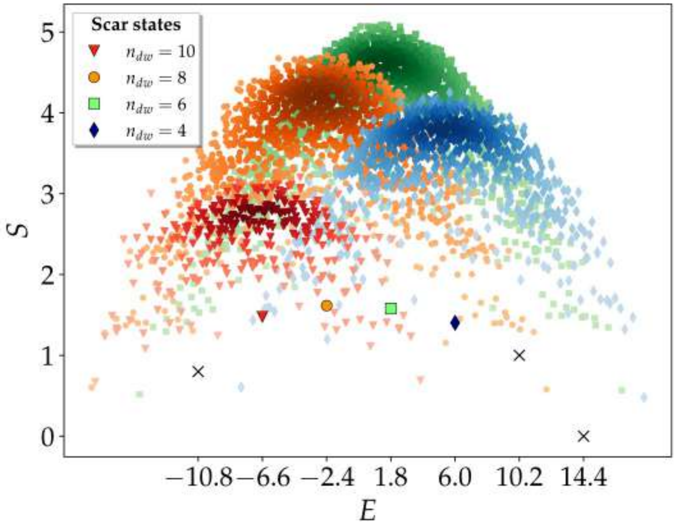
❖ **VQE-like** algorithm to target quantum many-body **scars:**

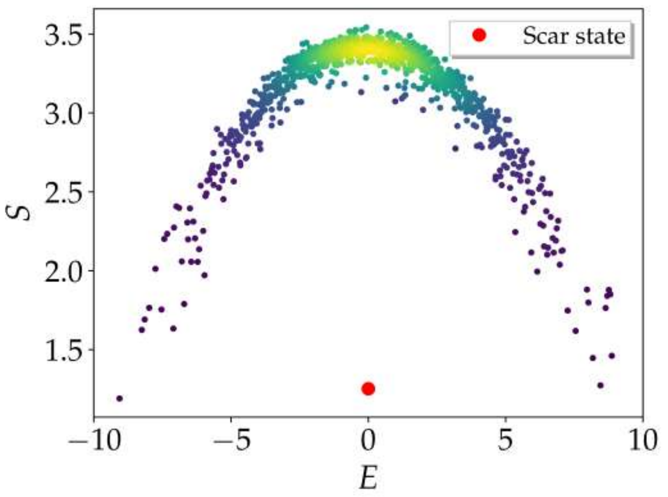
➢ **Pros:** conceptually easy,
straightforward to generalize to n-d systems

➢ **Cons:** noisy quantum gates could lead
to difficult optimization

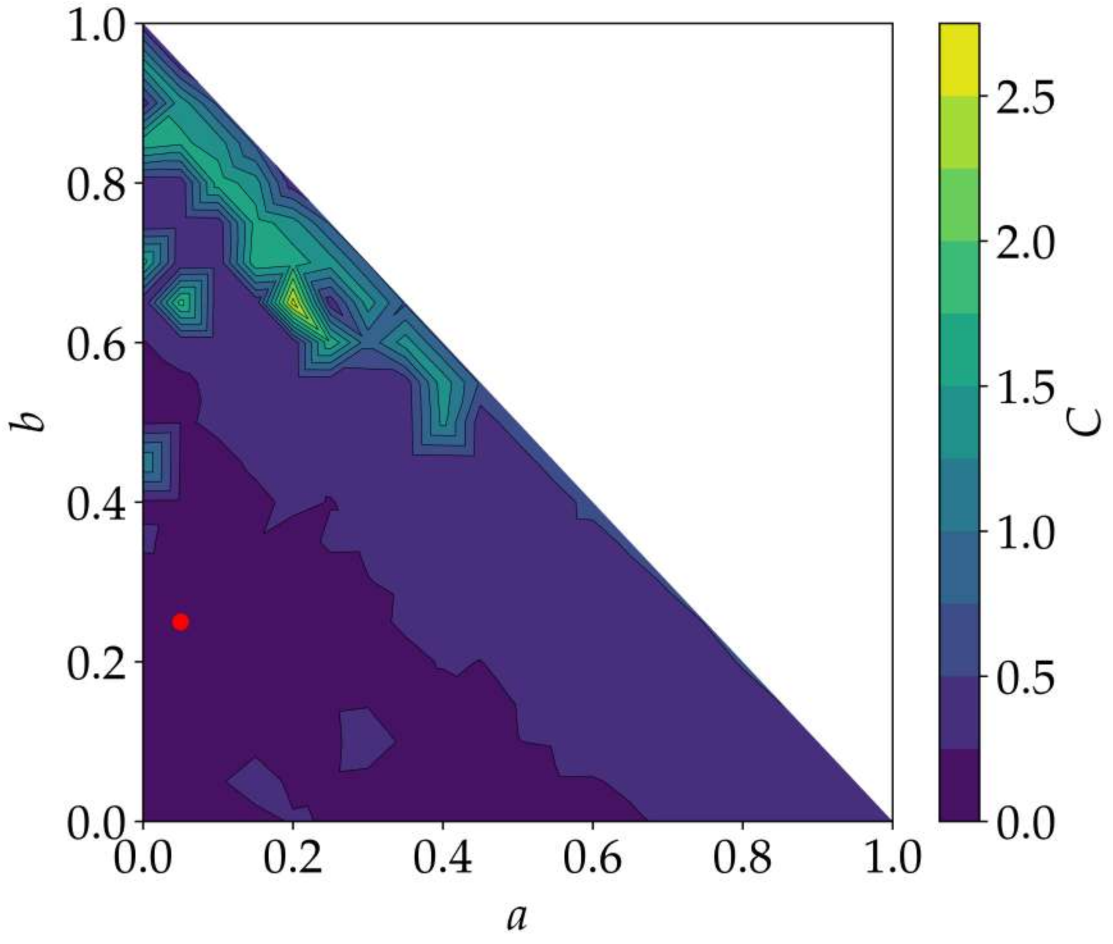


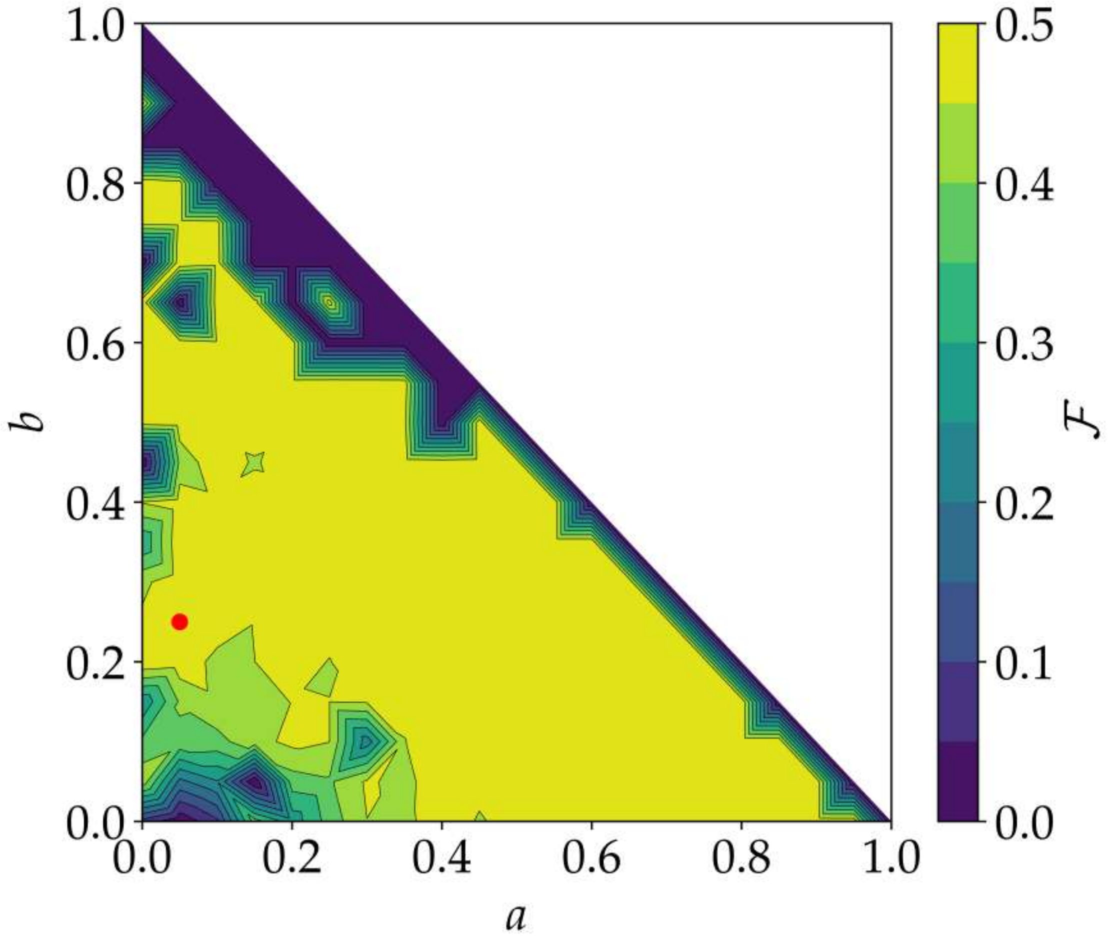






Bonus: Hyperparameters convergence





Bonus: Matrix Product States

A **Matrix product state** (**MPS**) is a **quantum state** of many particles (N sites), written in the following form:

$$|\Psi\rangle = \sum_{\vec{i}} Tr\left[A^{[1]i_1} A^{[2]i_2} \dots A^{[L]i_L}\right] |i_1, i_2, \dots, i_L\rangle$$

➔ Low bond dimension ➔ Area-law states

The **density matrix renormalization group** (**DMRG**) is a numerical variational technique devised to obtain the **MPS** representation of low-energy eigenstates of quantum many-body systems

S.-Y. Zhang, et al. Extracting Quantum Many-Body Scarred Eigenstates with Matrix Product States. *PRL*131.2:020402 (2023).

➔ **DMRG-S:** matrix product state (MPS) algorithm to extract scar states

It works well, but **no easy generalization for more than 1-D** systems, the algorithm **needs initial overlap** with the scar state, the energy must be fine tuned