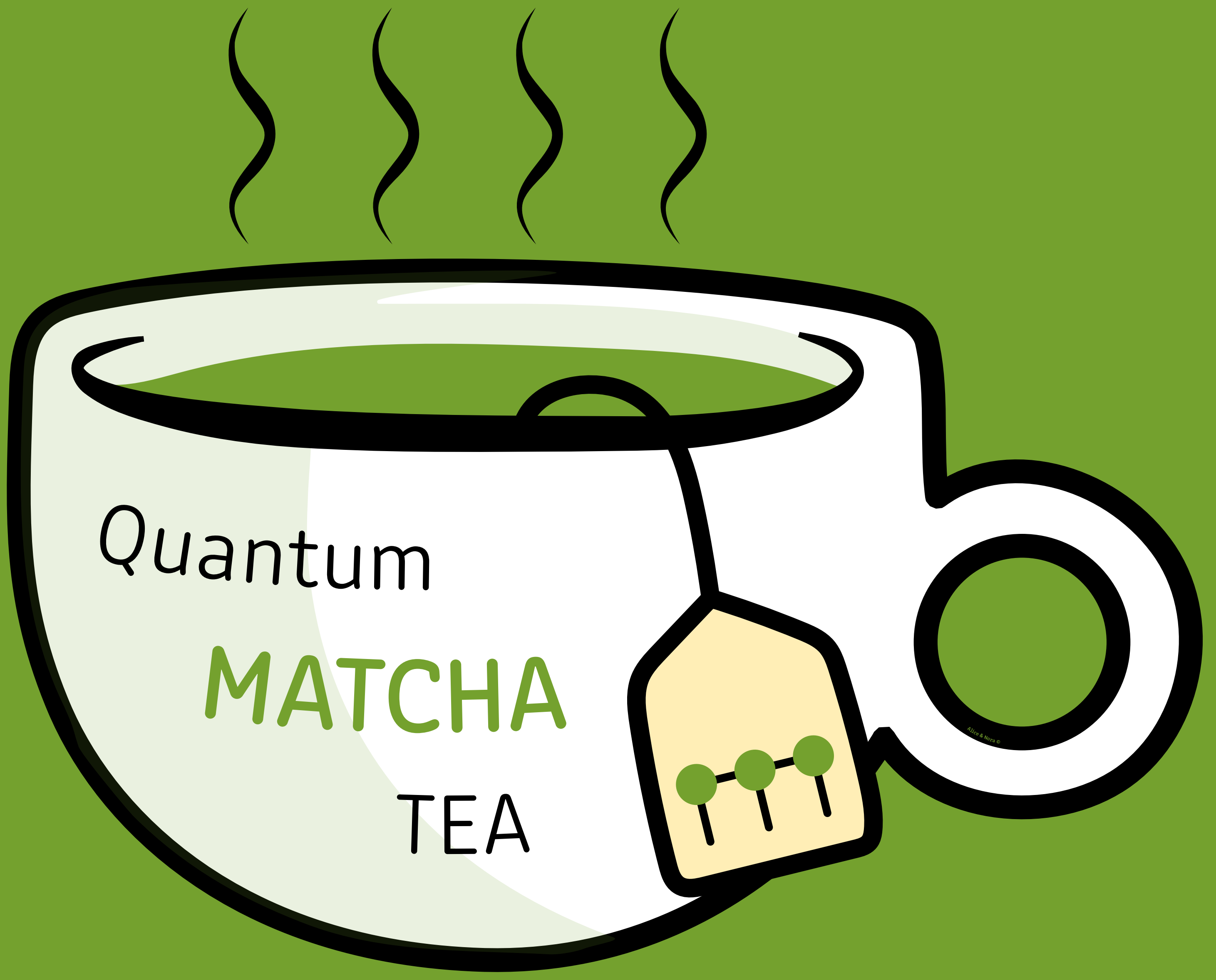
HPCQC
December 15 2023

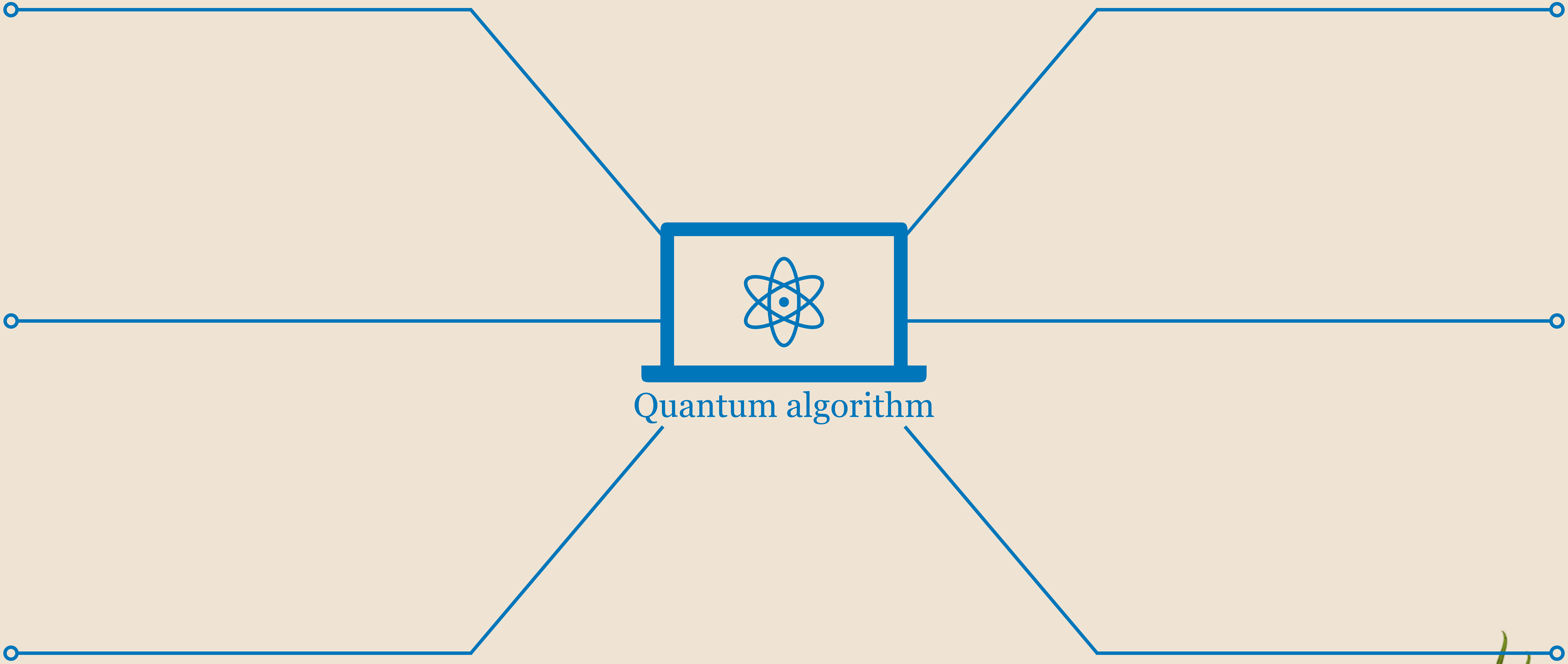
Quantum Matcha Tea

A tensor network emulator for quantum circuits on Leonardo



Marco Ballarin
Università degli studi di Padova

Running quantum algorithms



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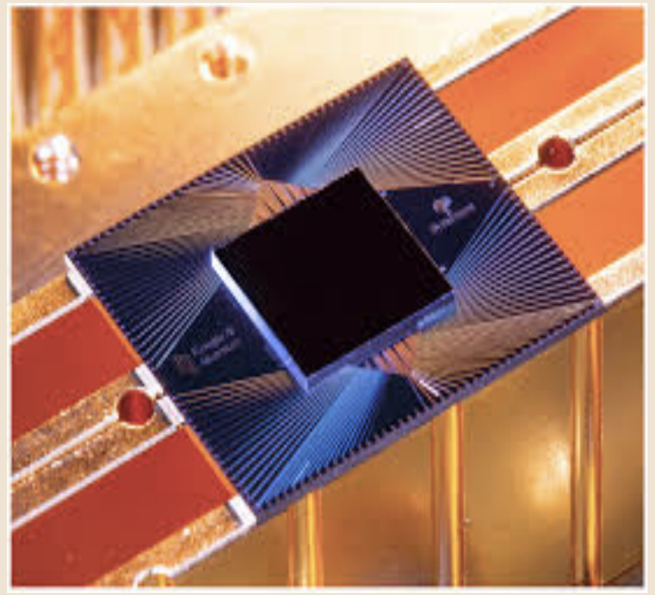
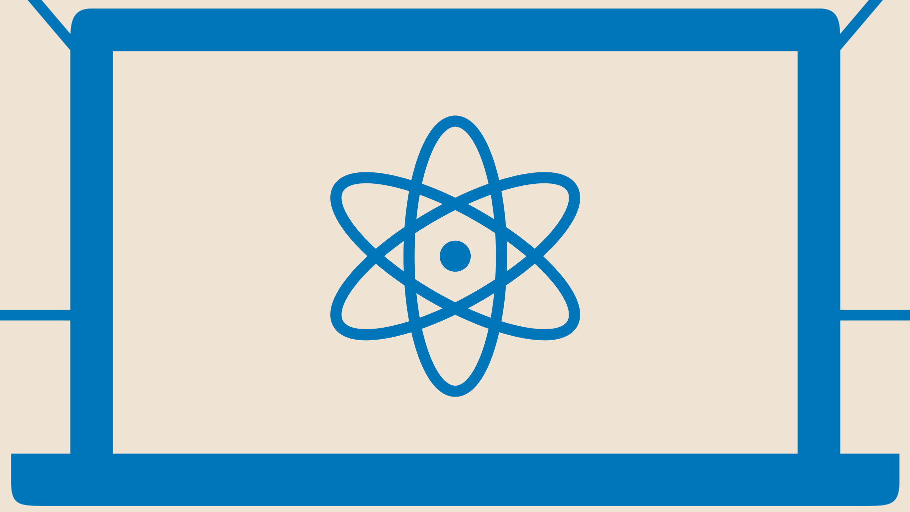


+ Real hardware
- Noisy
- Limited number of qubits

Quantum hardware



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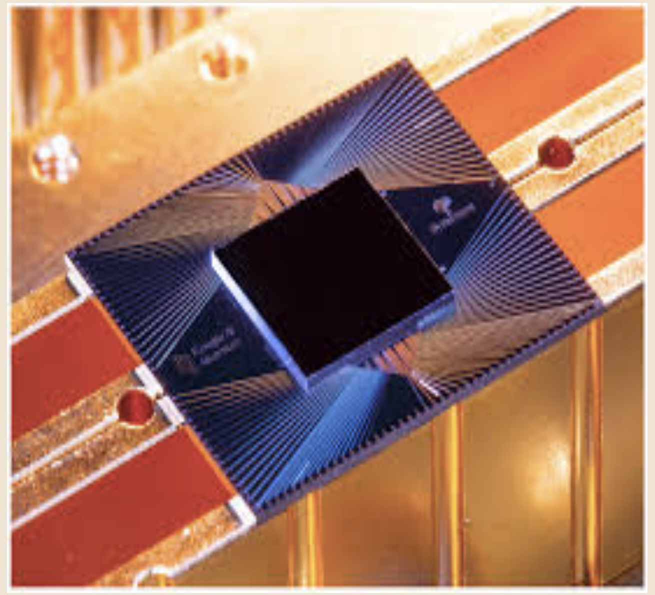

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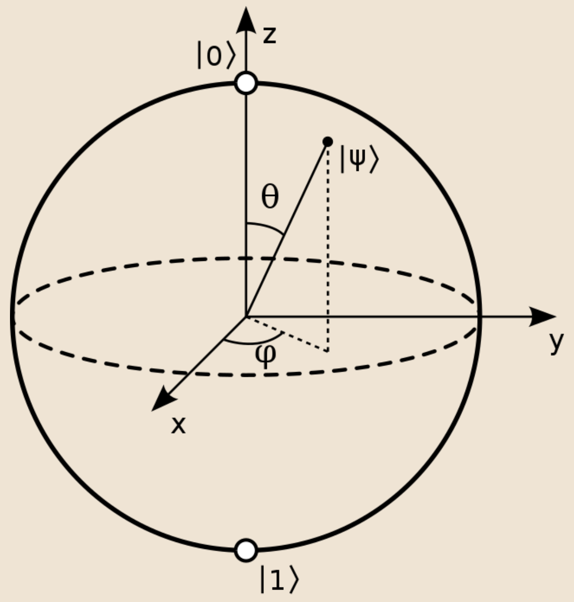
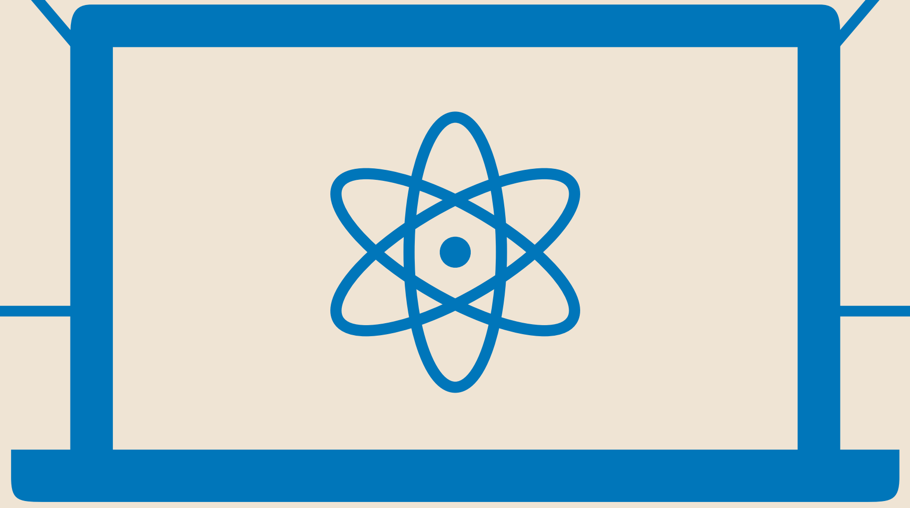


+ Access to exact state
- Limited number of qubits

Exact simulator



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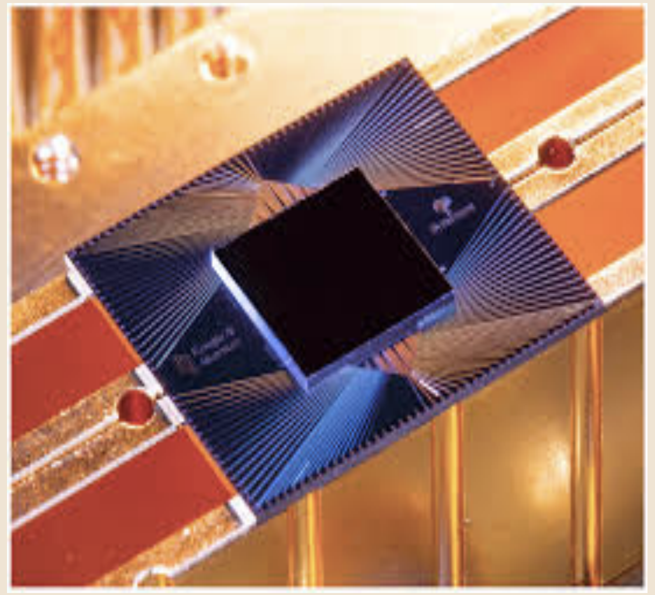

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Quantum hardware

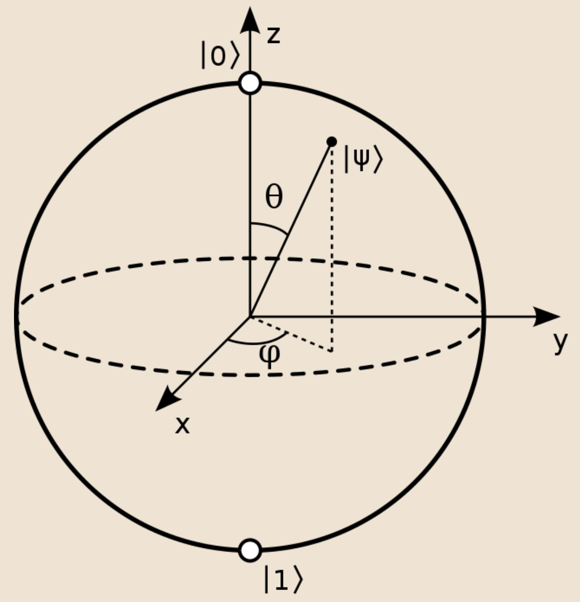
High # of qubits +

Flexibility (observables) -

Depth of the circuit -

cuQuantum

Quantum algorithm



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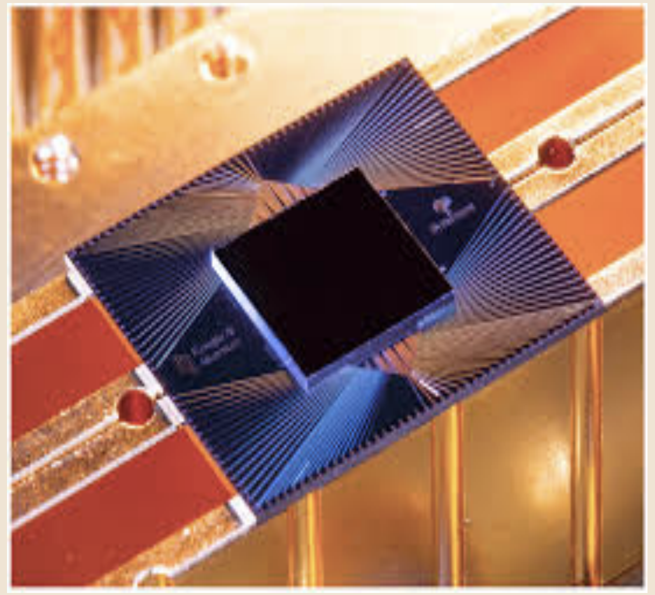

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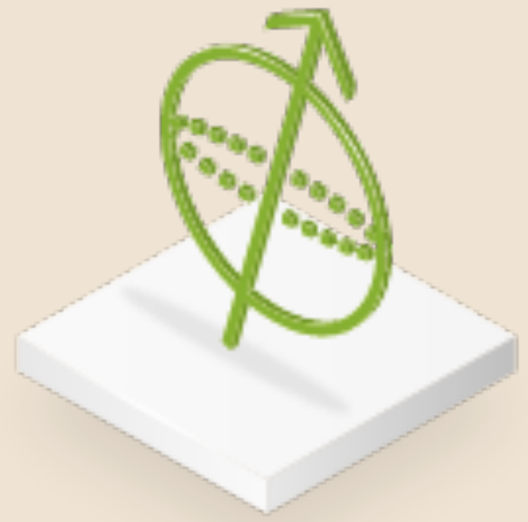

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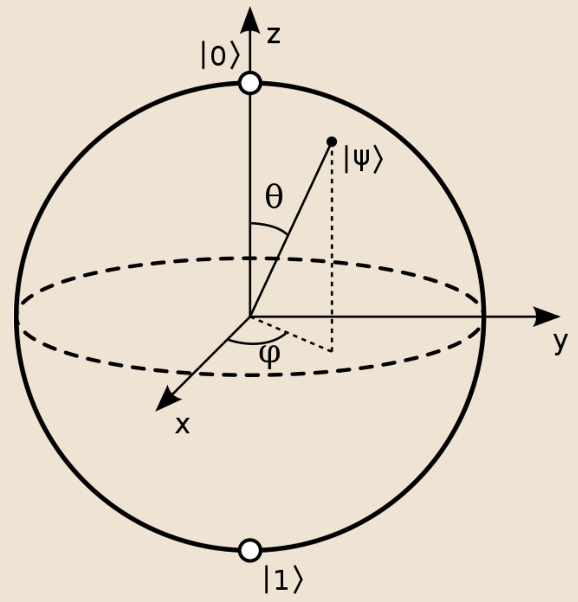

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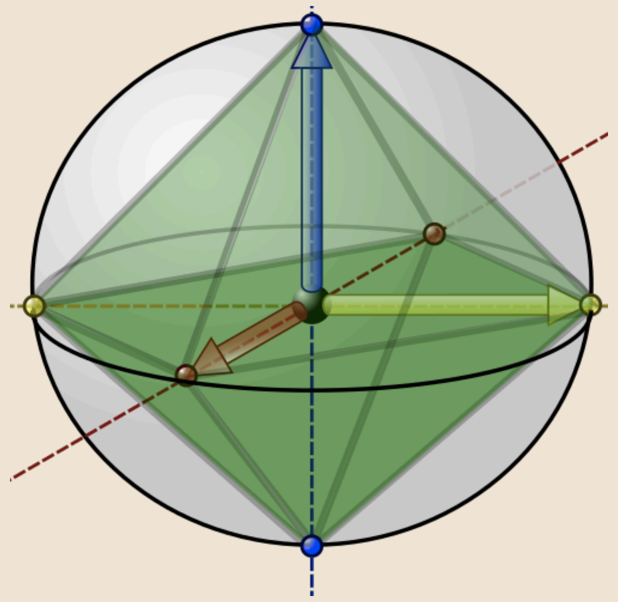
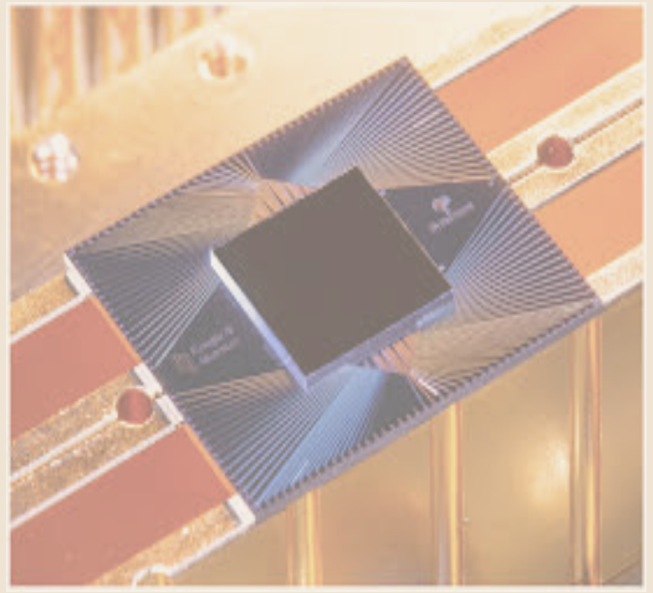


High # of qubits +

Flexibility (# of T gates) -

Clifford simulator

Running quantum algorithms



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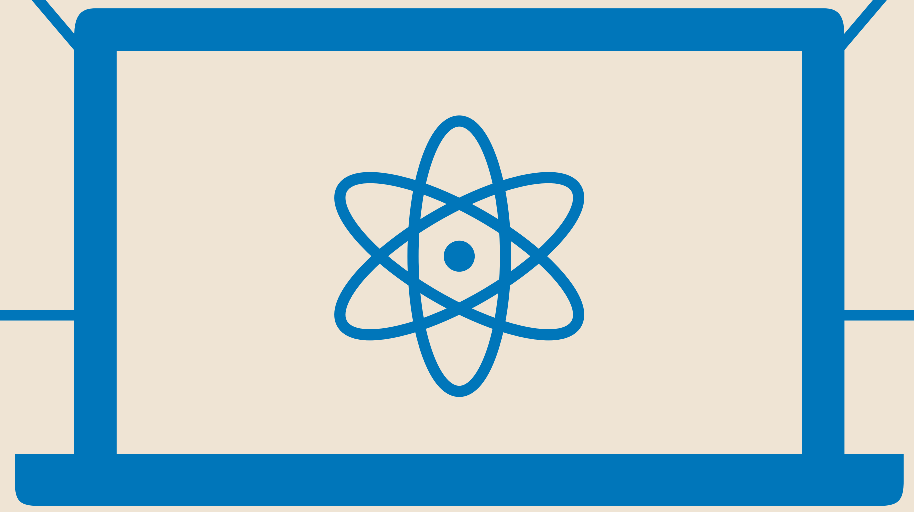

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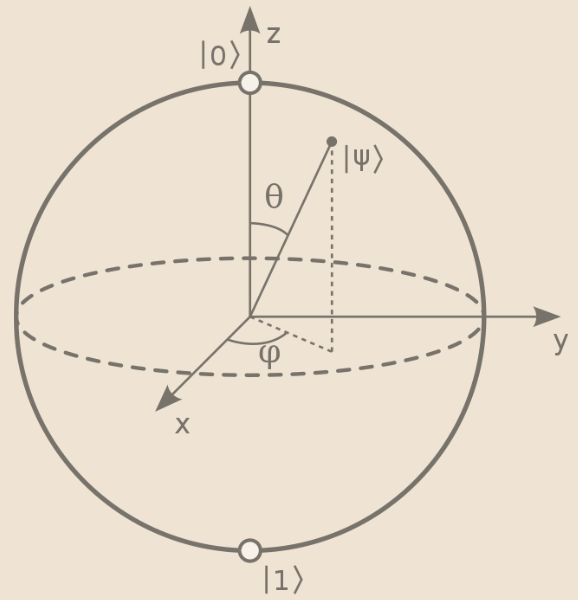

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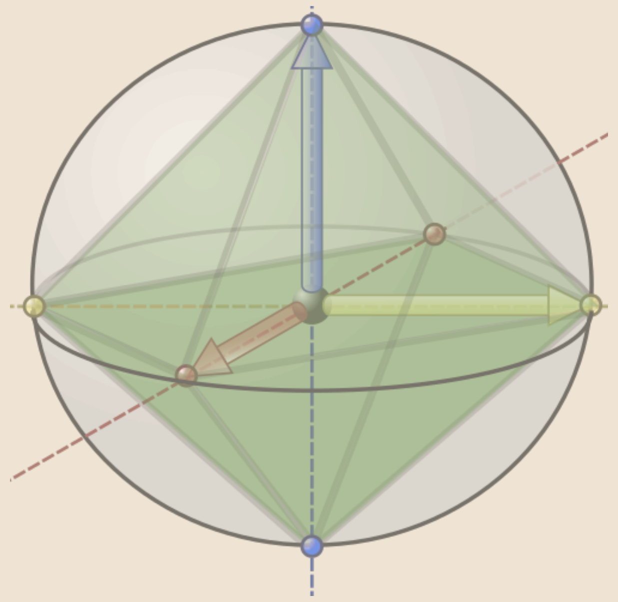
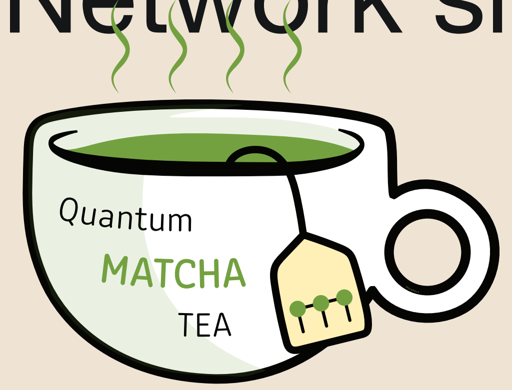


Clifford simulator

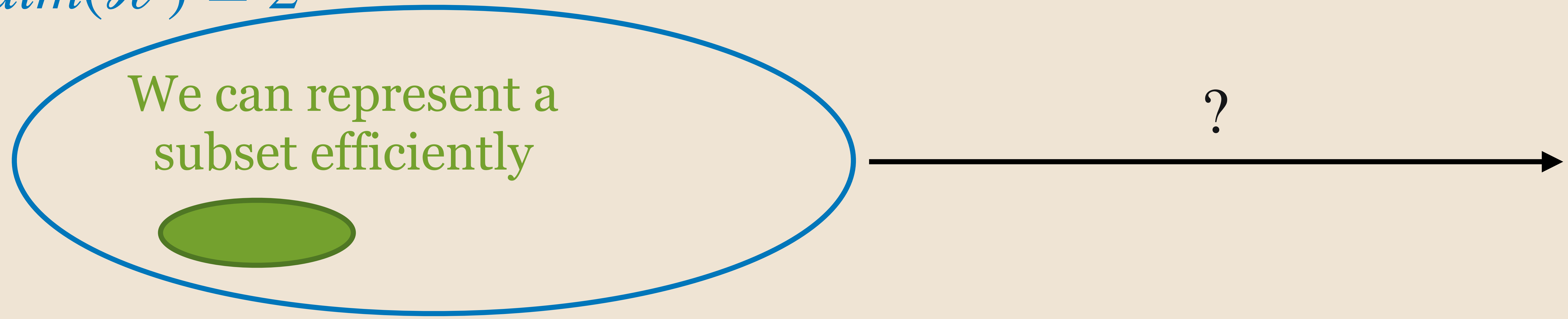
Tensor Network simulator



+ High # of qubits
- Flexibility (entanglement)

Why tensor networks

$dim(\mathscr{H}) = 2^n$



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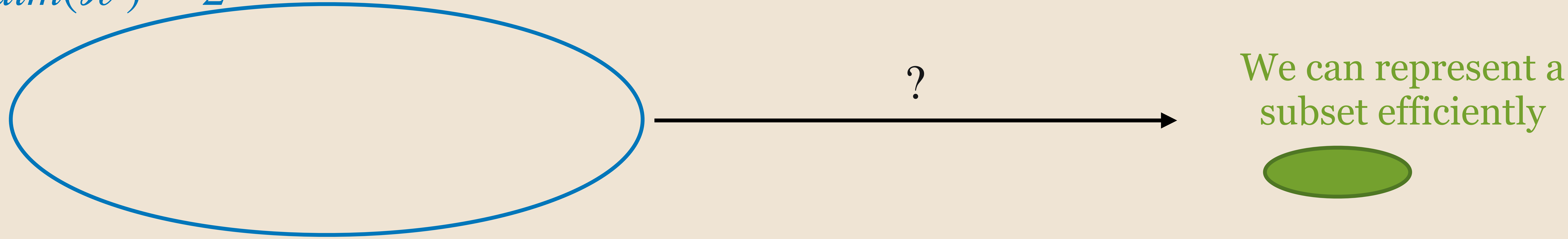

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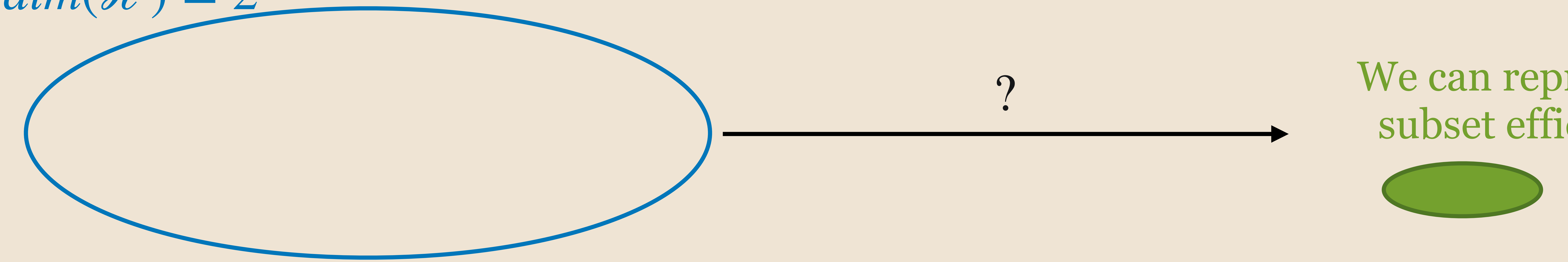
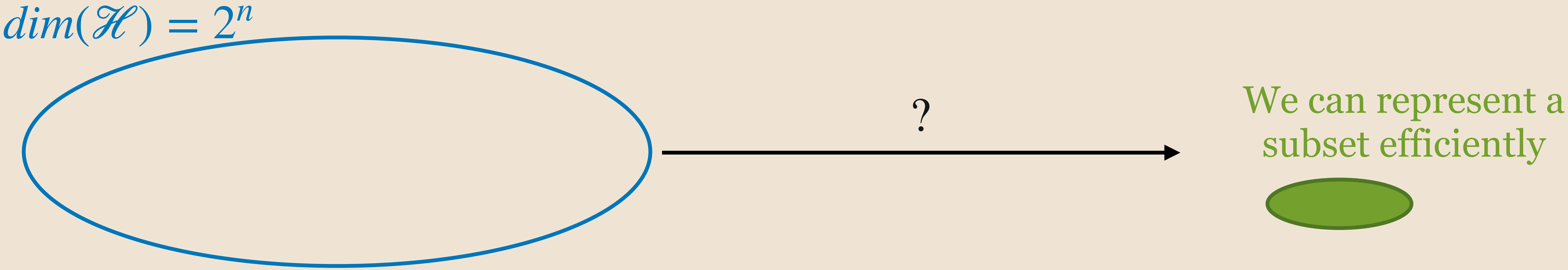


We can represent a subset efficiently

$$|\psi\rangle = \sum_{\alpha=1}^{\chi} \boxed{|A_\alpha\rangle} \overset{\lambda_\alpha}{—} \boxed{|B_\alpha\rangle}$$

Why tensor networks



Tensor networks compress the quantum correlations between subsystems ⇒ **compress entanglement**

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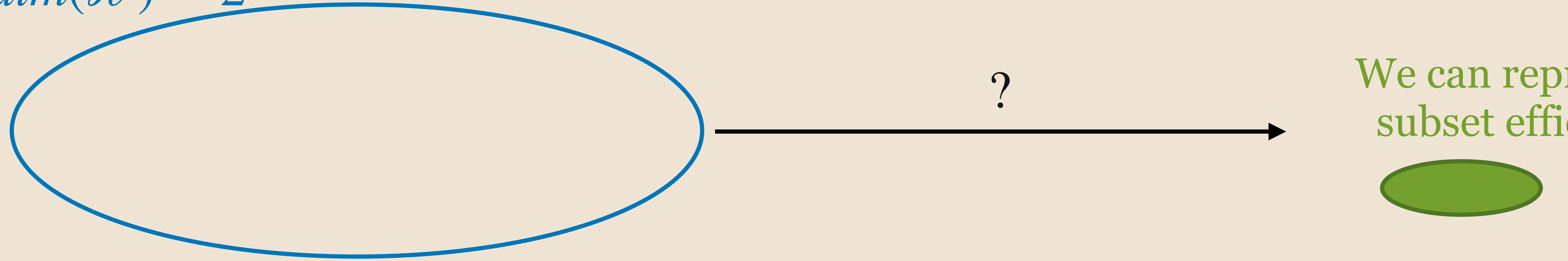

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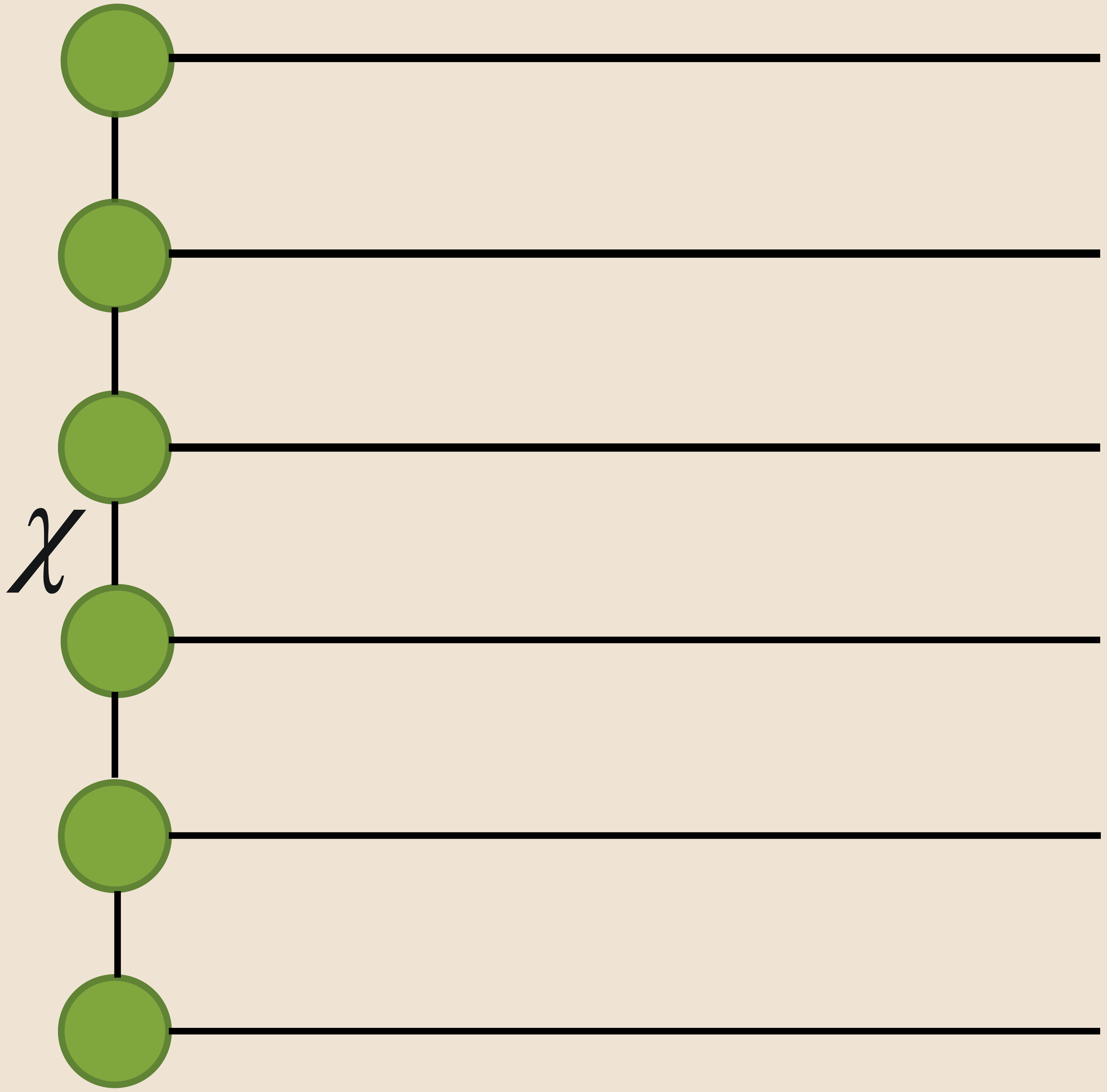
Only keep highest χ Schmidt values

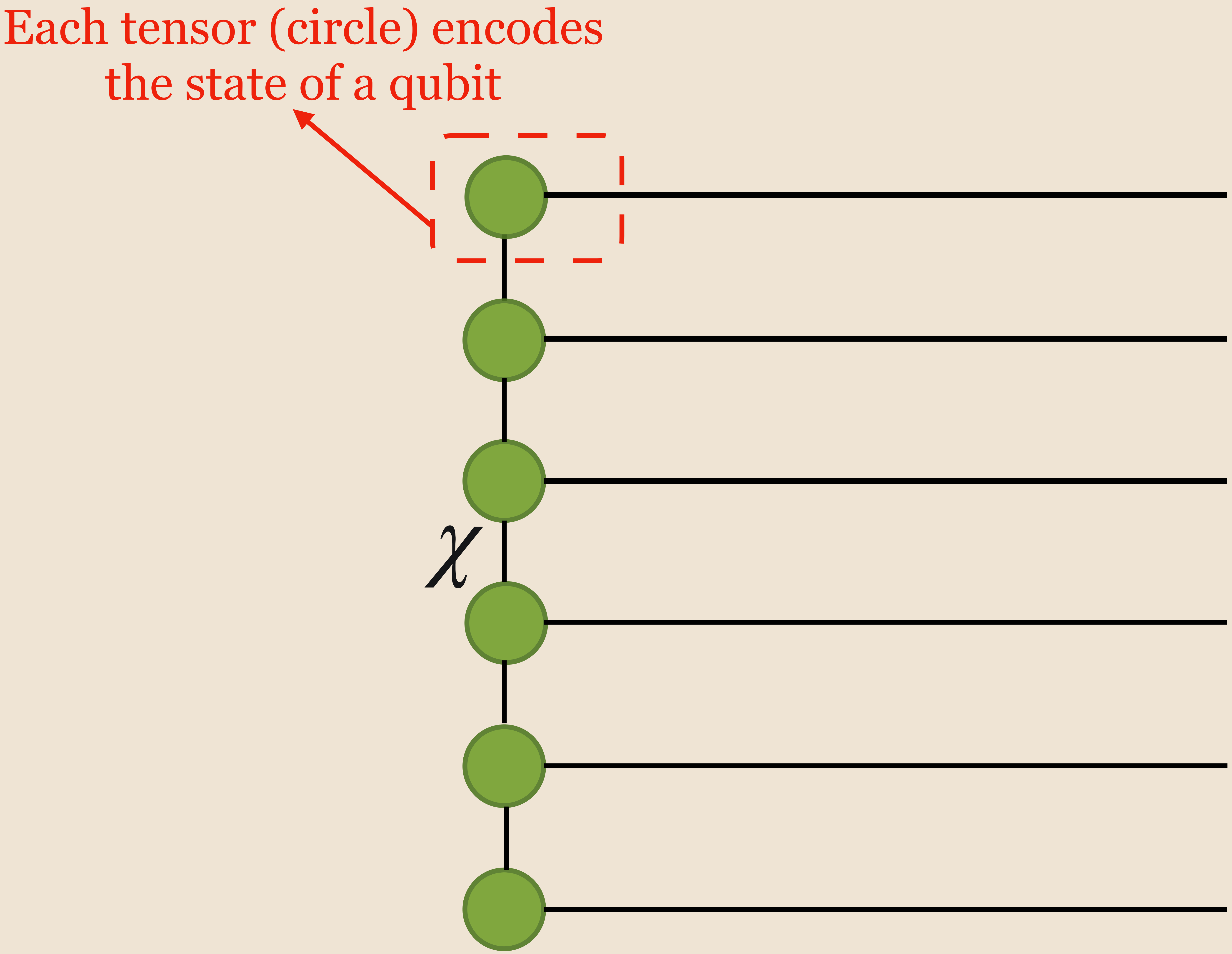
Matrix product states



Memory requirements

$$O(2^n) \to O(2n\chi^2)$$

Matrix product states



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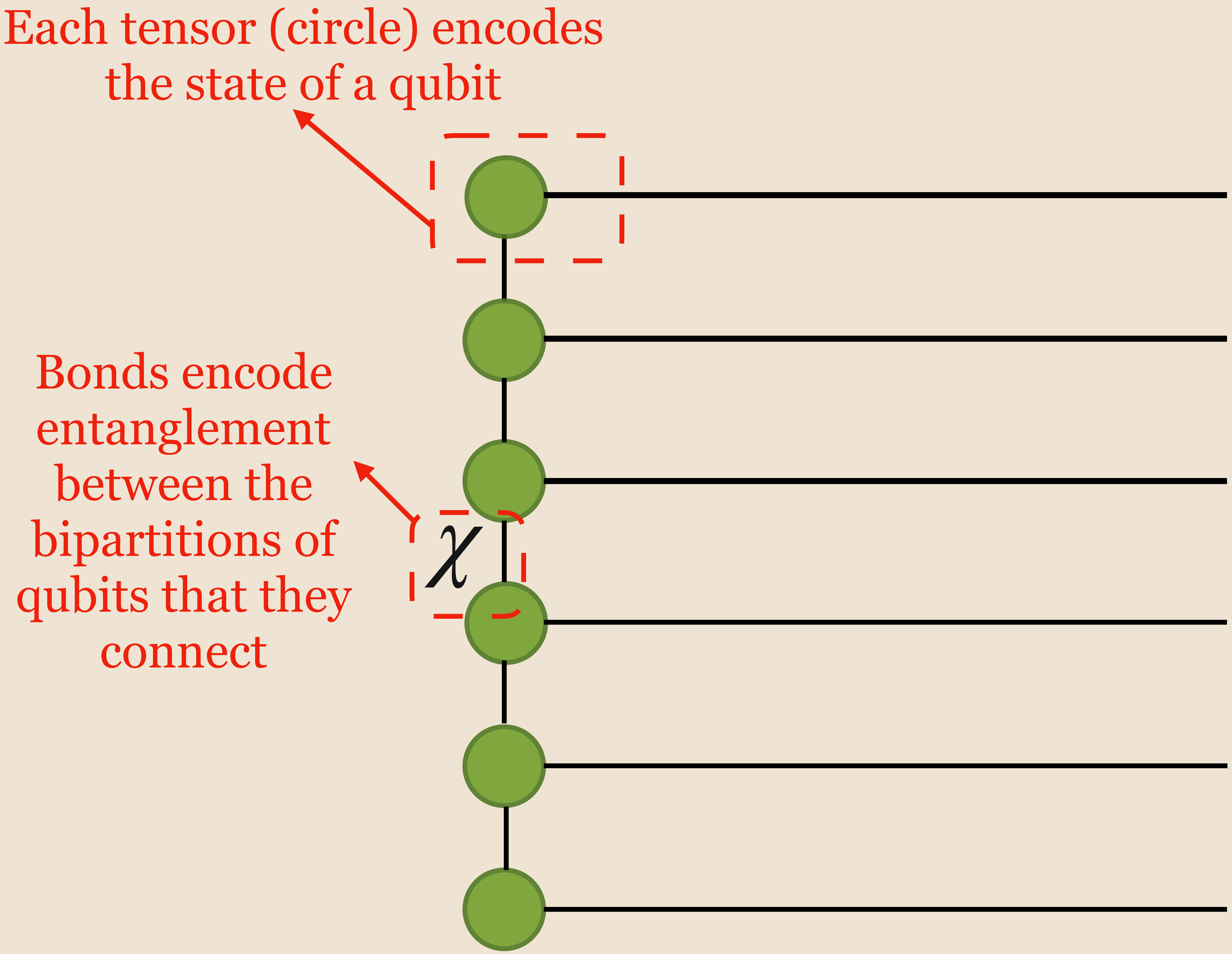

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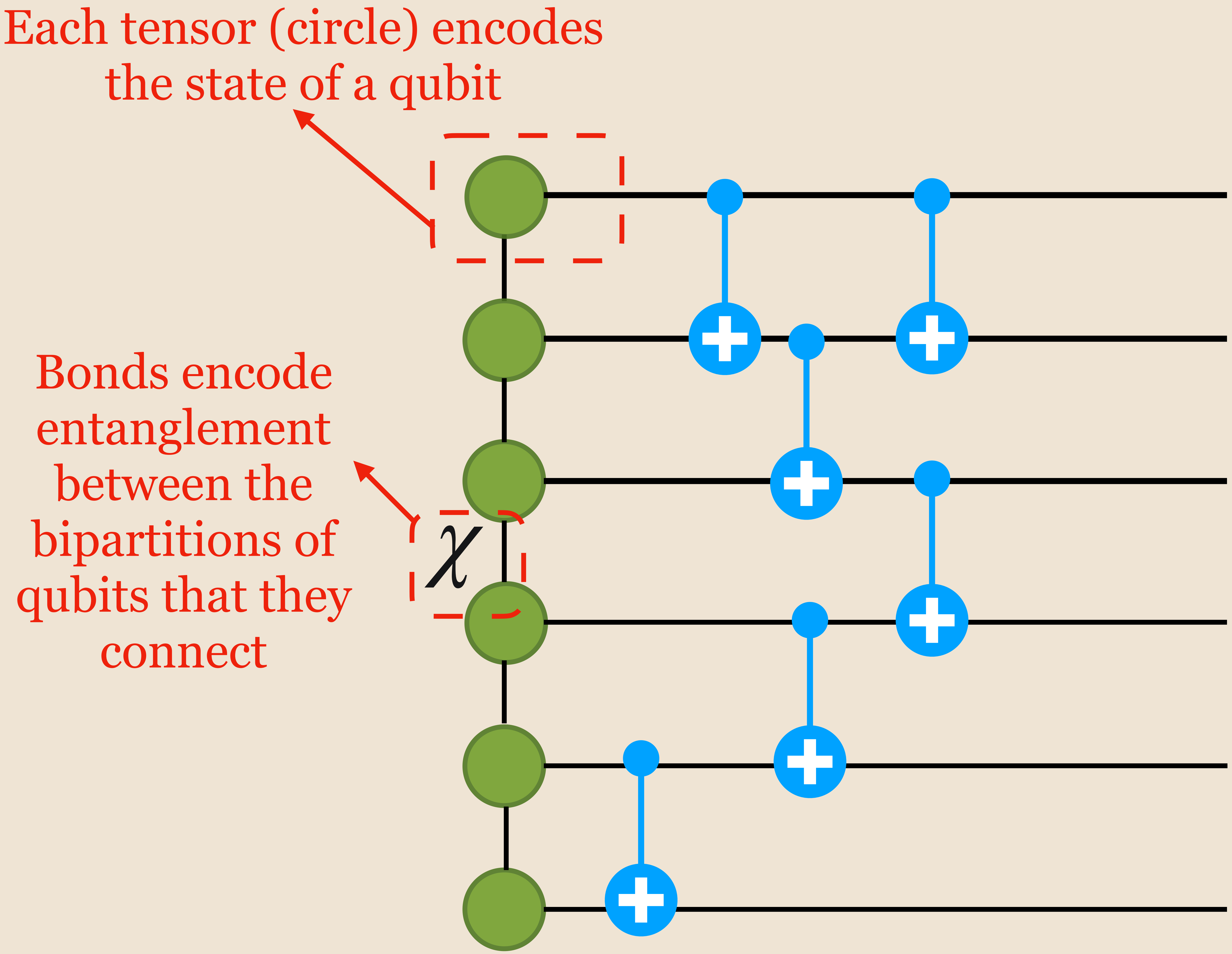

Memory requirements

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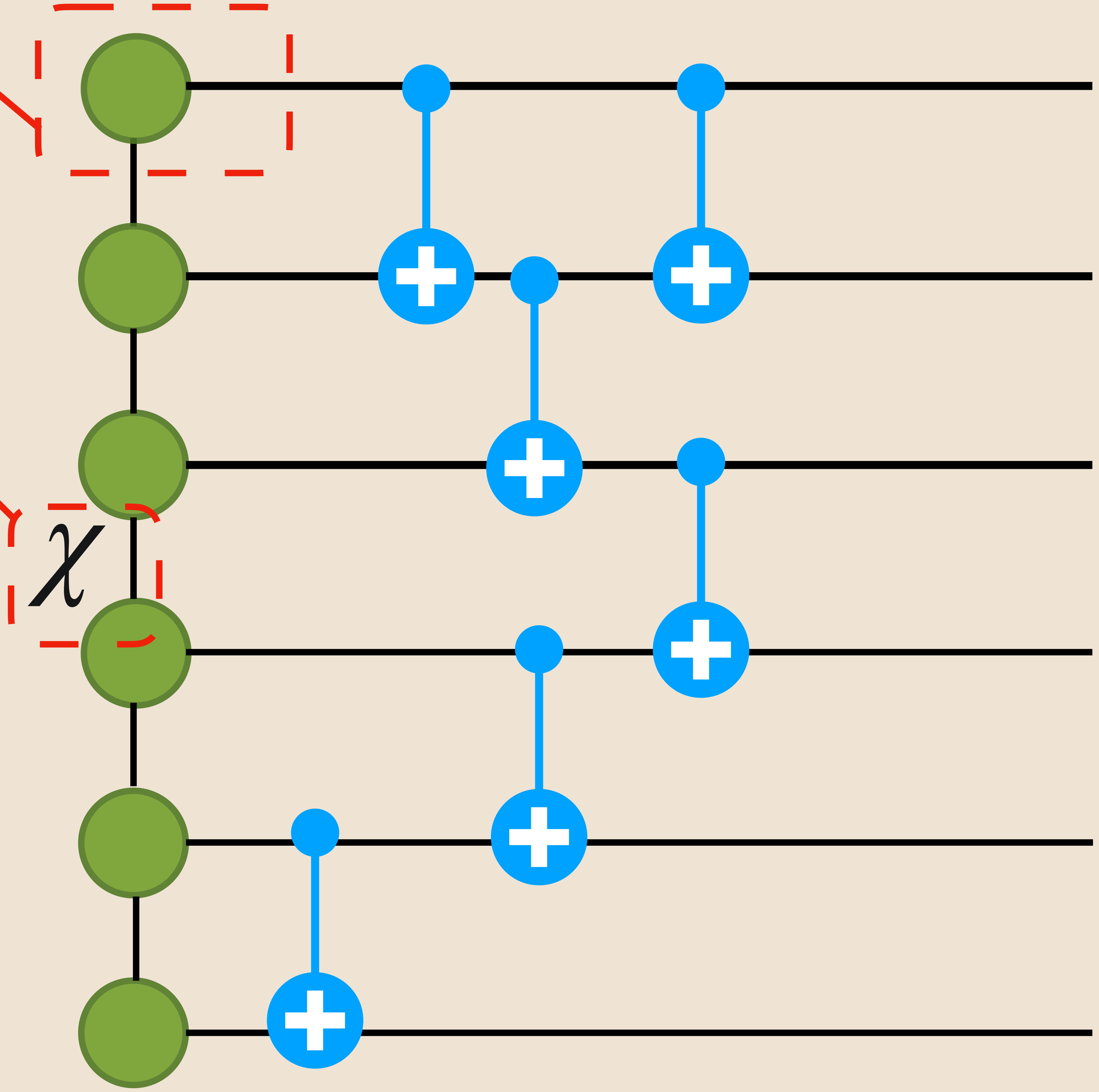
Matrix product states

Each tensor (circle) encodes the state of a qubit

Bonds encode entanglement between the bipartitions of qubits that they connect



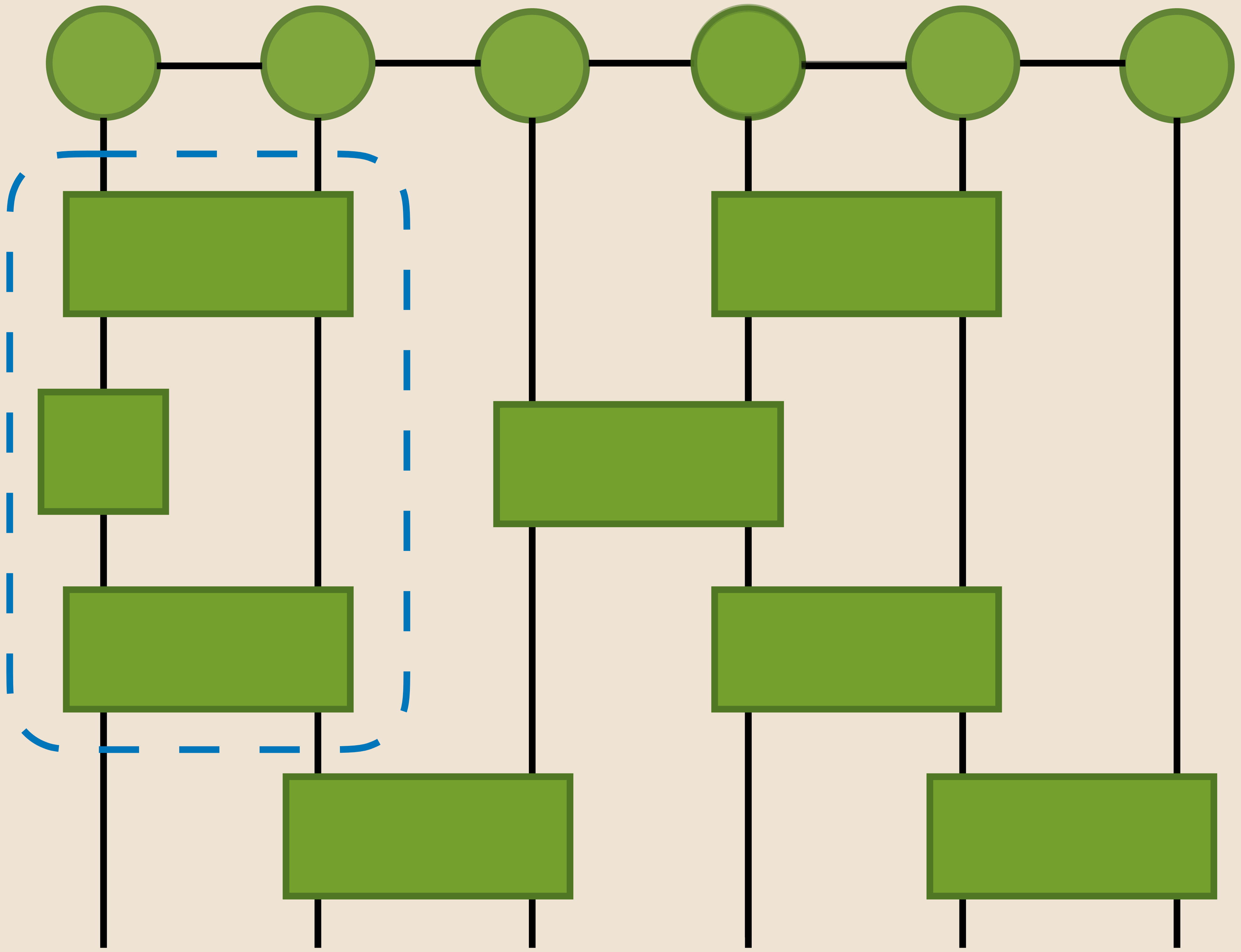
Memory requirements

$$O(2^n) \rightarrow O(2n\chi^2)$$

MPS simulations are not limited by the number of qubits but by the entanglement

Optimisation & parallelism

Gates acting on
the same qubits
are first contracted
together, then with
the state



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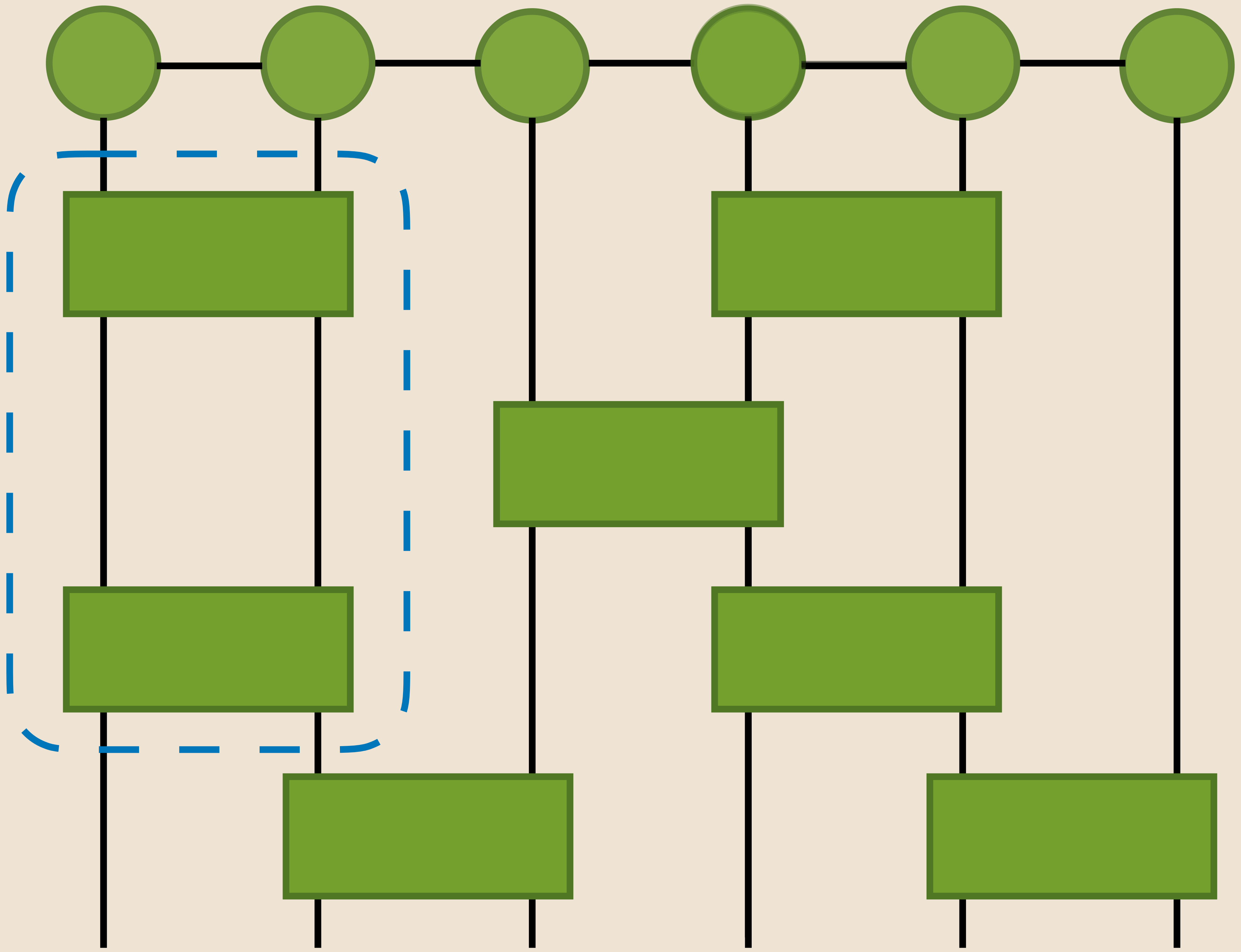

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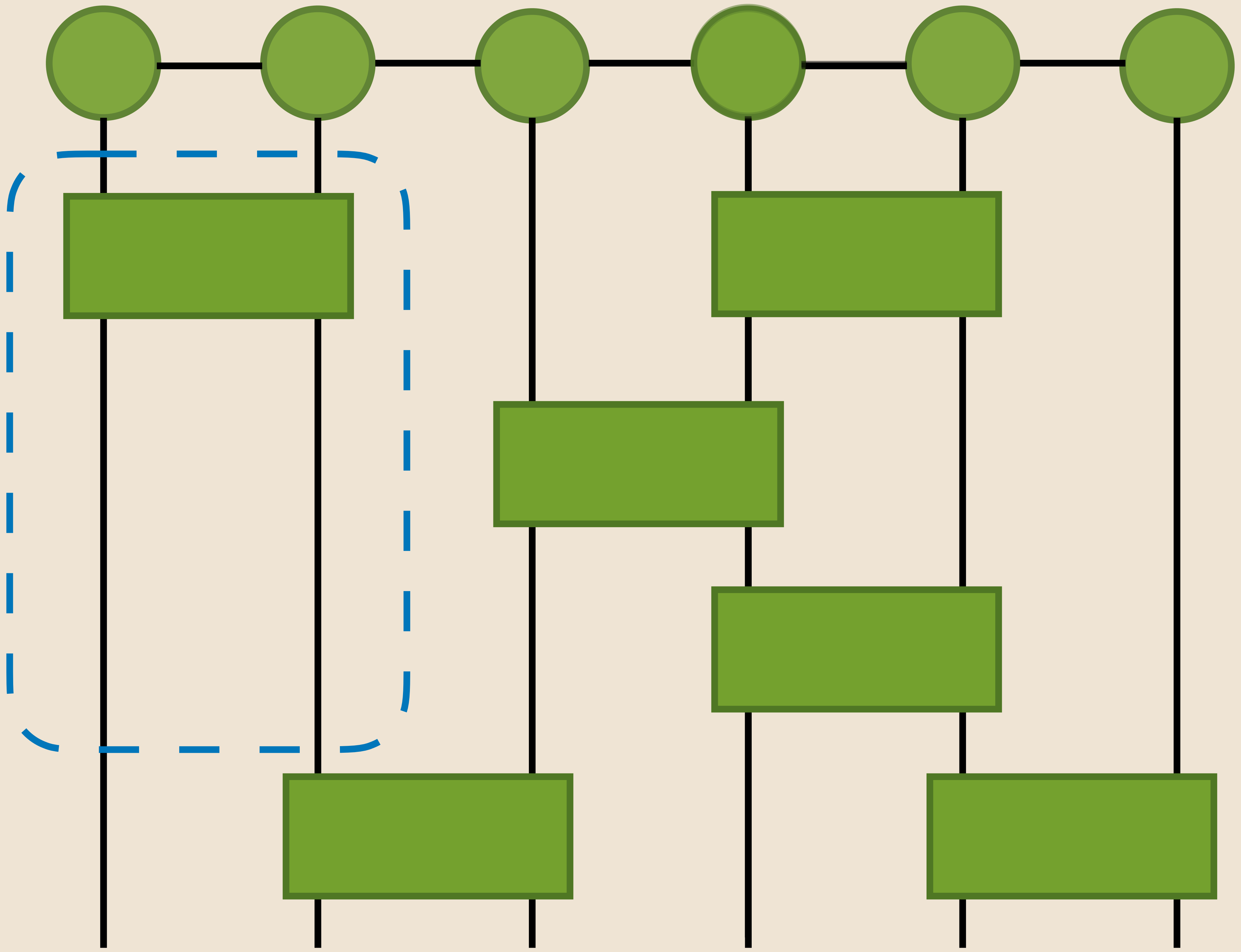

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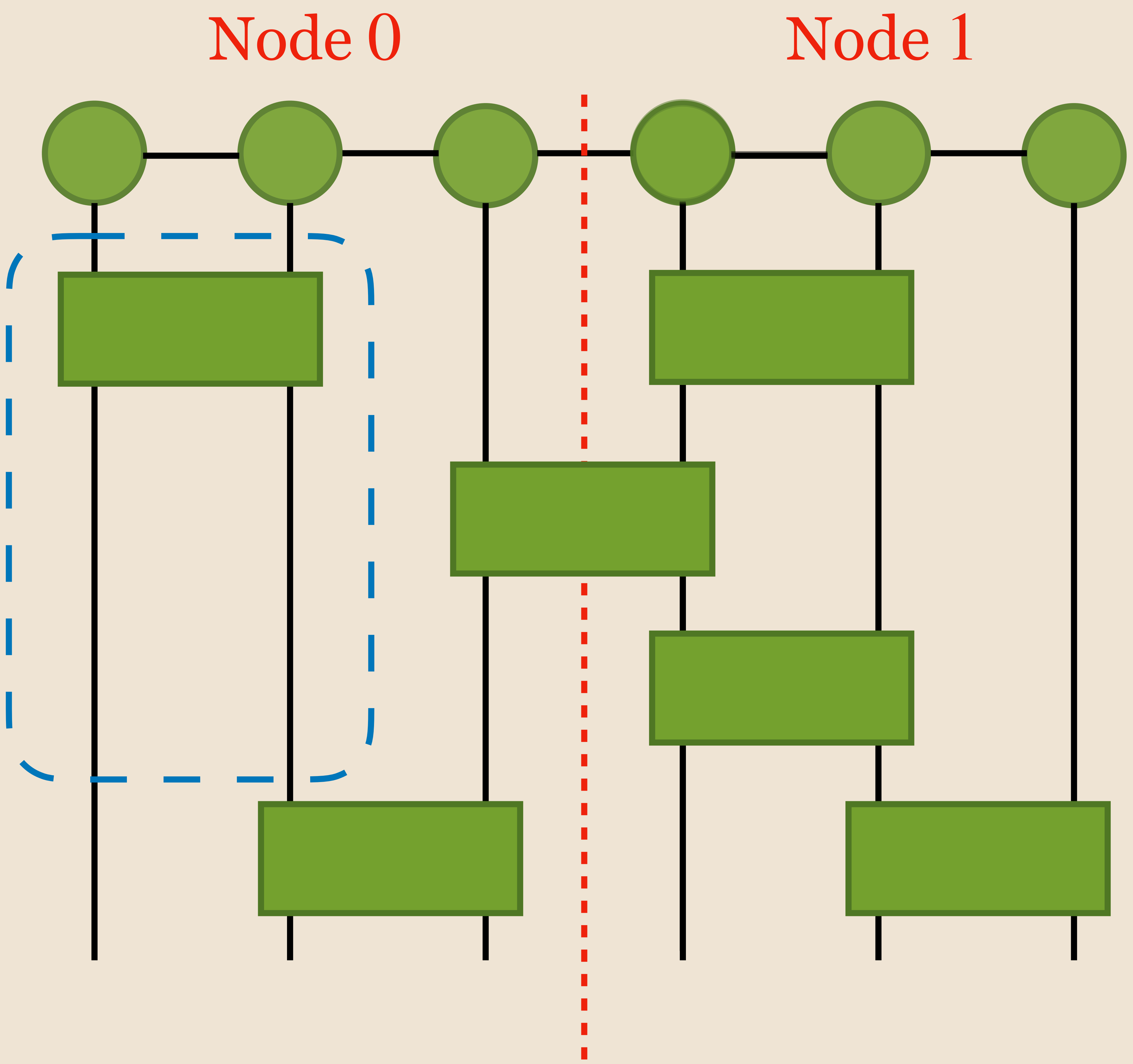

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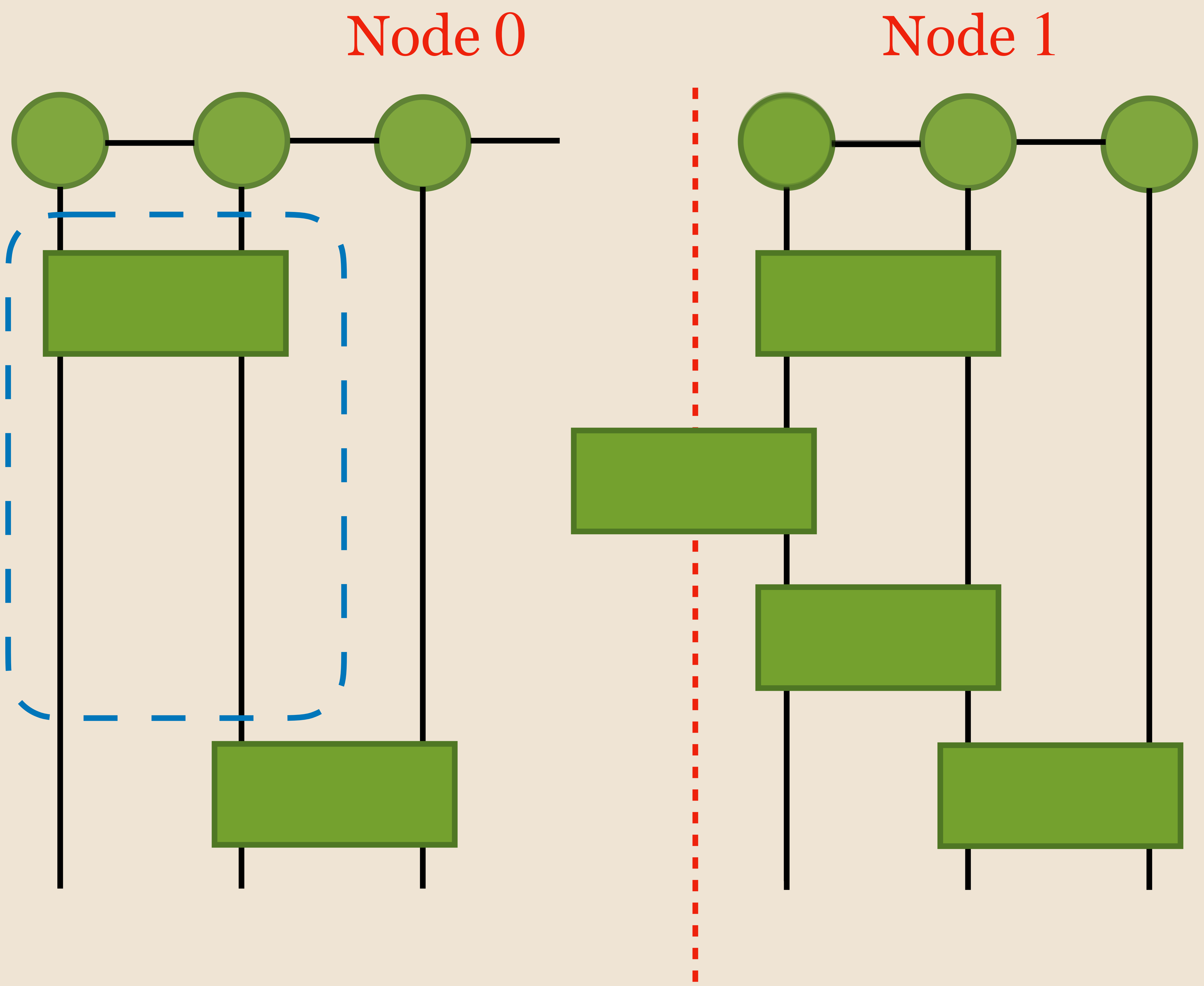

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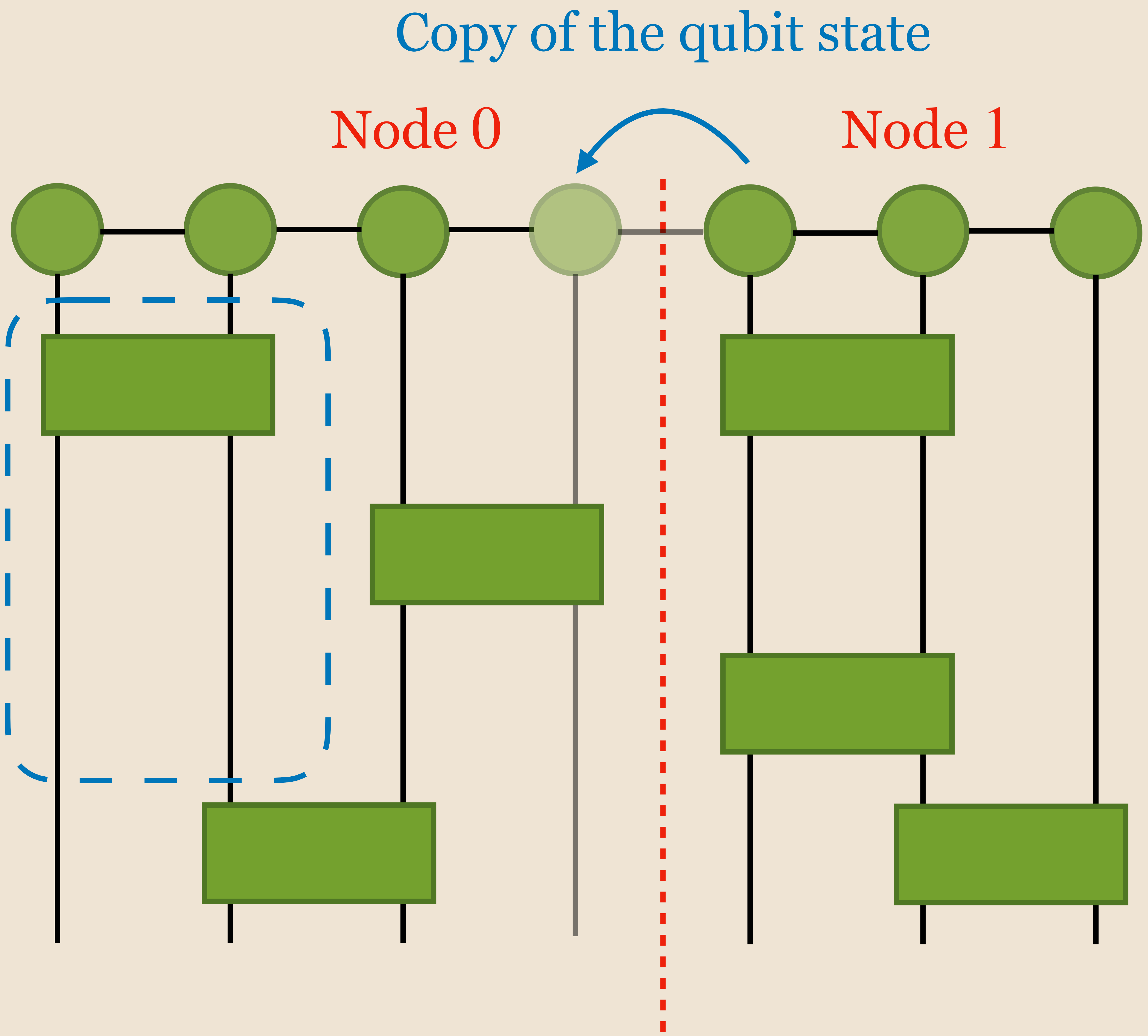

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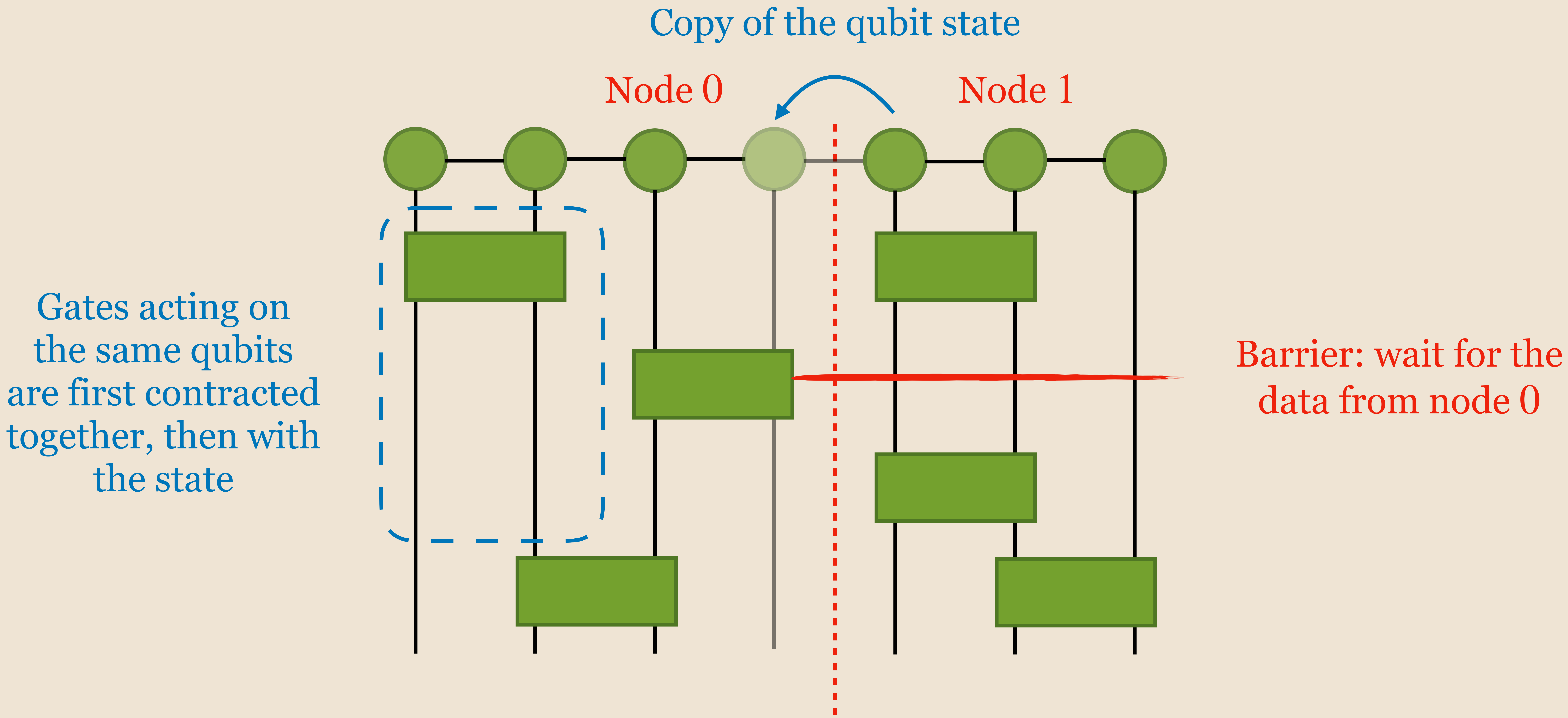

Quantum Matcha Tea on Leonardo

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[[redacted]@login02 ~]$ module load profile/quantum
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Loading qmatcha_tea/0.3.11
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CuPy NumPy

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Run the simulation

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Load the module on Leonardo

Define quantum circuit in qiskit

Define the observables

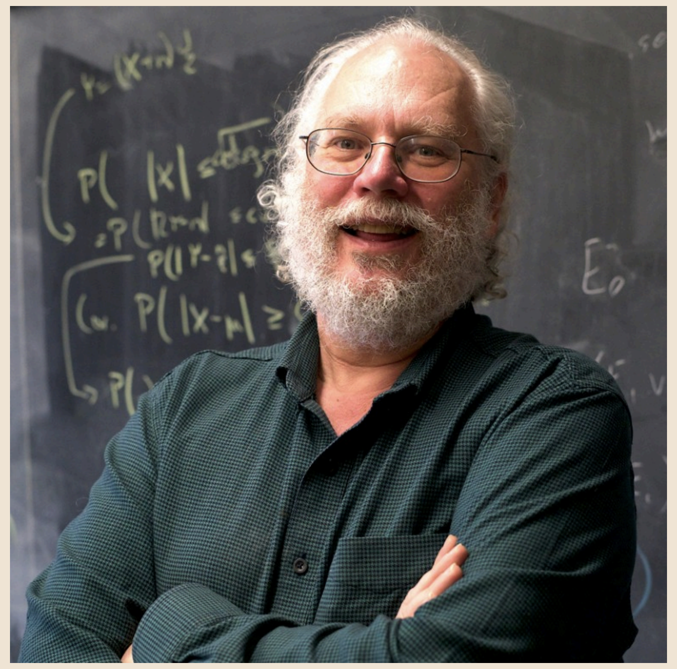
Define the convergence parameters

Define the backend

Run the simulation

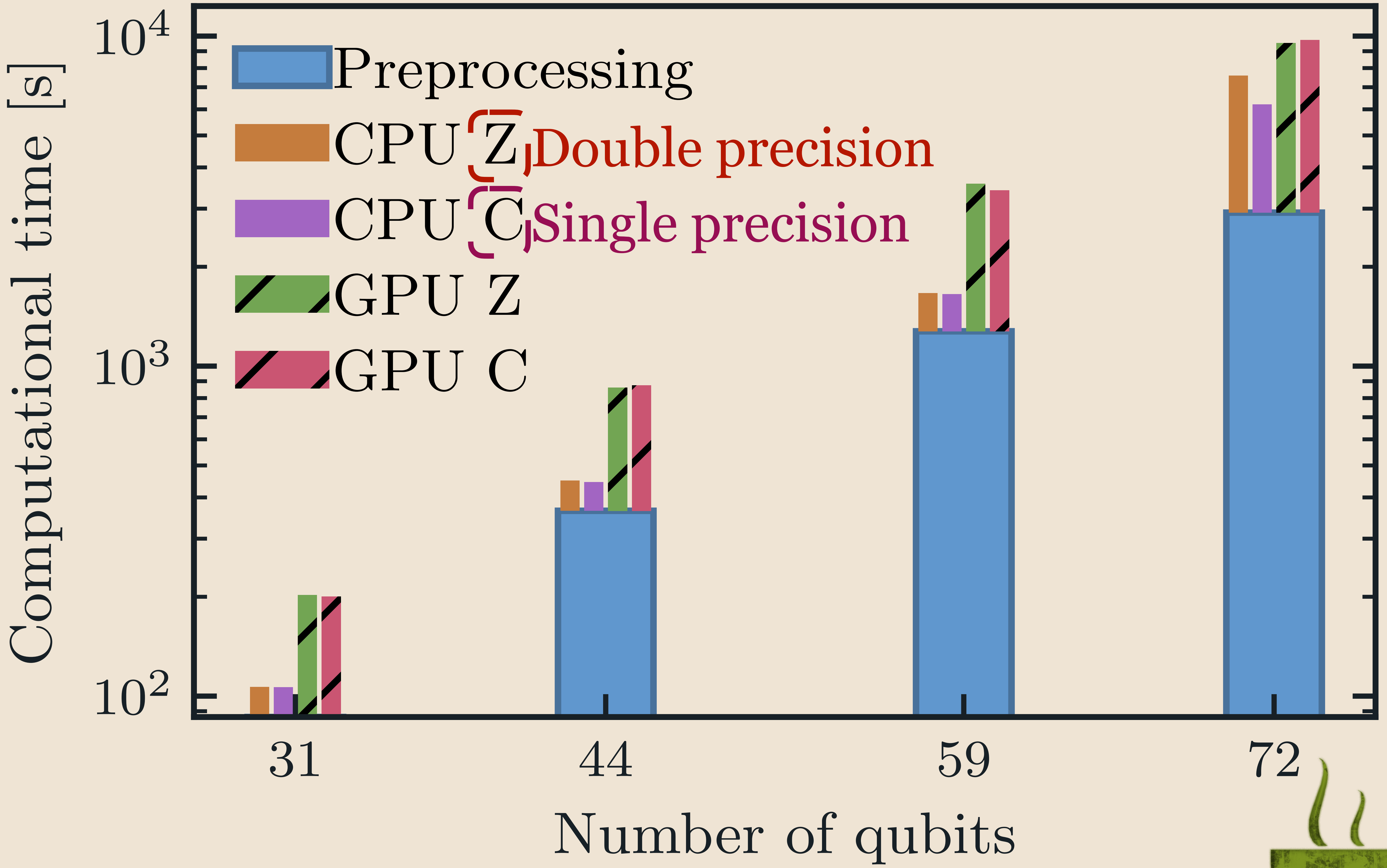
Obtain and analyse the results

Benchmarks: Shor algorithm



Factorization of prime numbers p, q

$$N = pq \rightarrow p, q$$



Thanks to Alessandro Cavion for providing the algorithm

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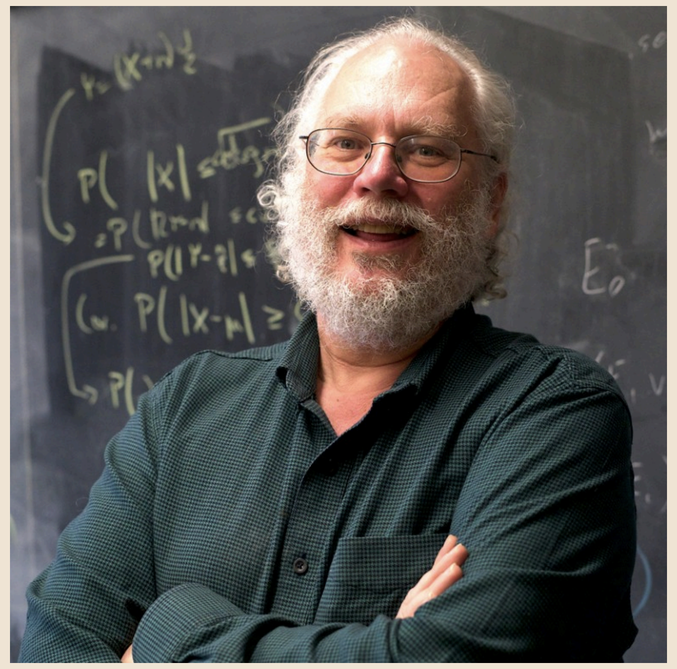

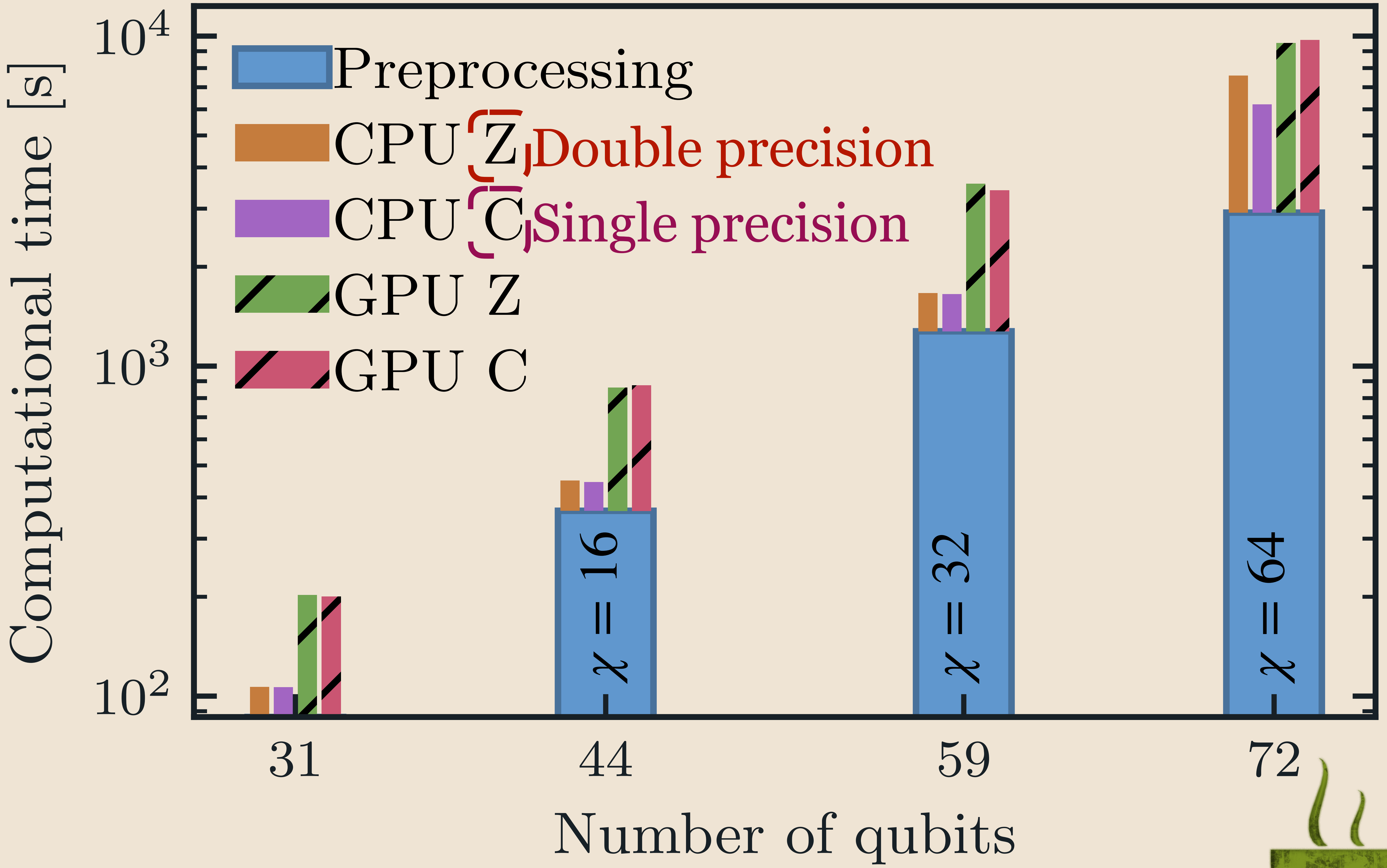

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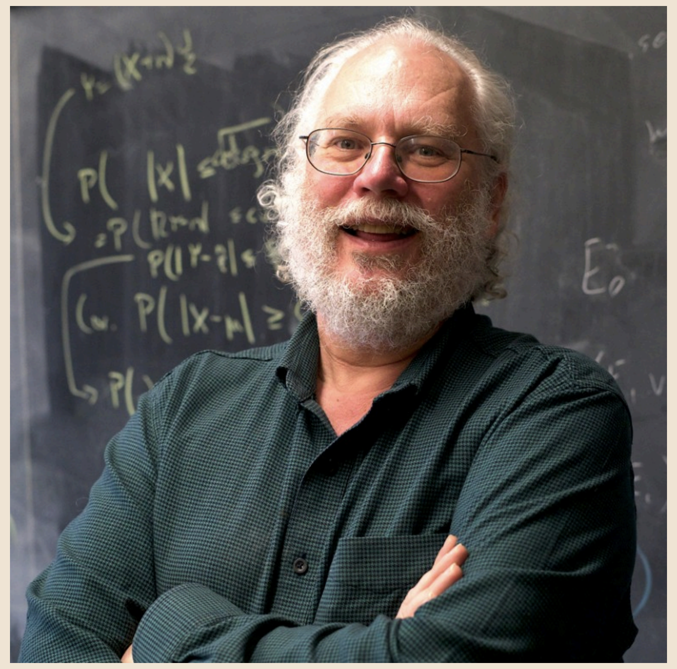

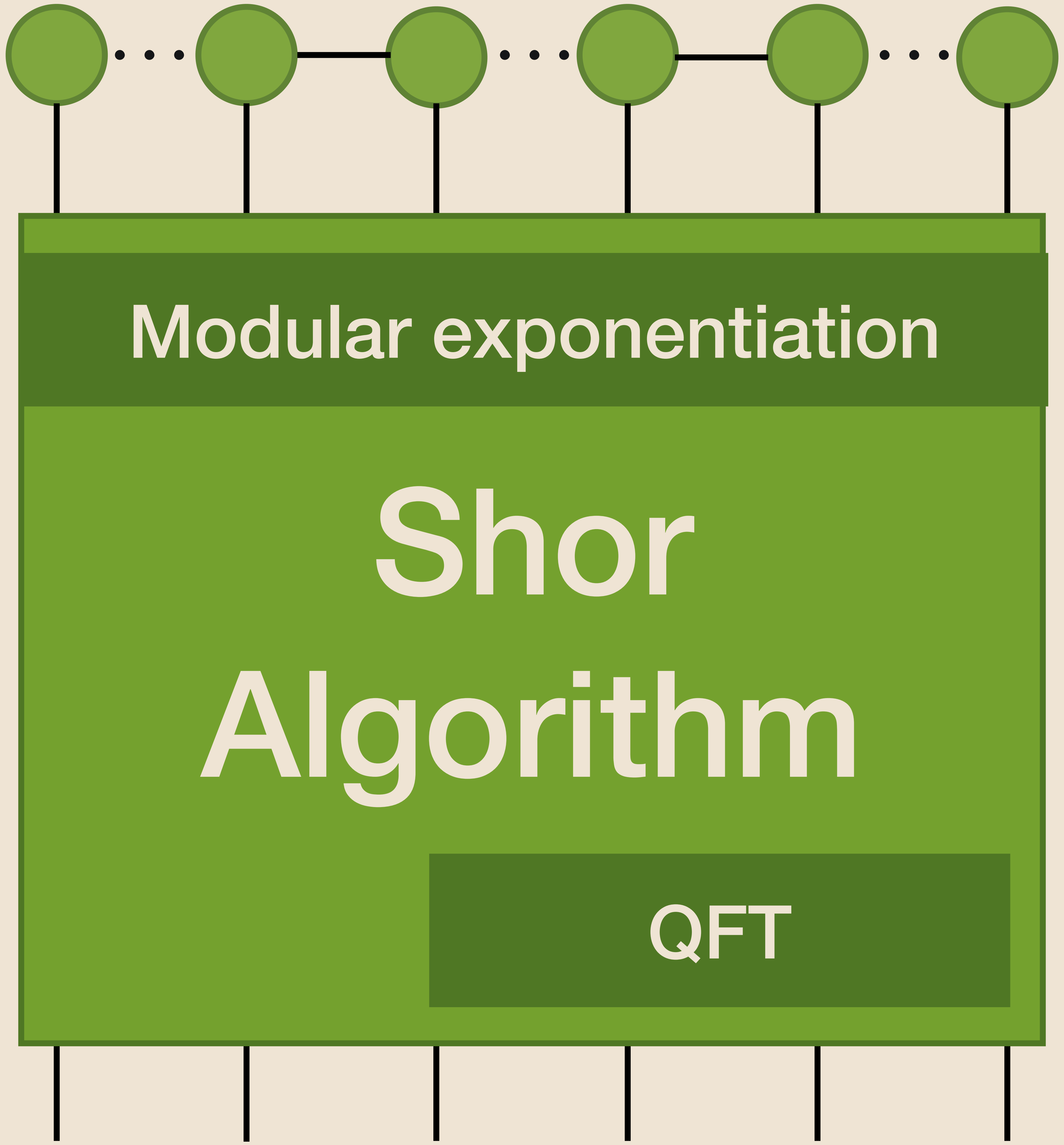

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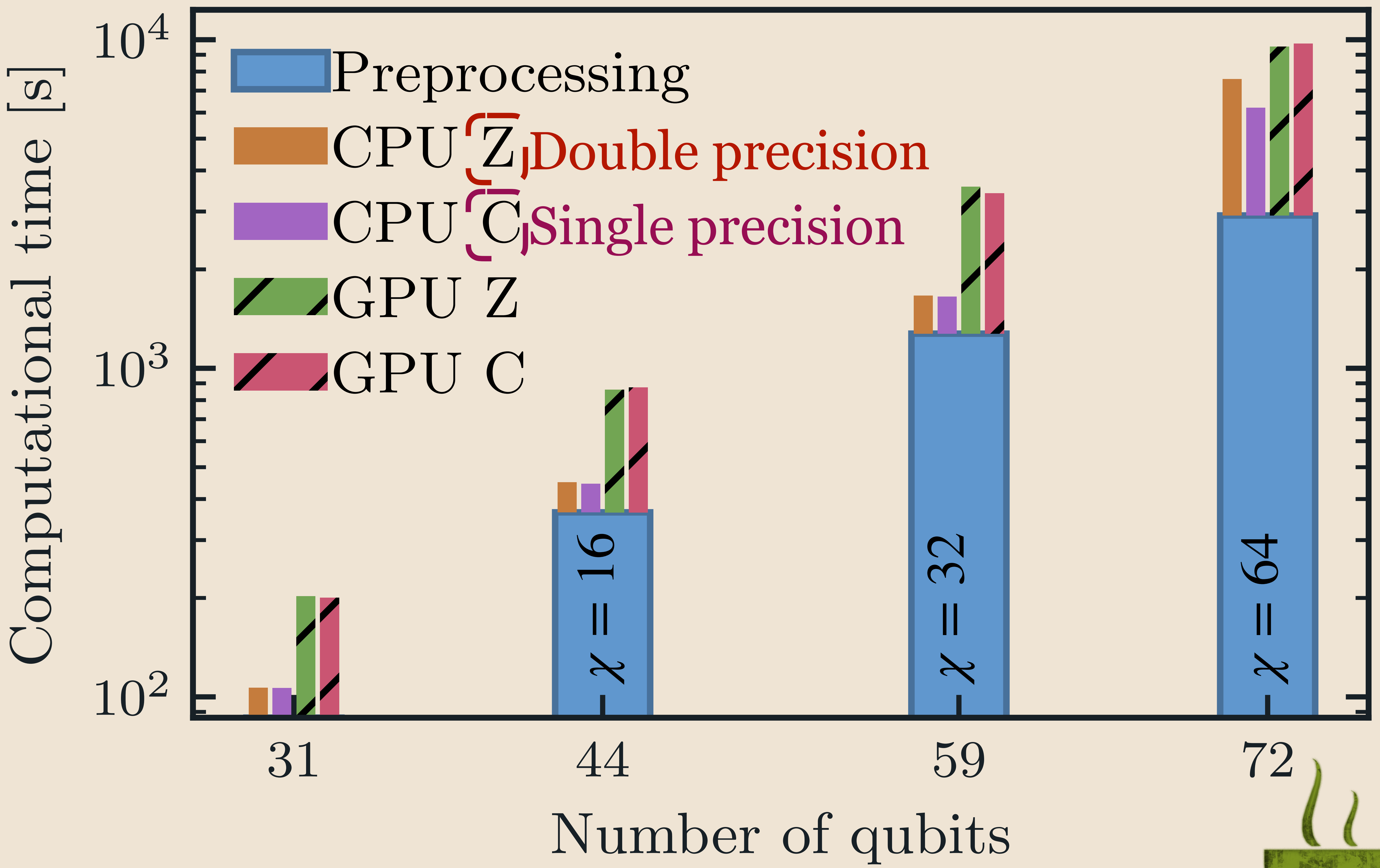
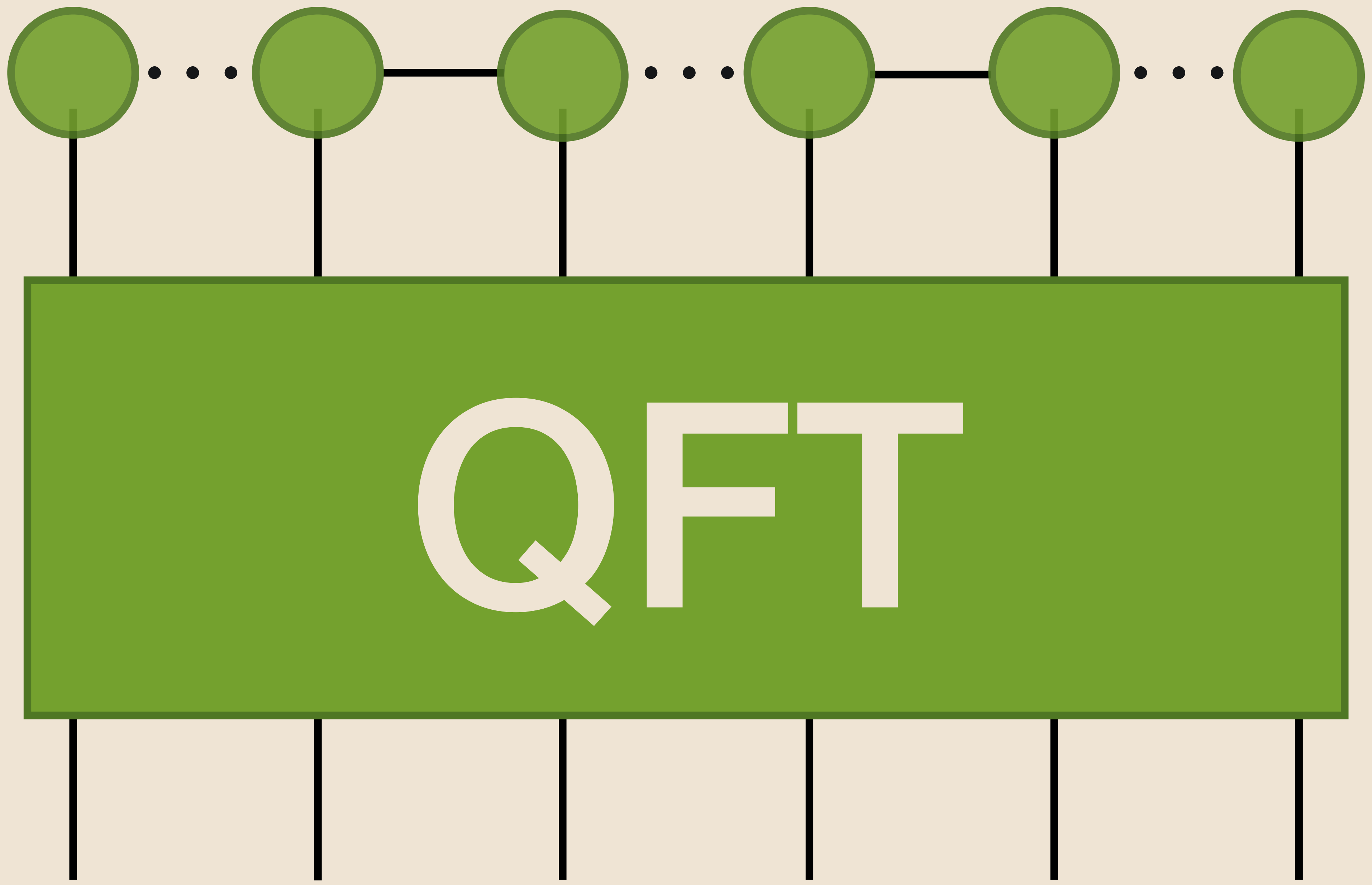
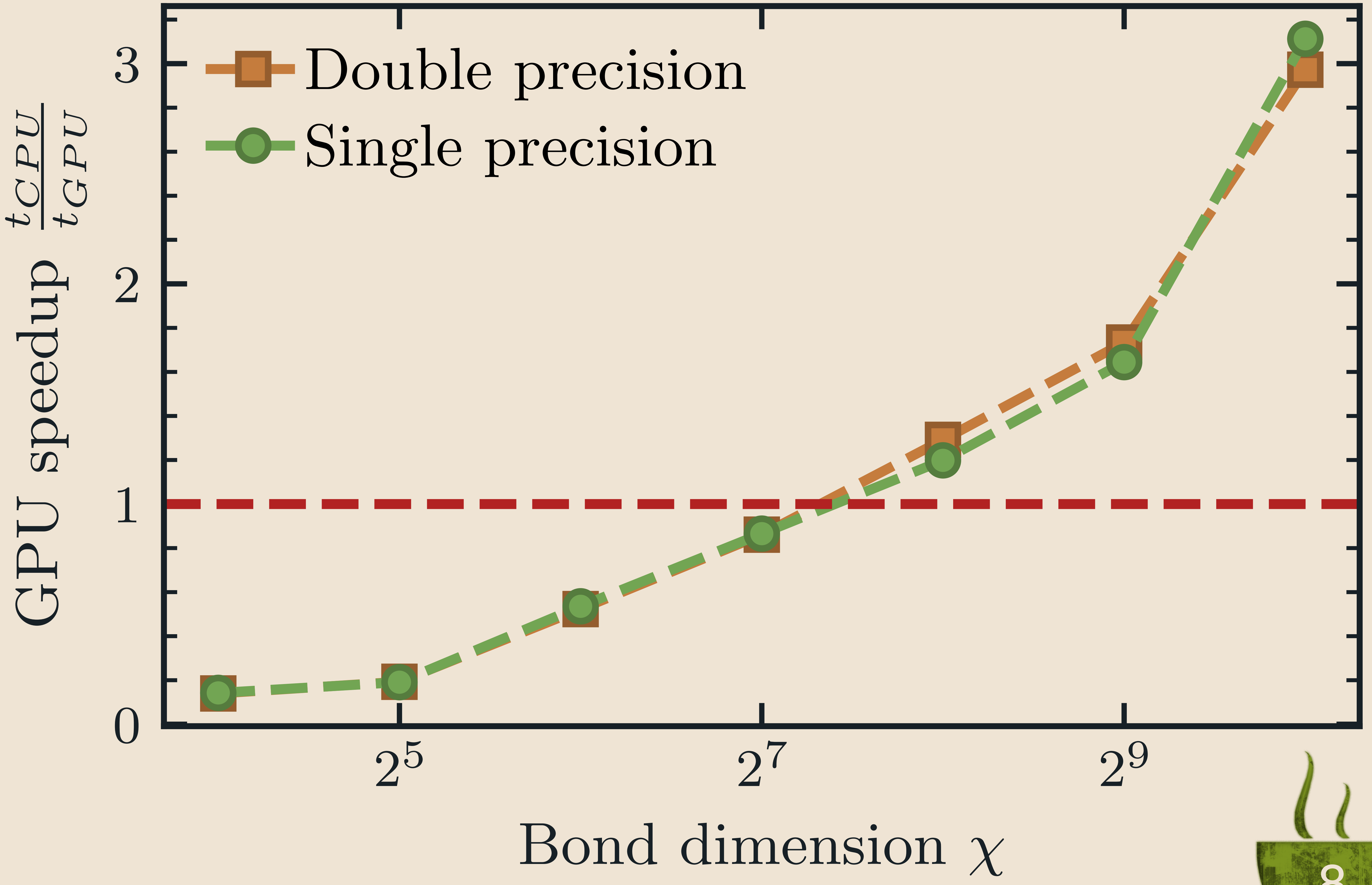


Thanks to Alessandro Cavion for providing the algorithm

Benchmarks: QFT on entangled state



$n = 100$ qubits



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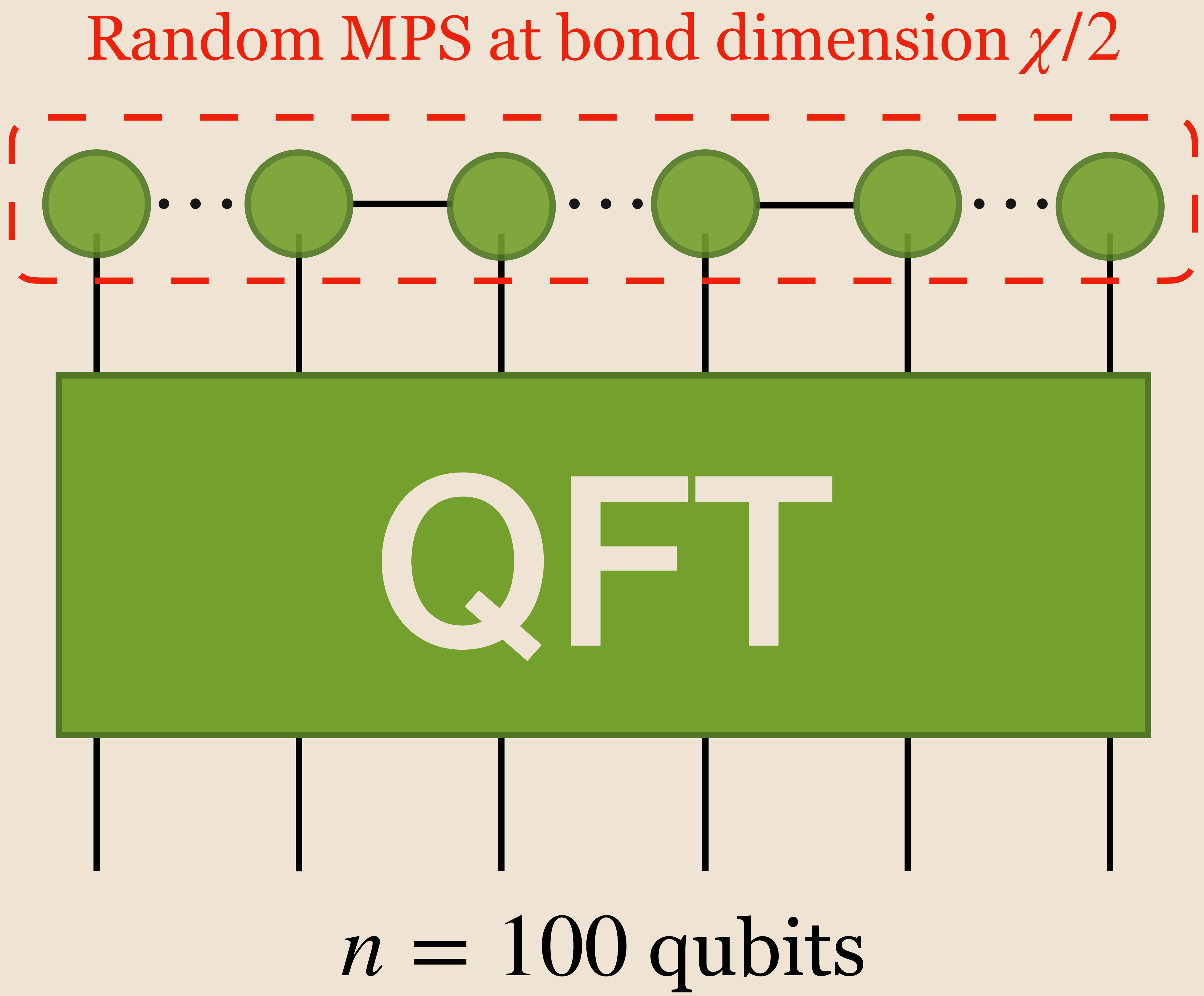
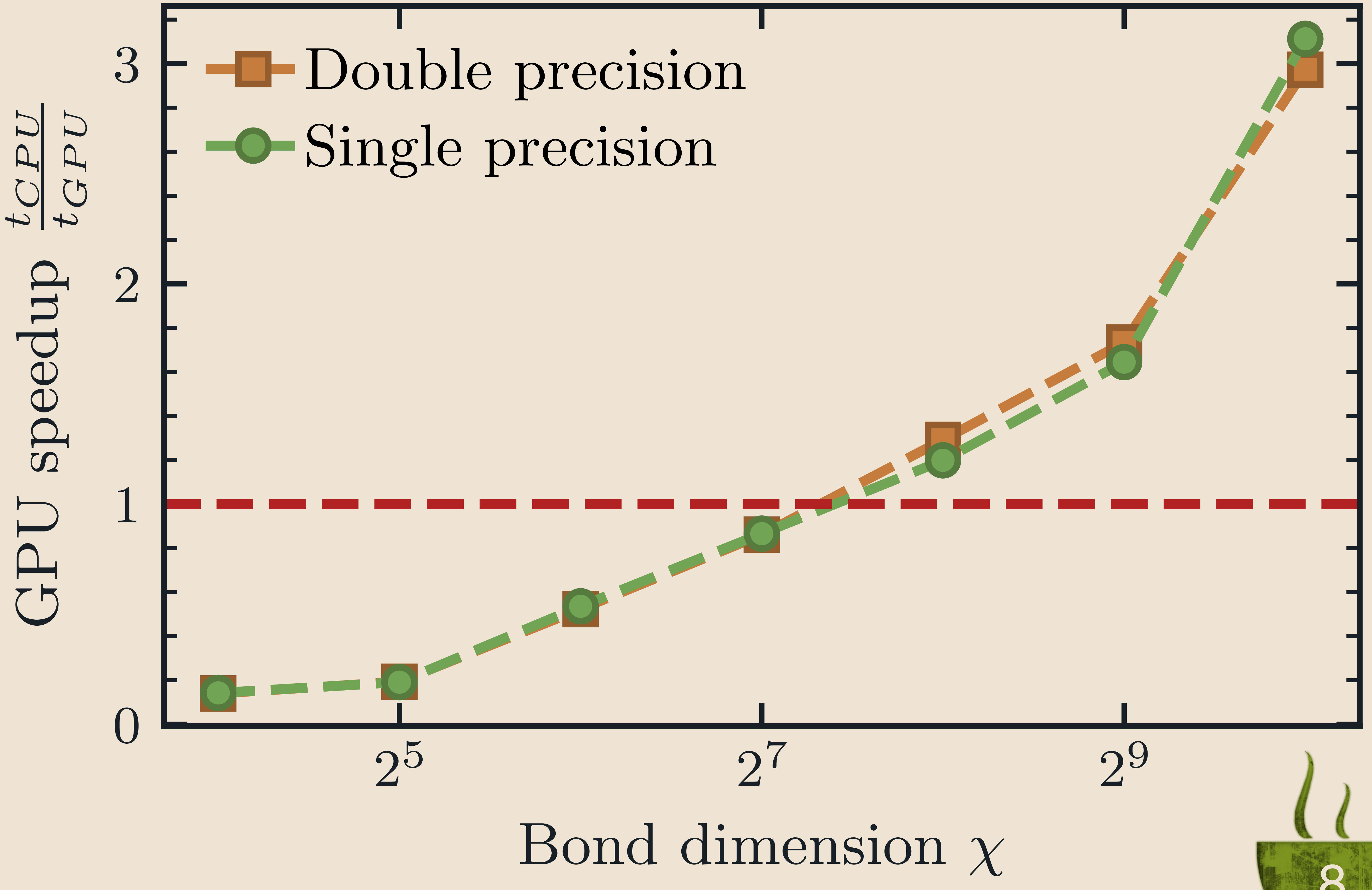
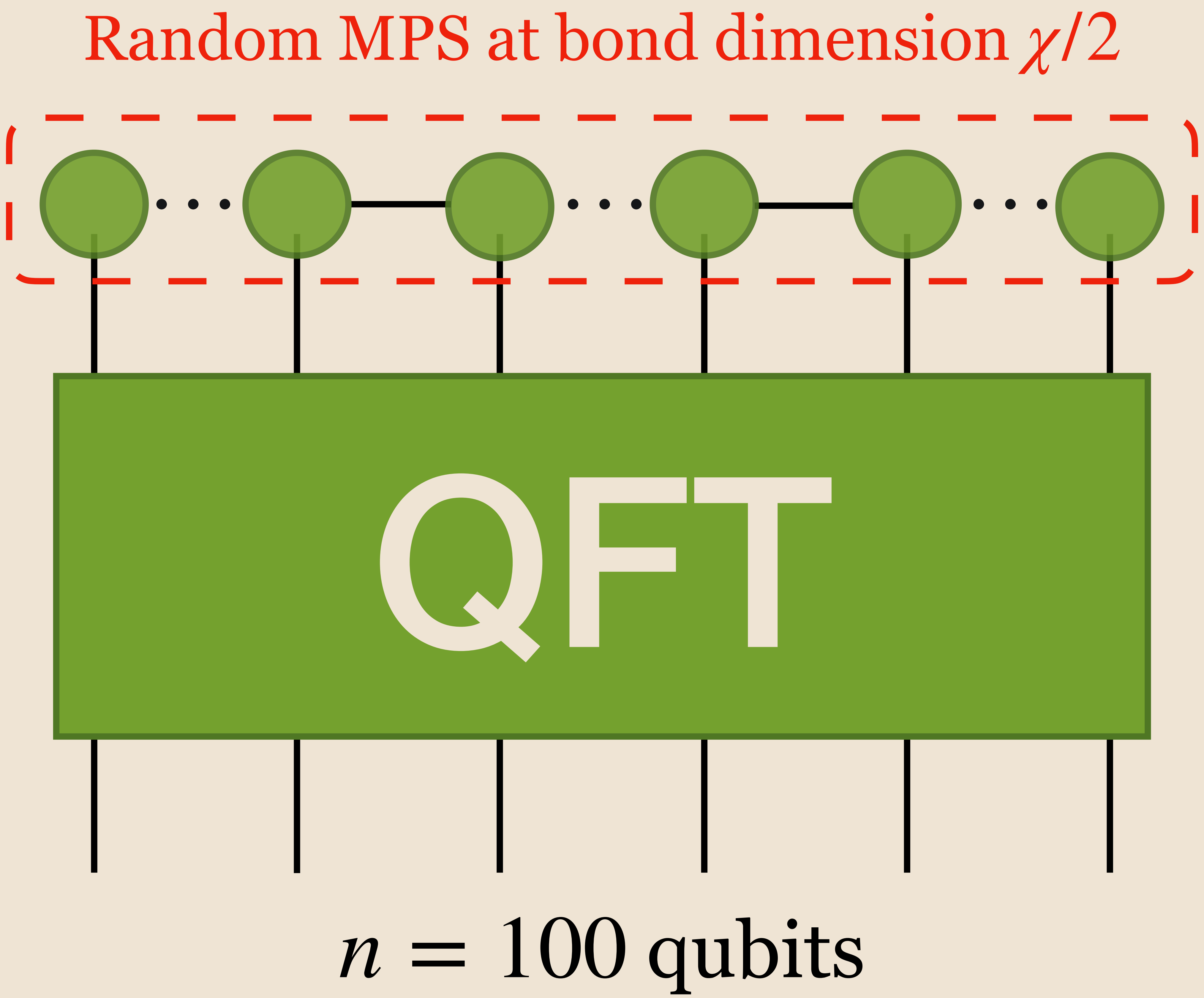
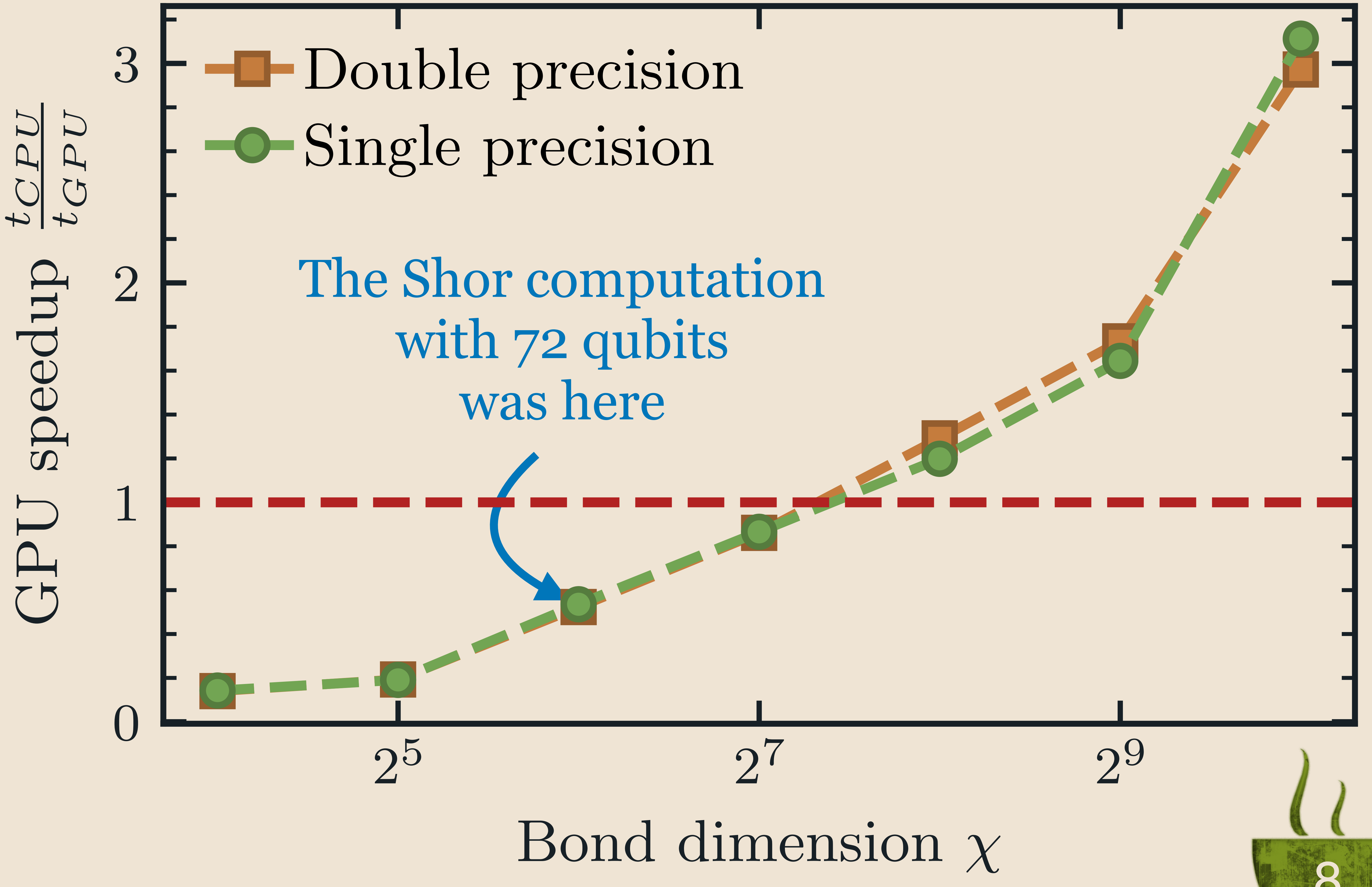


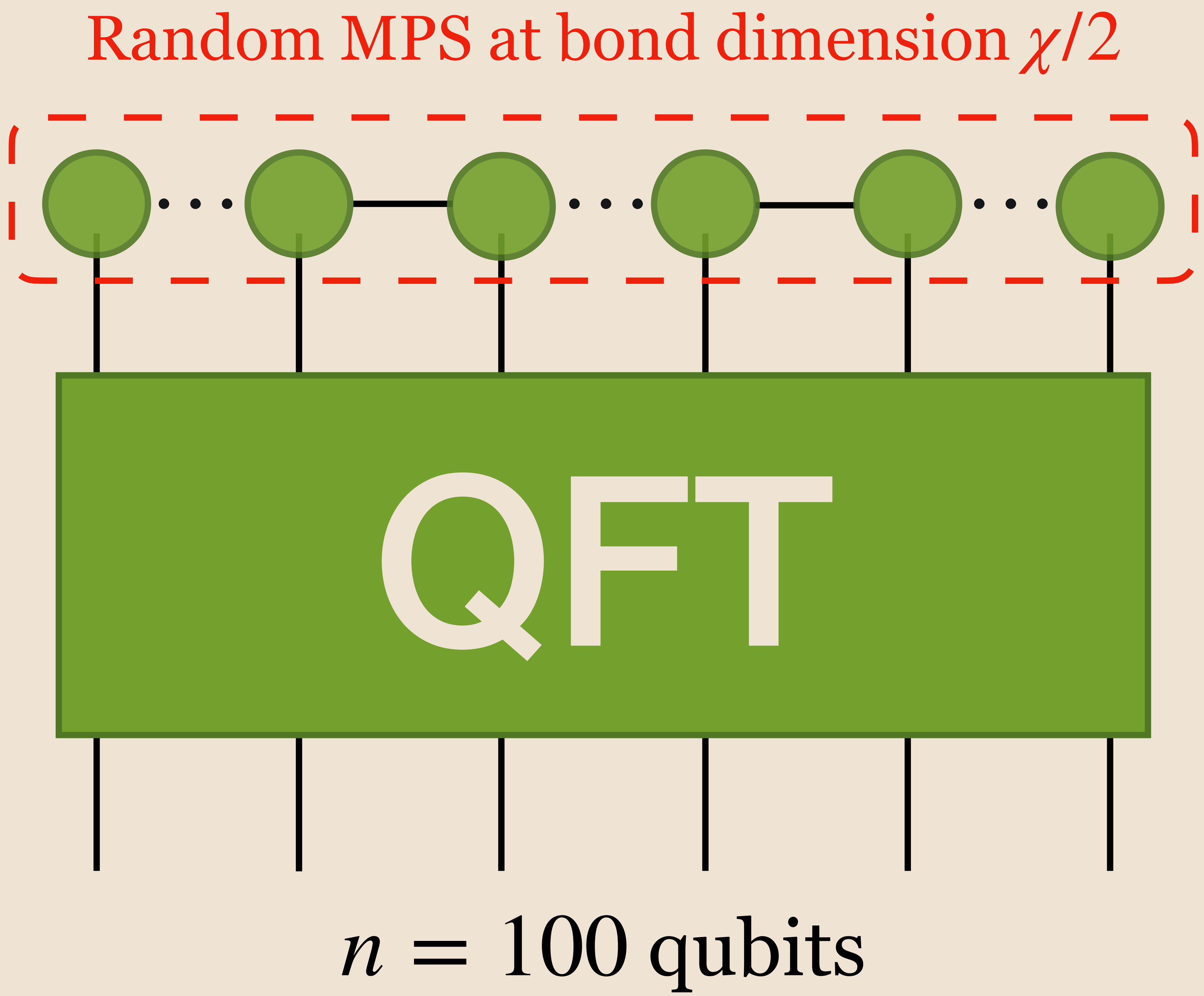


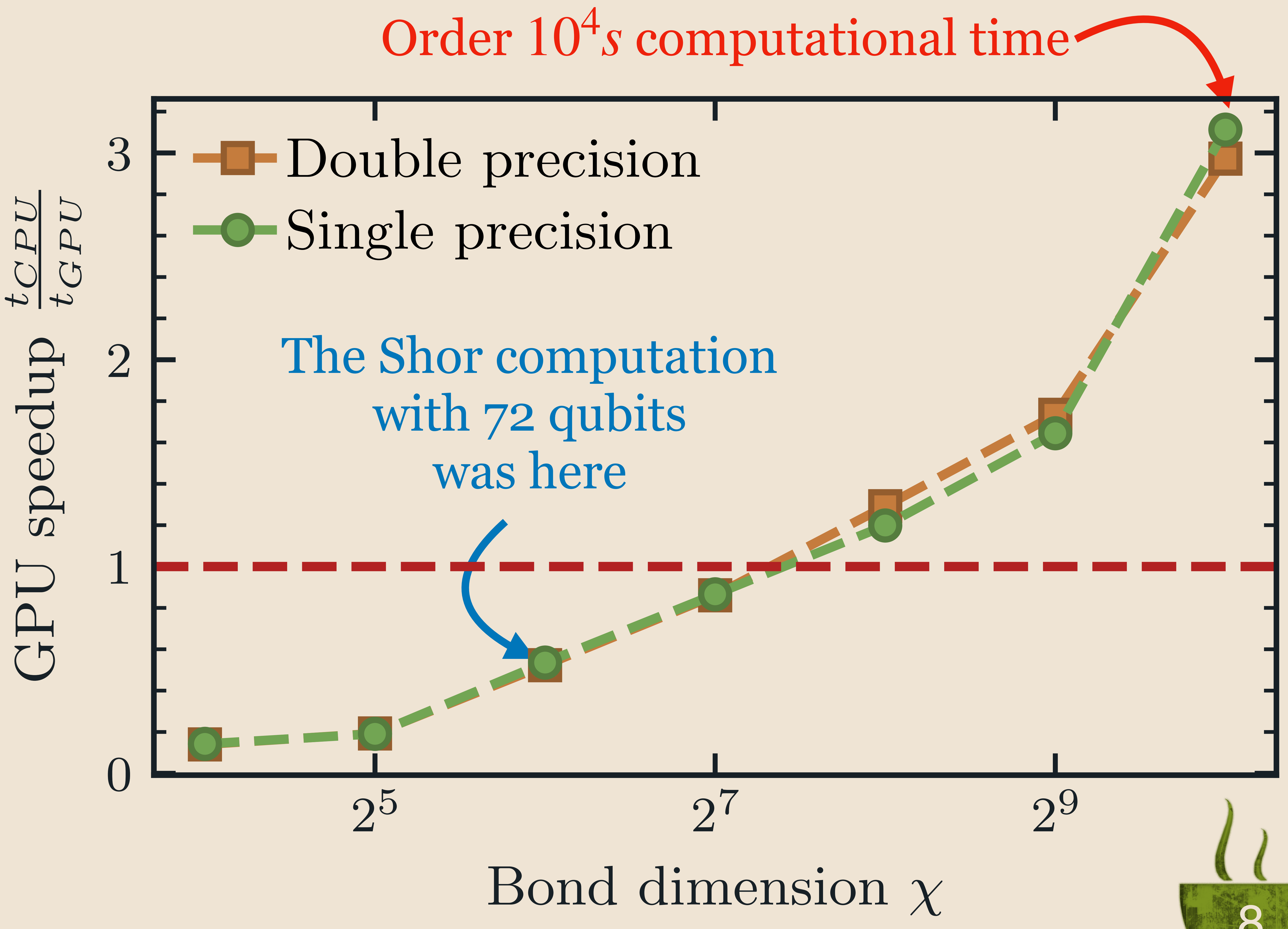
Benchmarks: QFT on entangled state





Benchmarks: QFT on entangled state

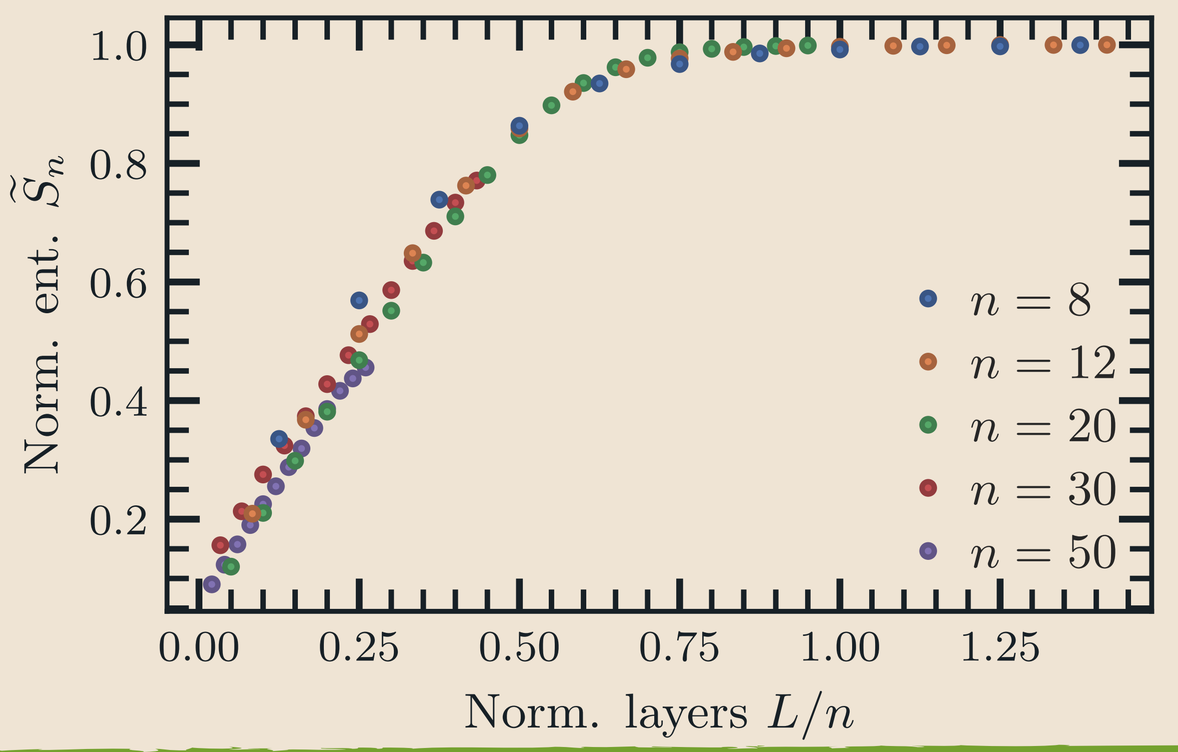




Applications

Entanglement entropy production in QNN

Ballarin, Marco, et al. Quantum 7, 1023 (2023)

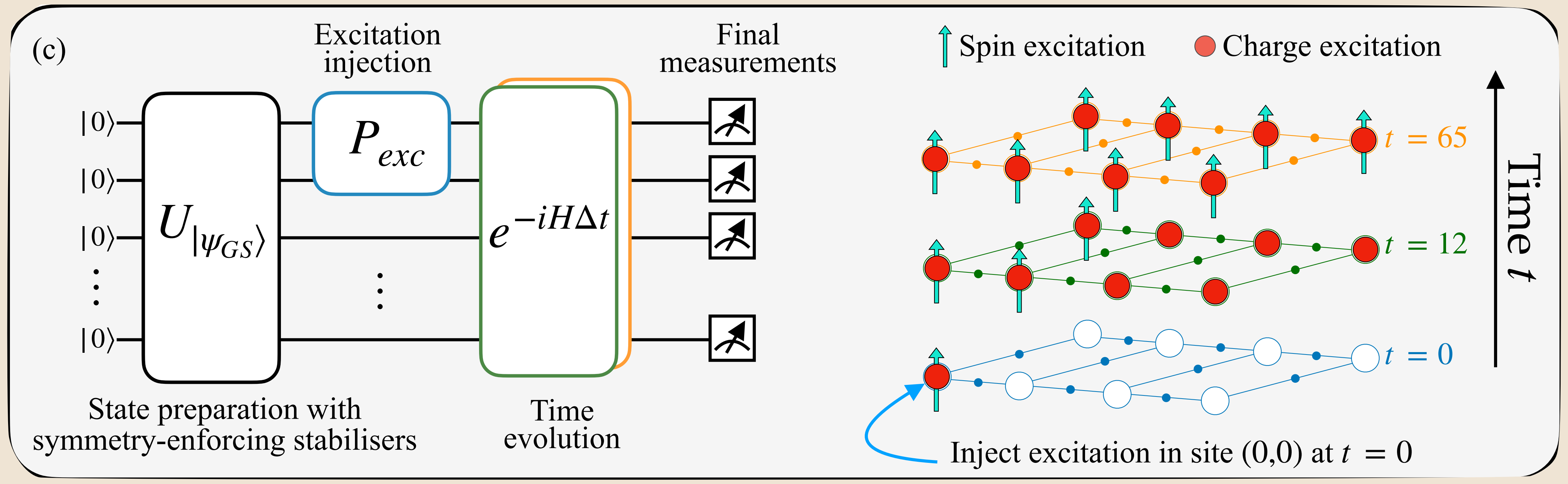


Ab initio two-dimensional digital twin for quantum computer

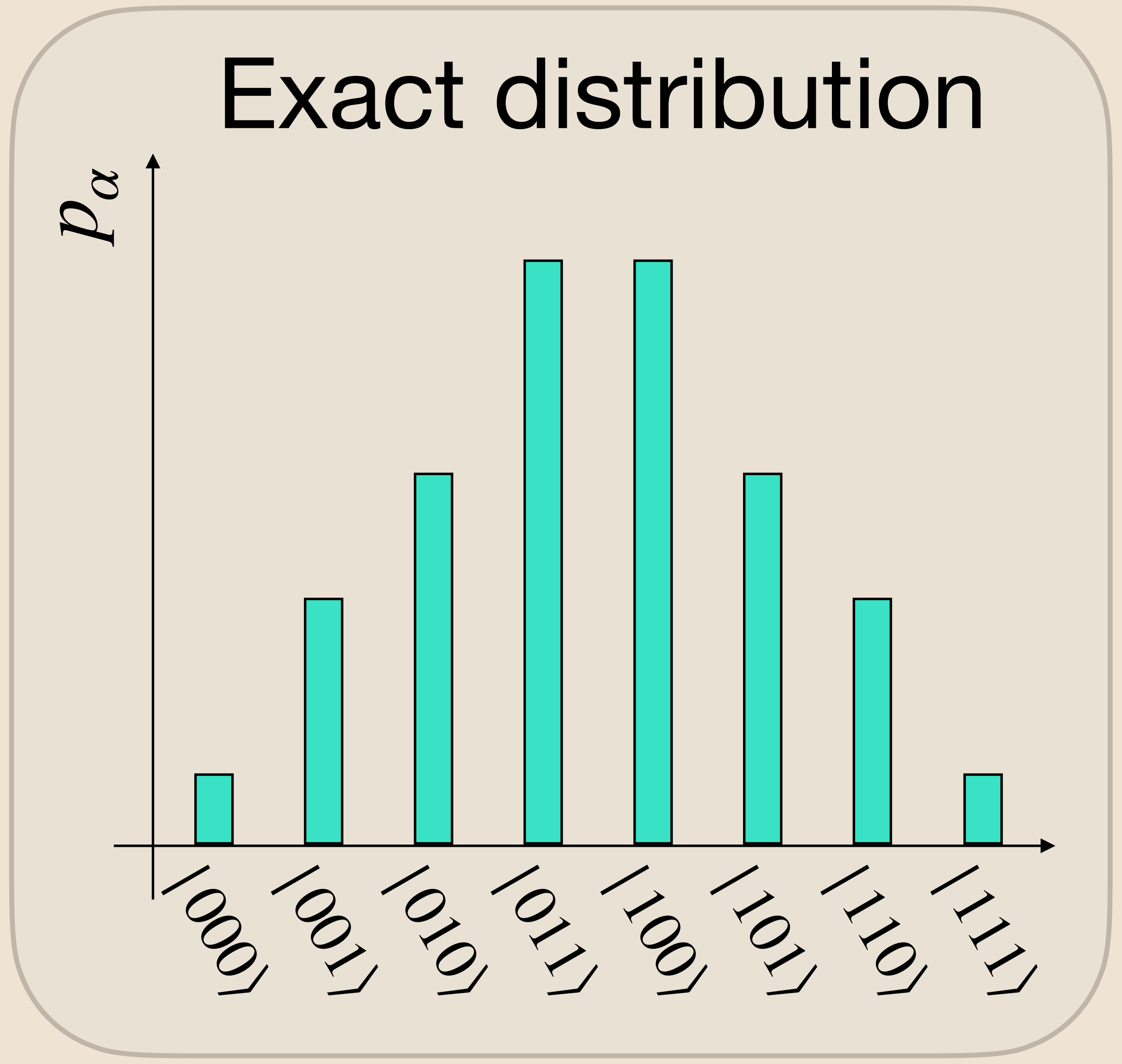
Jaschke, Daniel, et al. arXiv:2210.03763

Scalable digital quantum simulation of lattice fermion theories with local encoding

Ballarin, Marco, et al. arXiv:2310.15091



Optimal Exact Sampling of Tensor Networks



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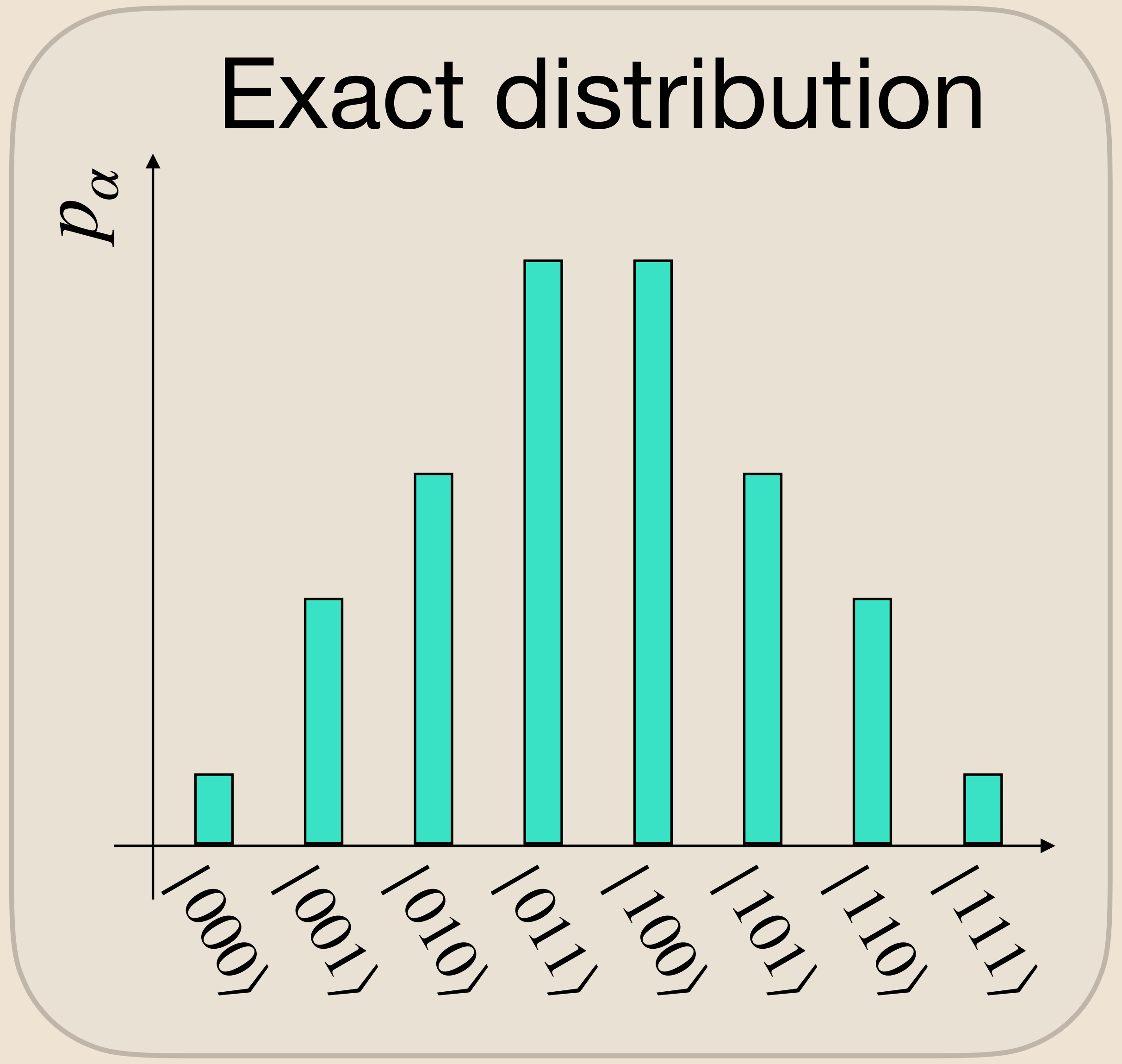

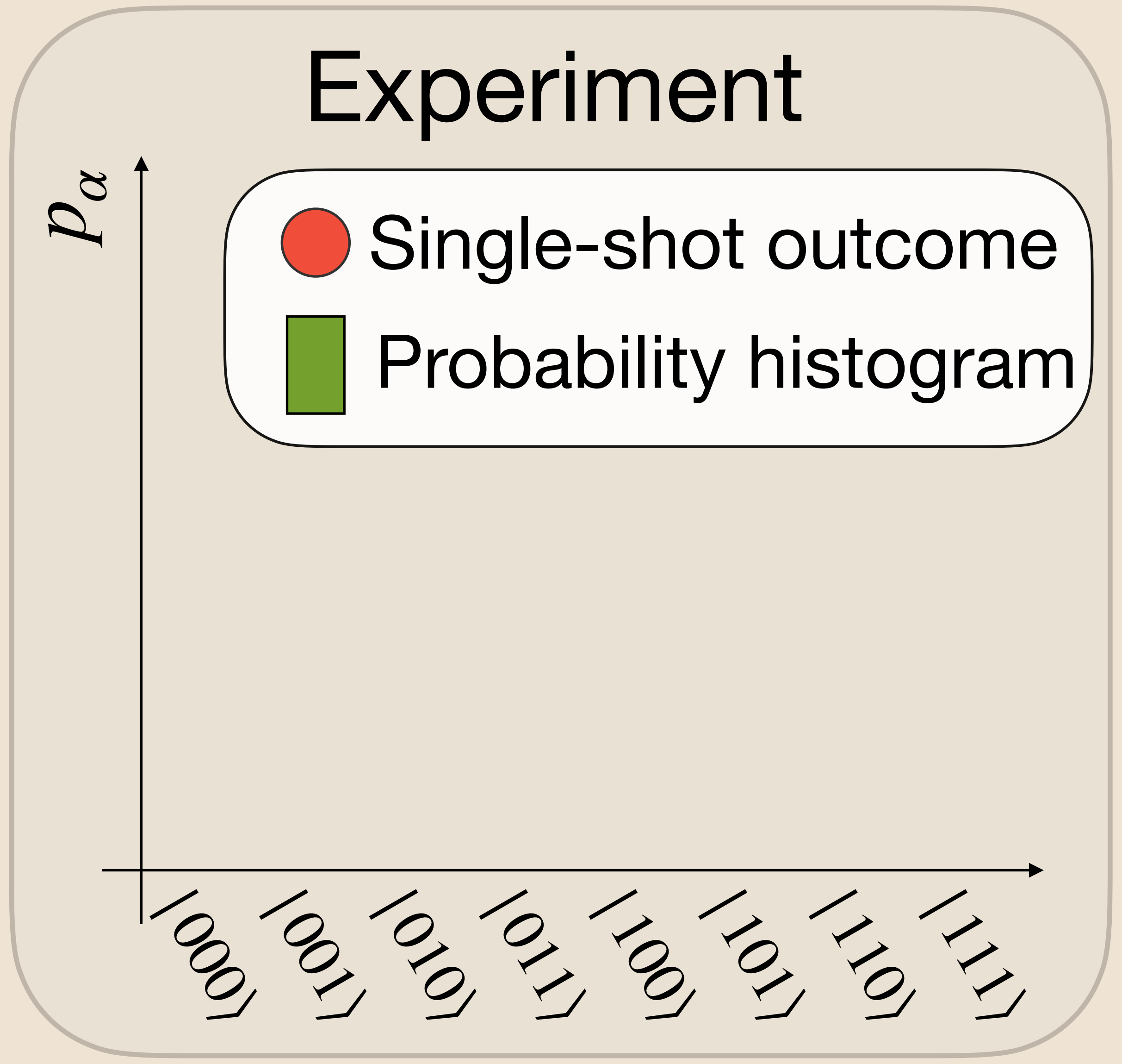

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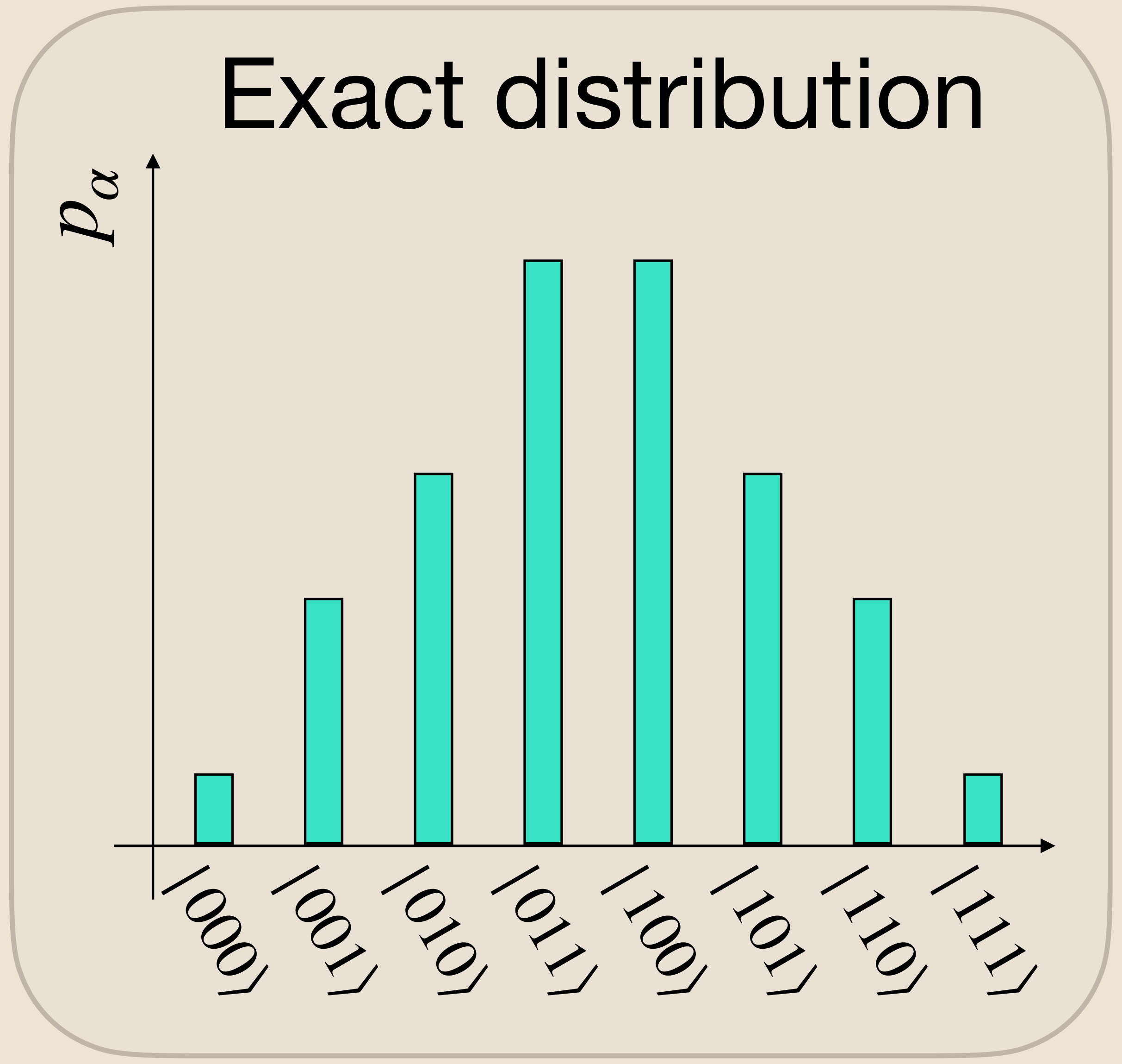

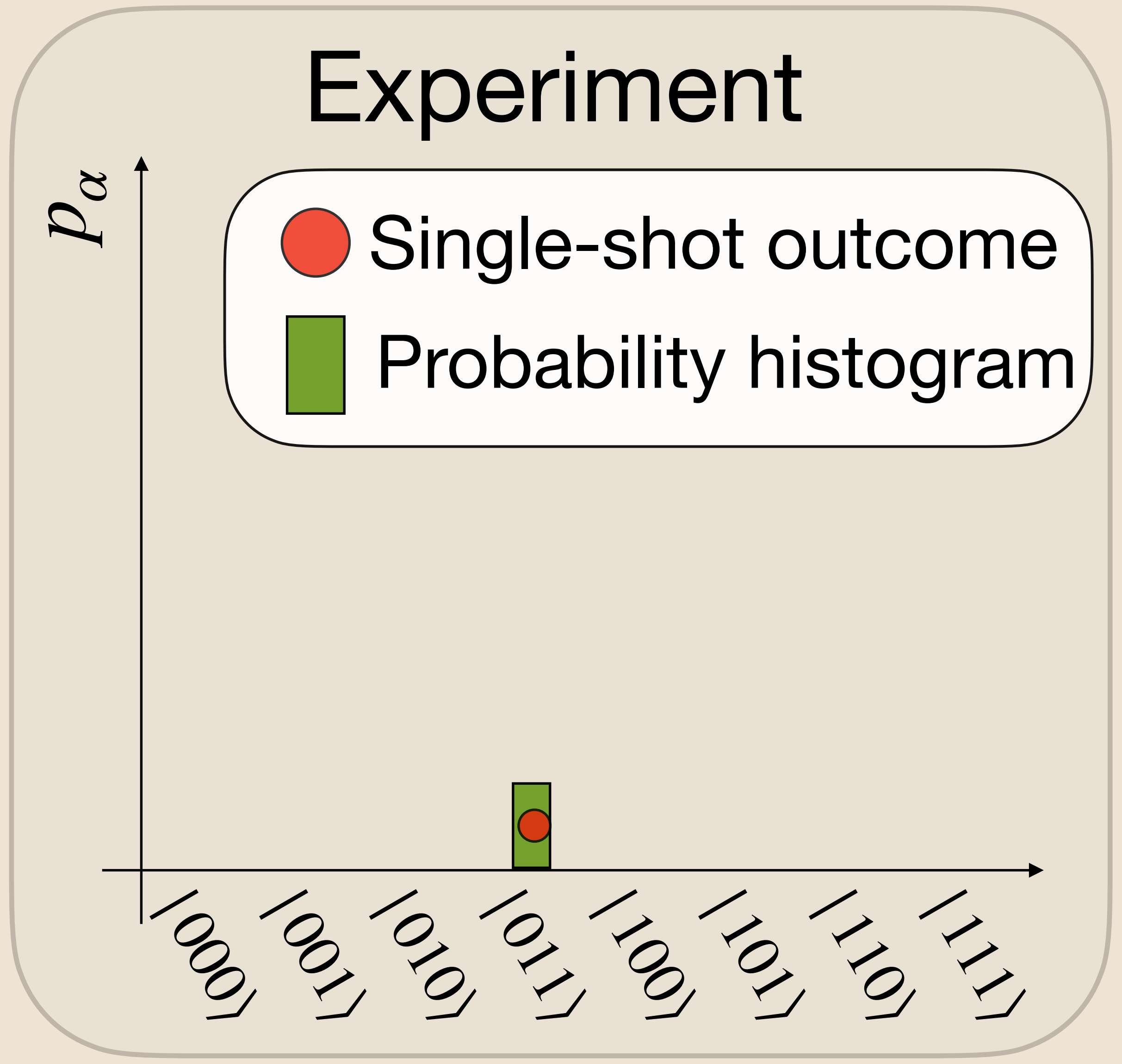

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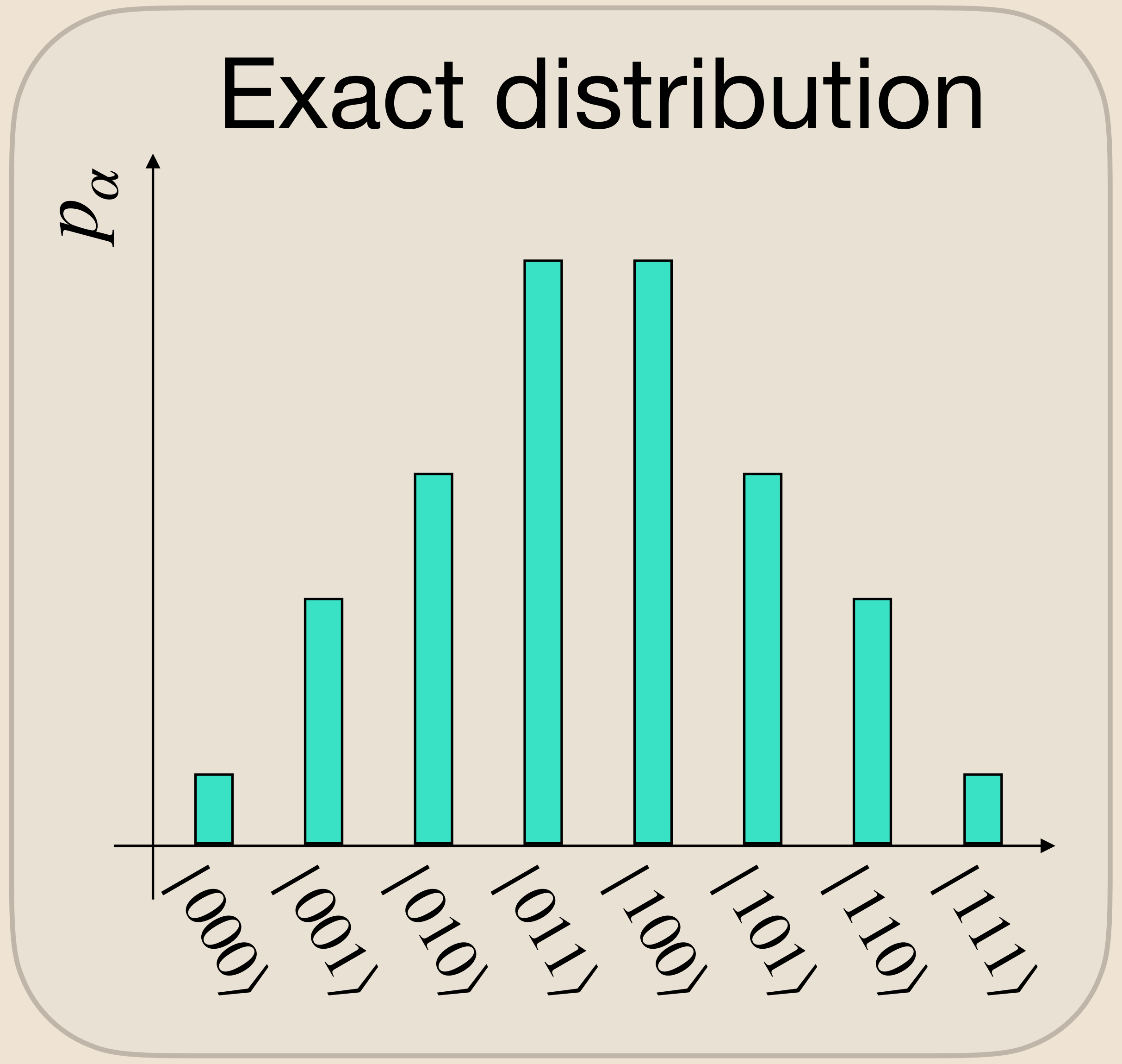

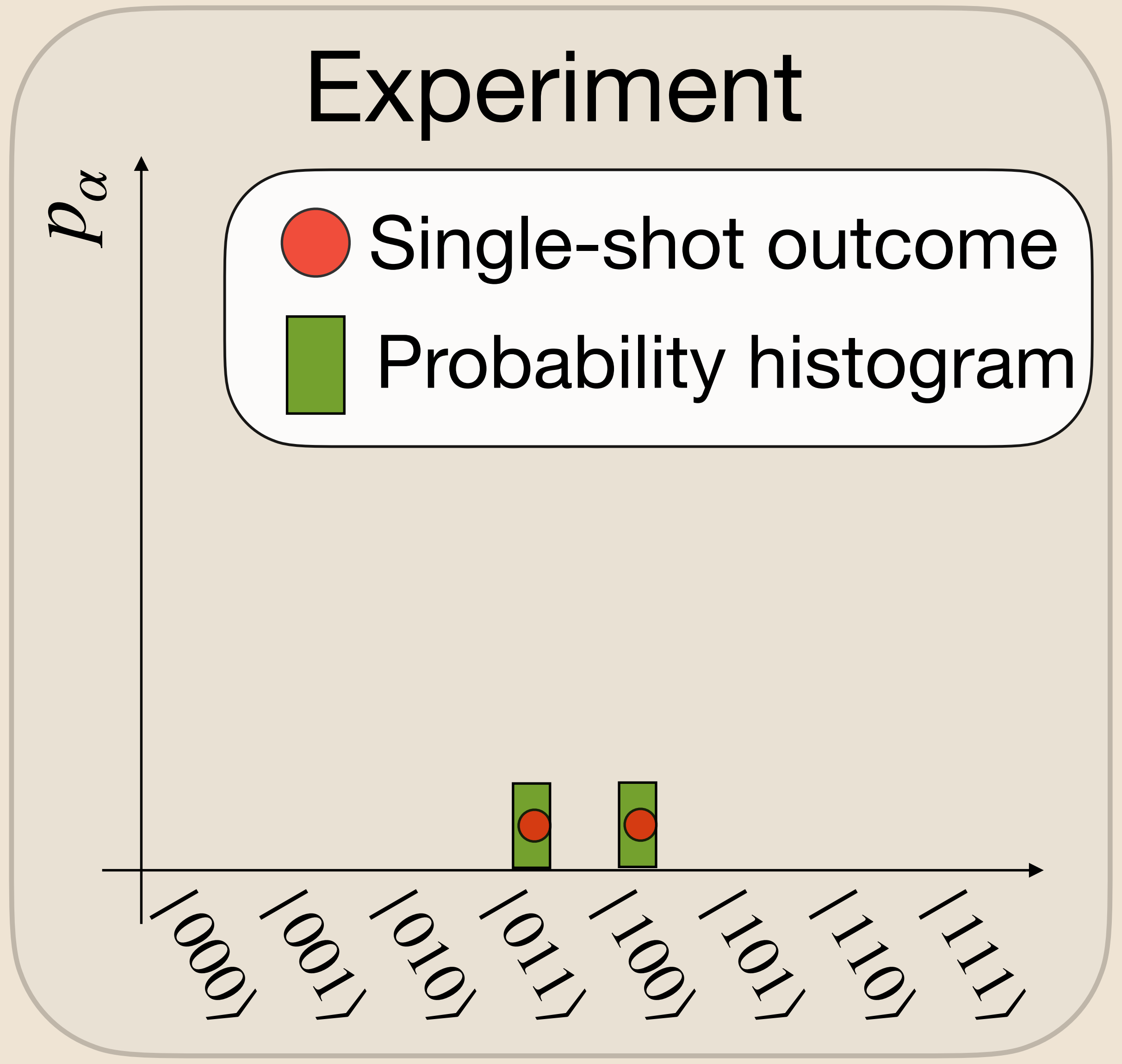

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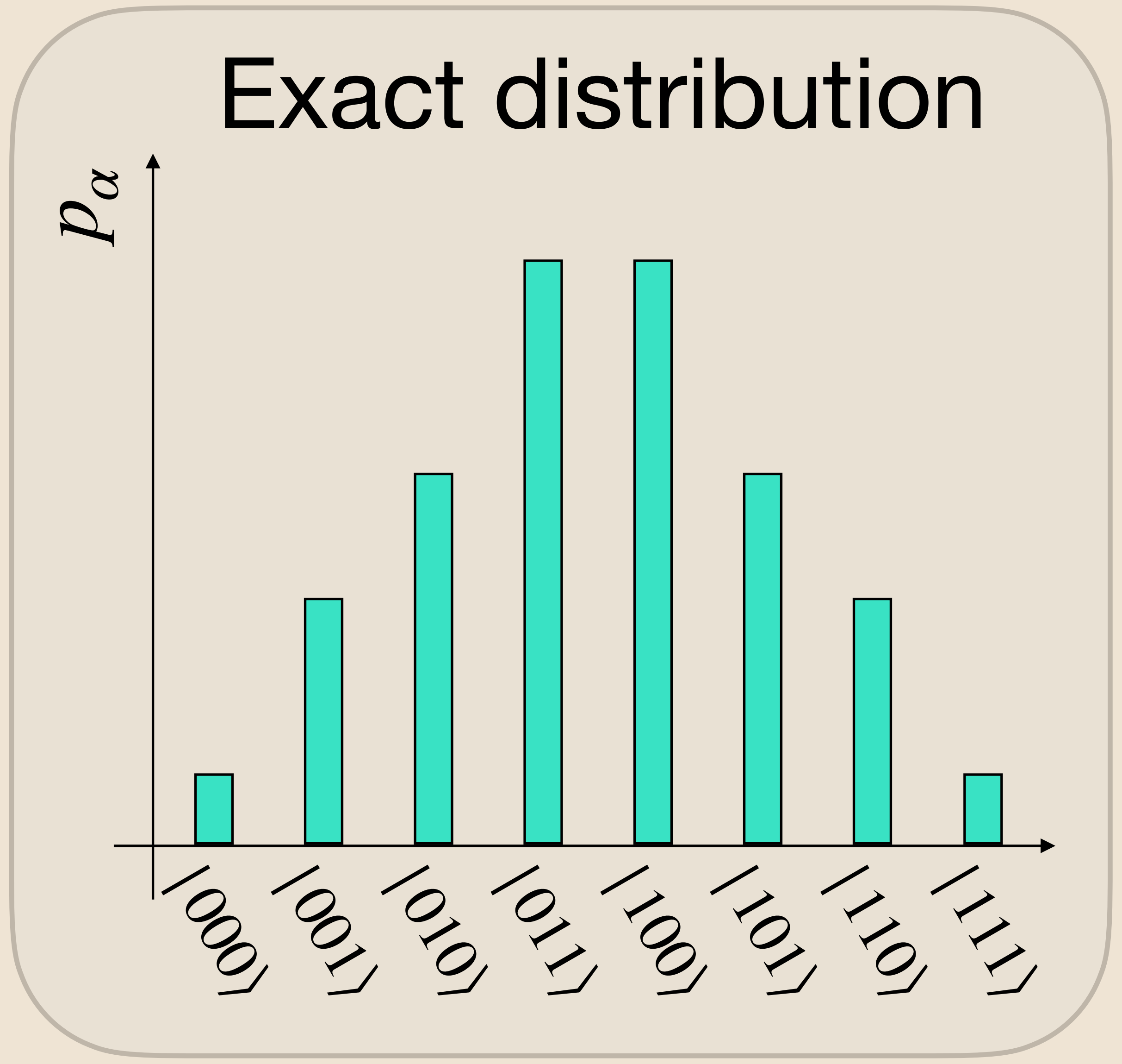

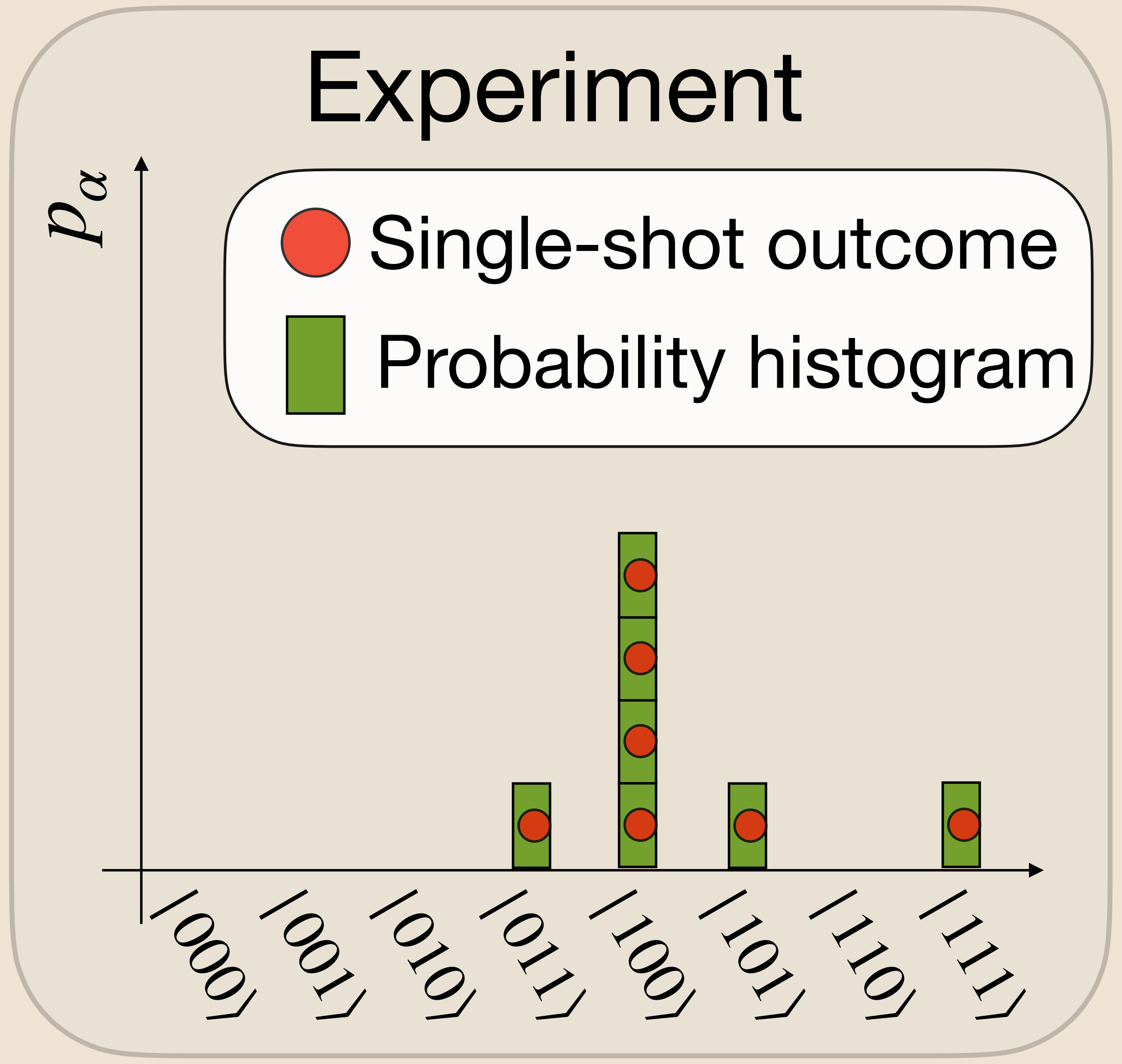

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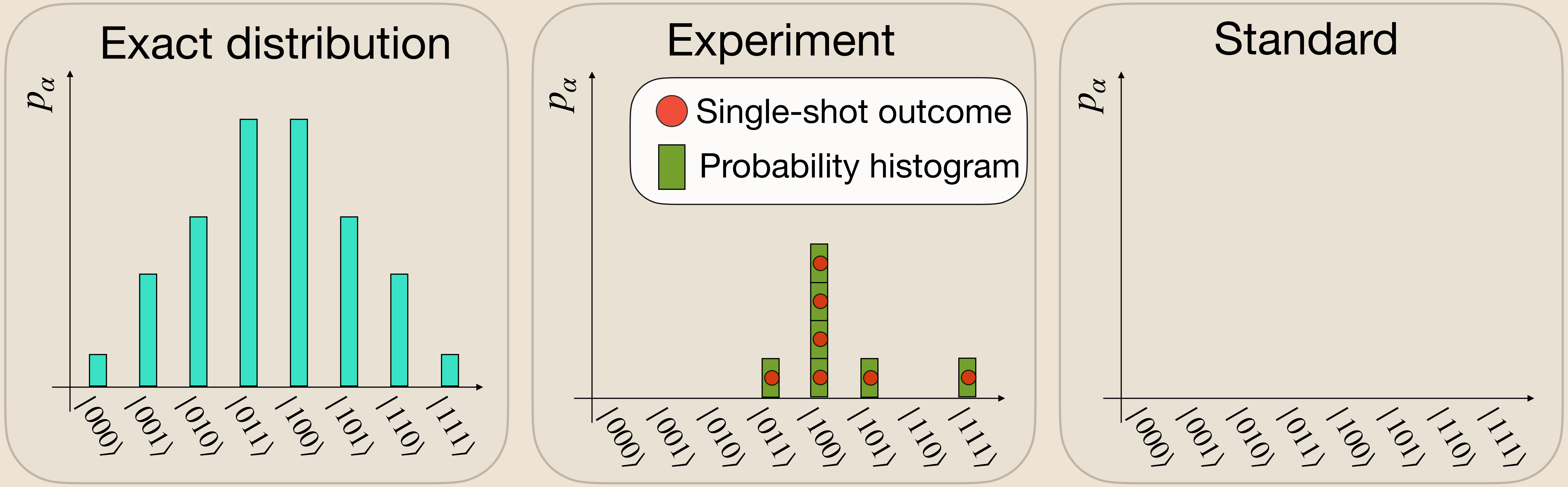

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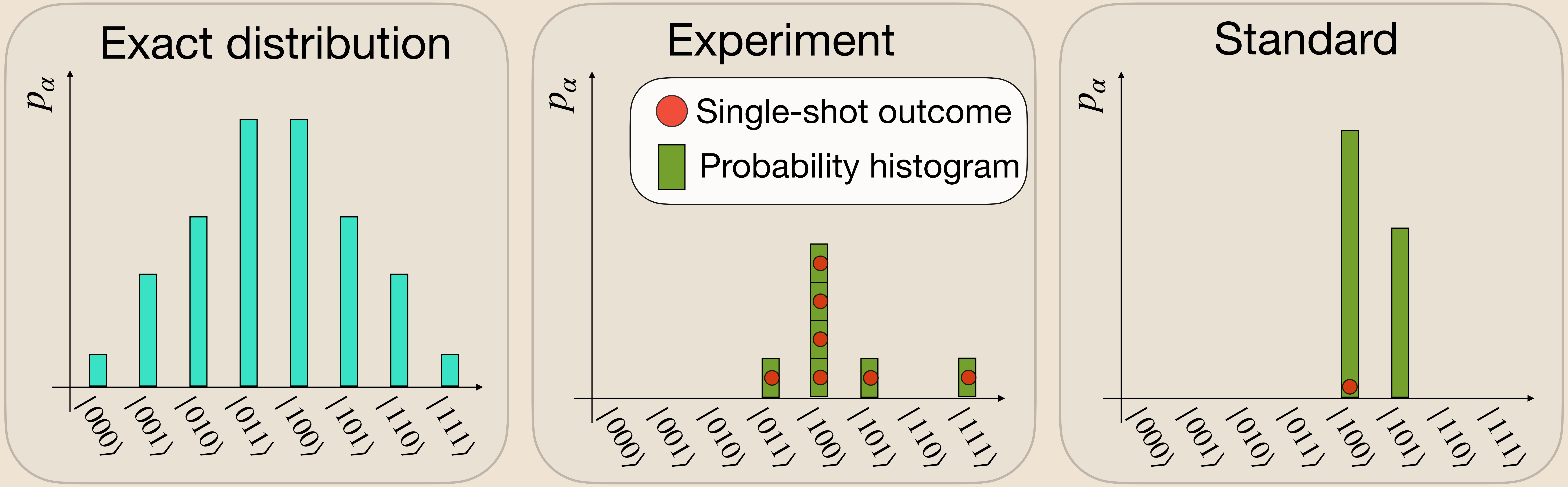

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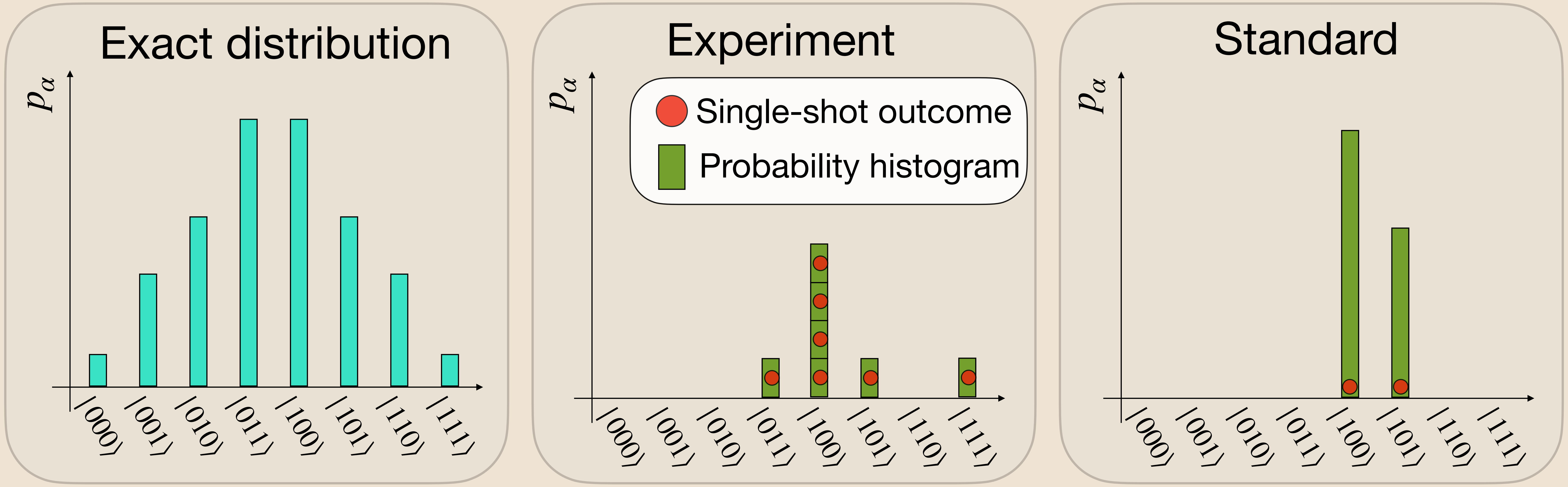

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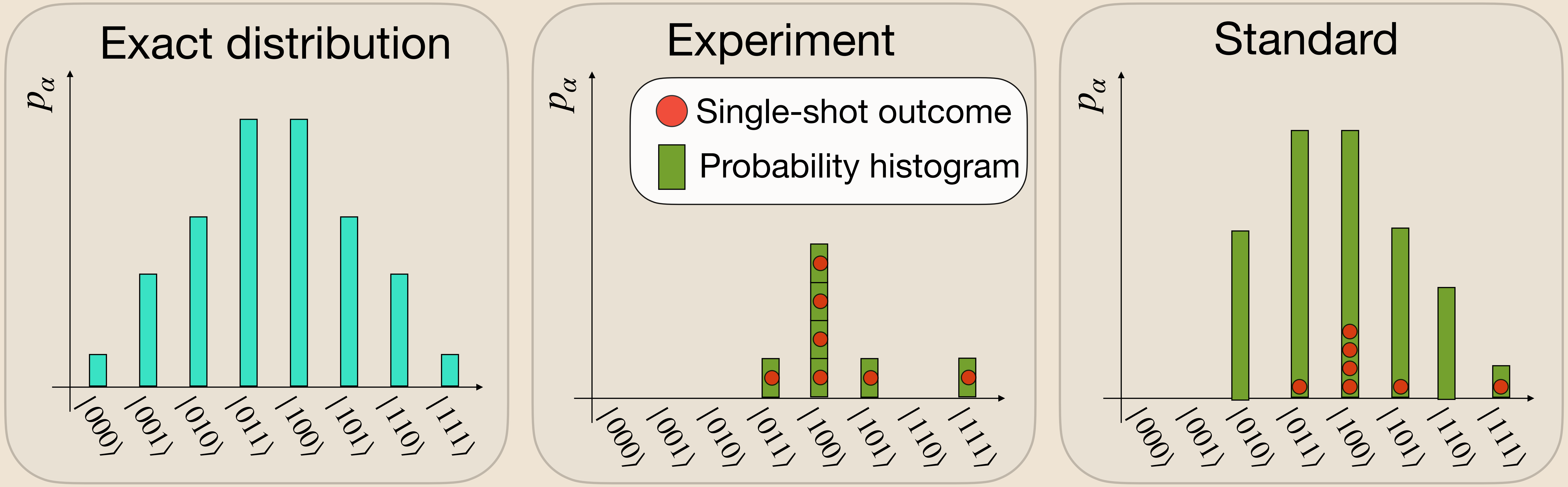

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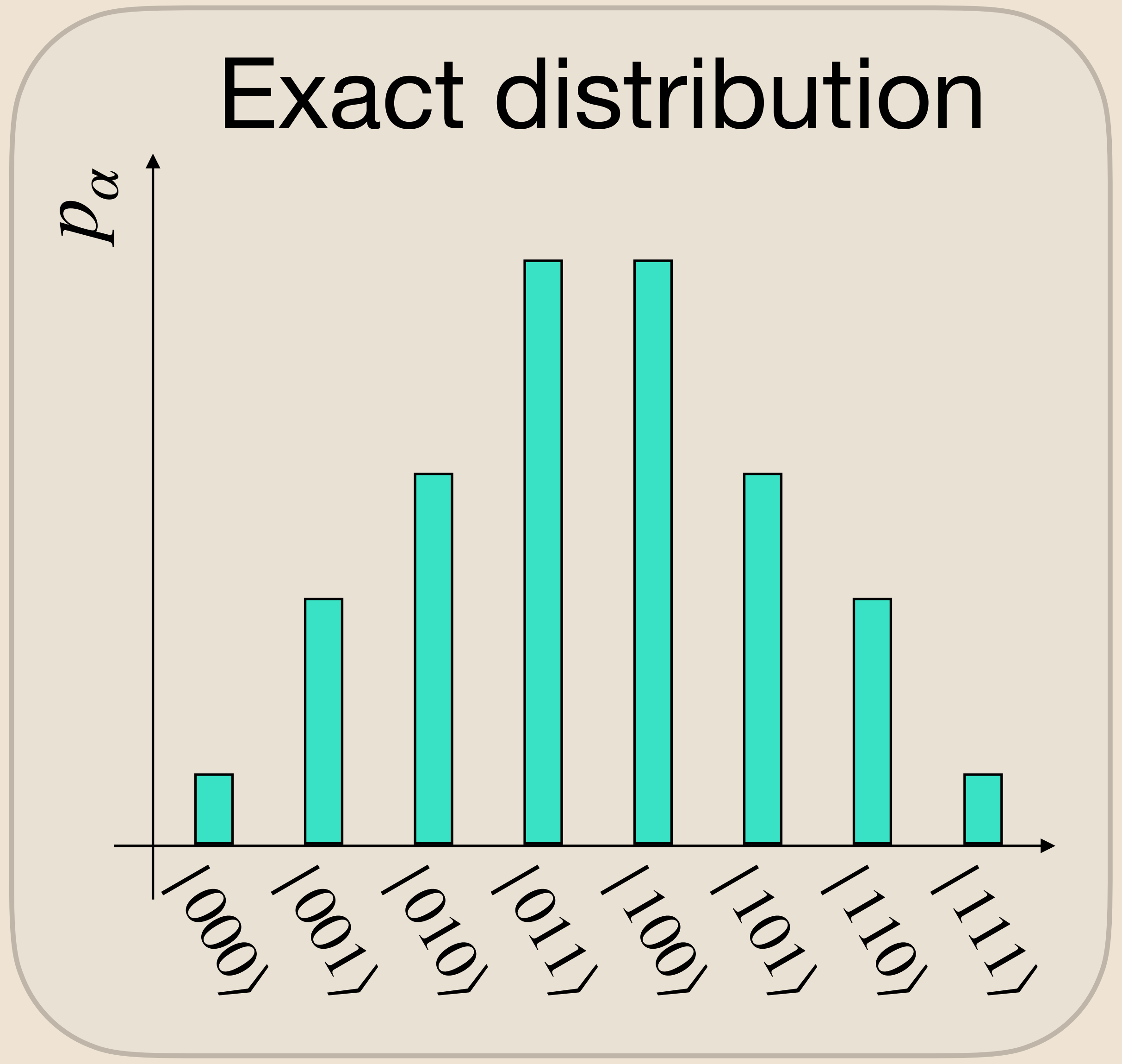

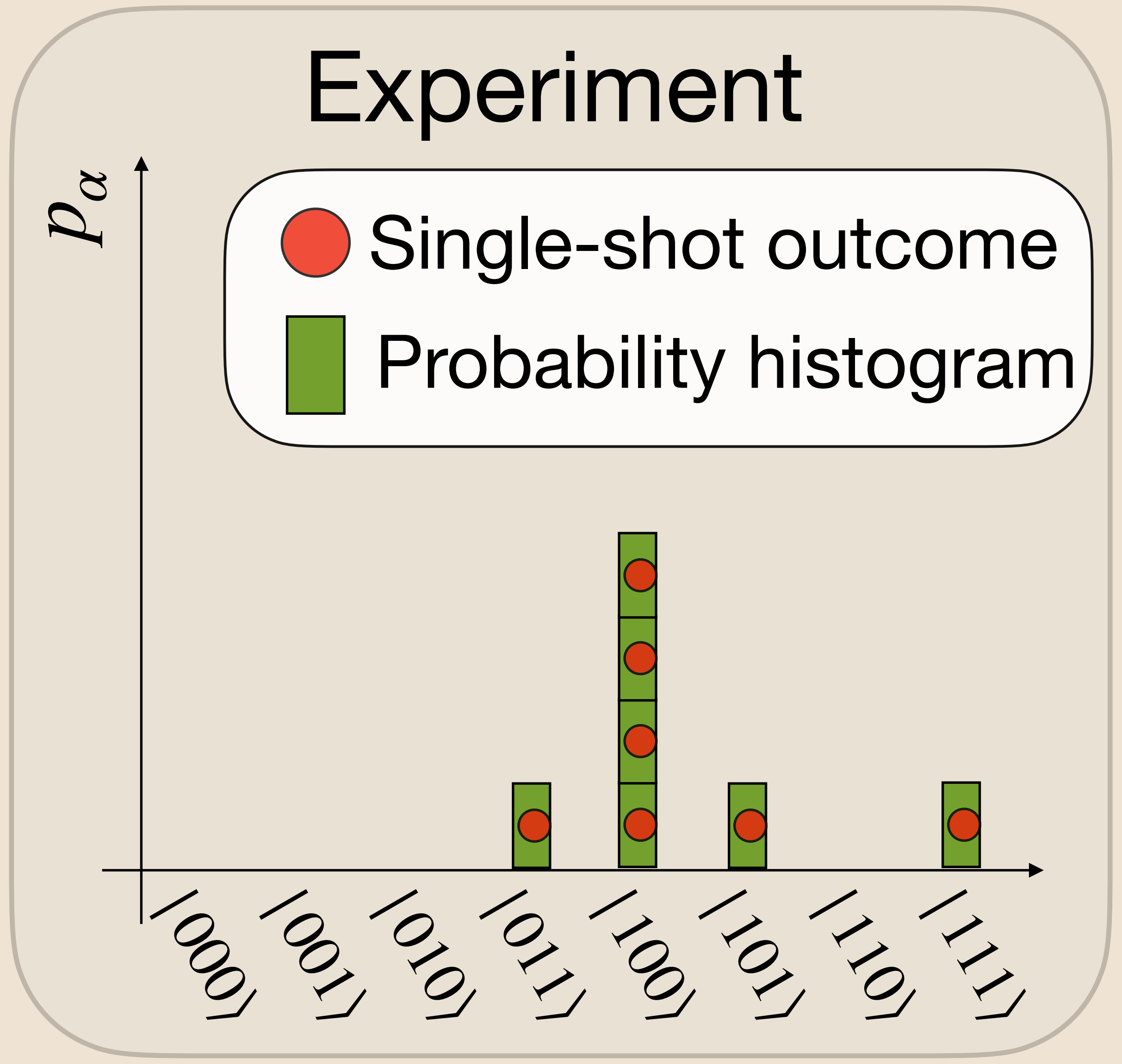

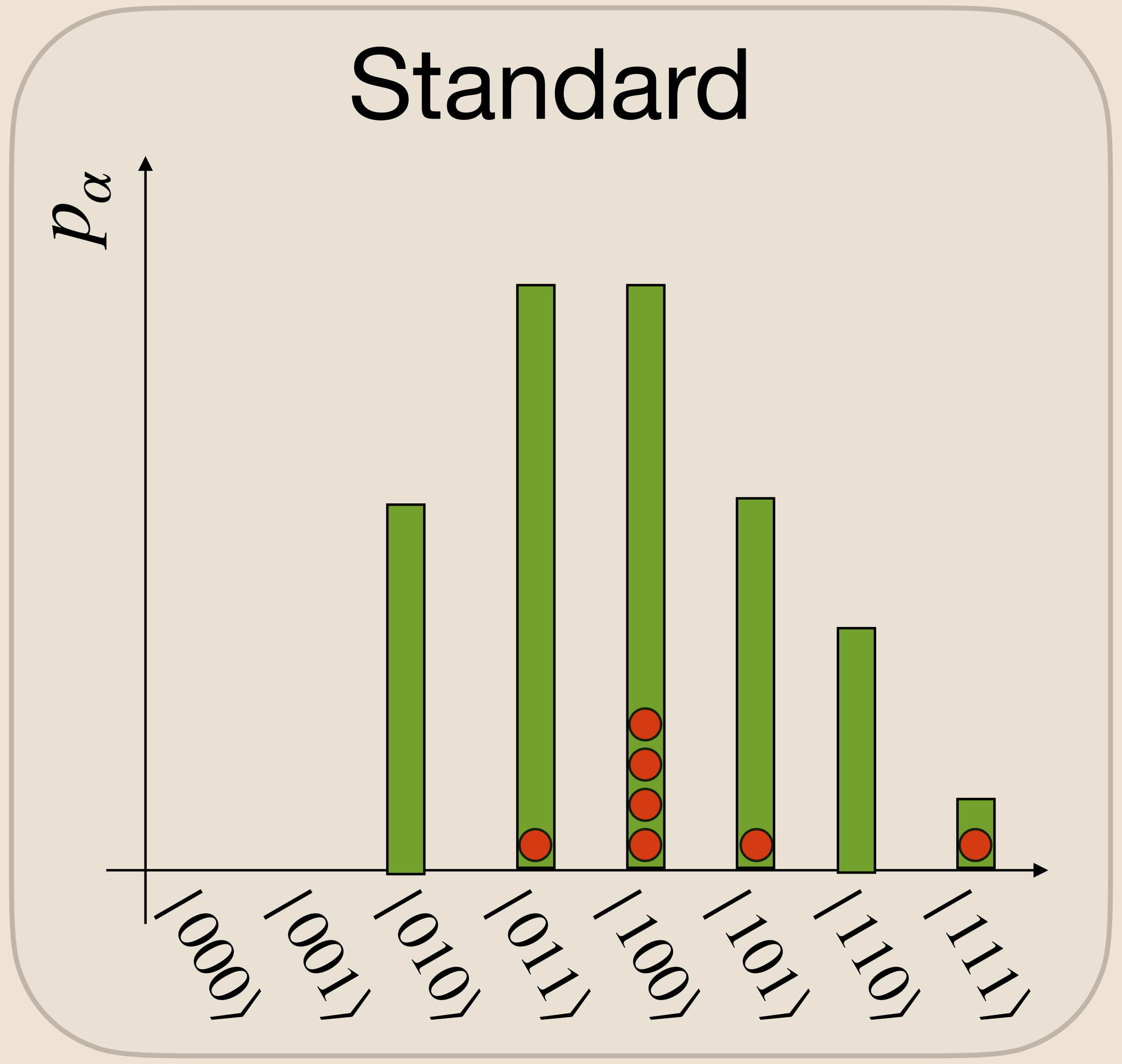

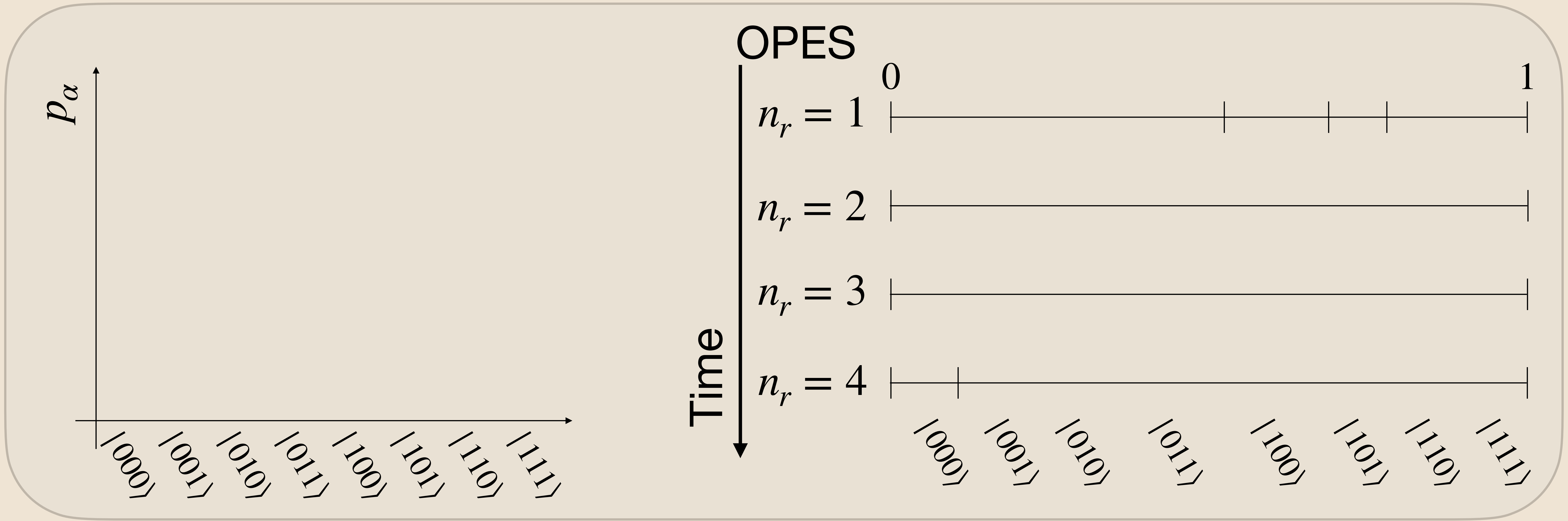

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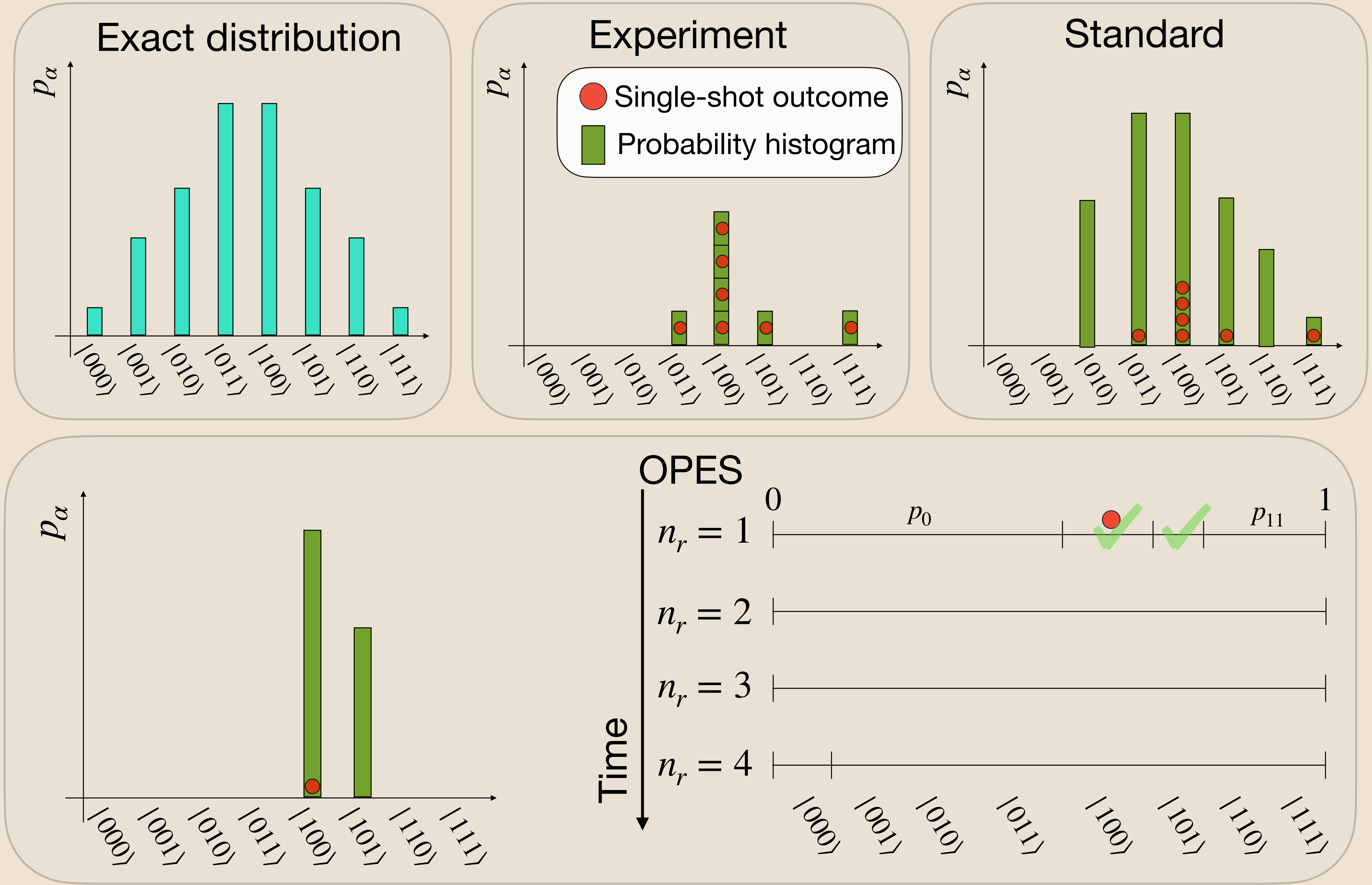

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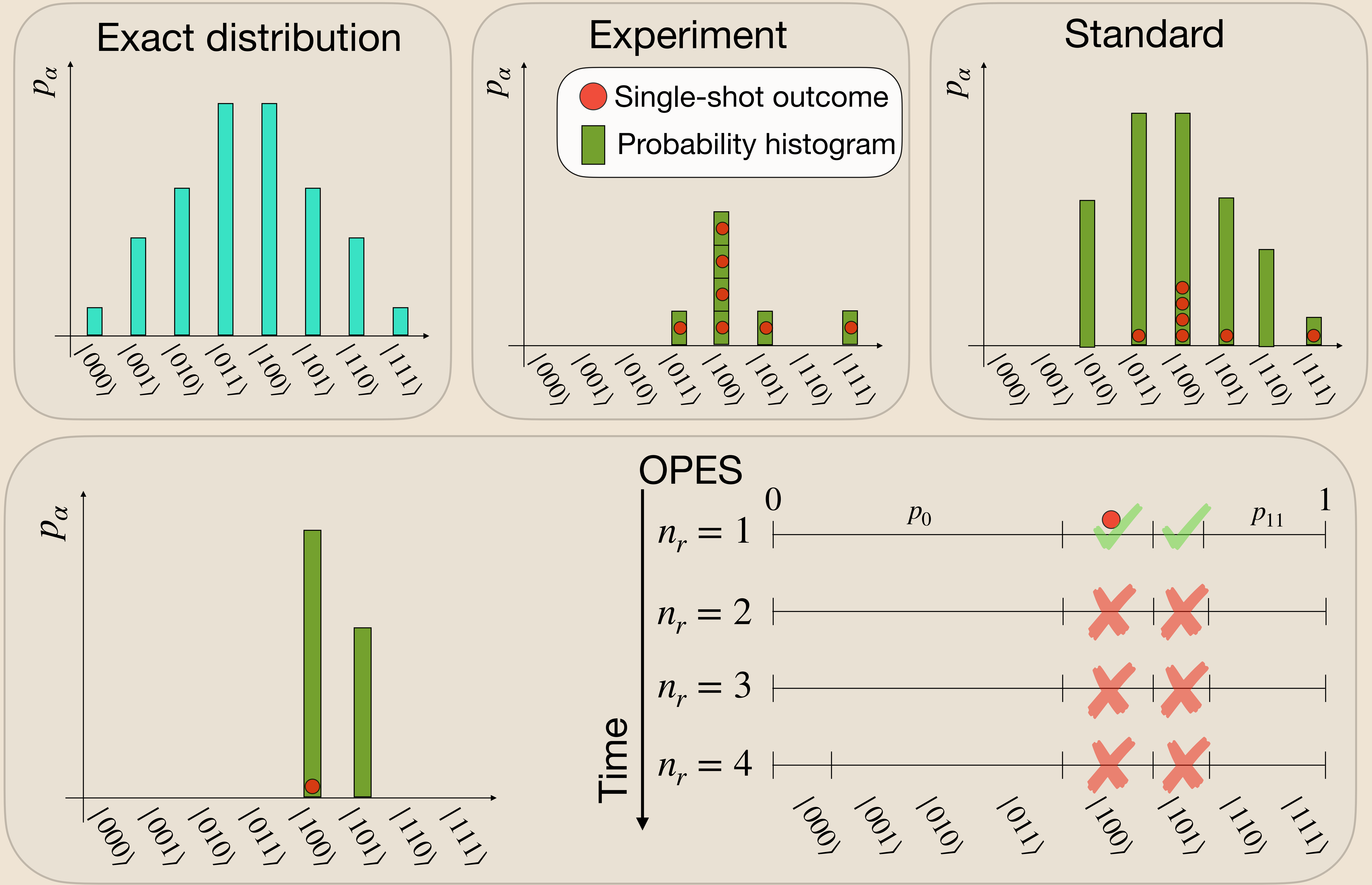

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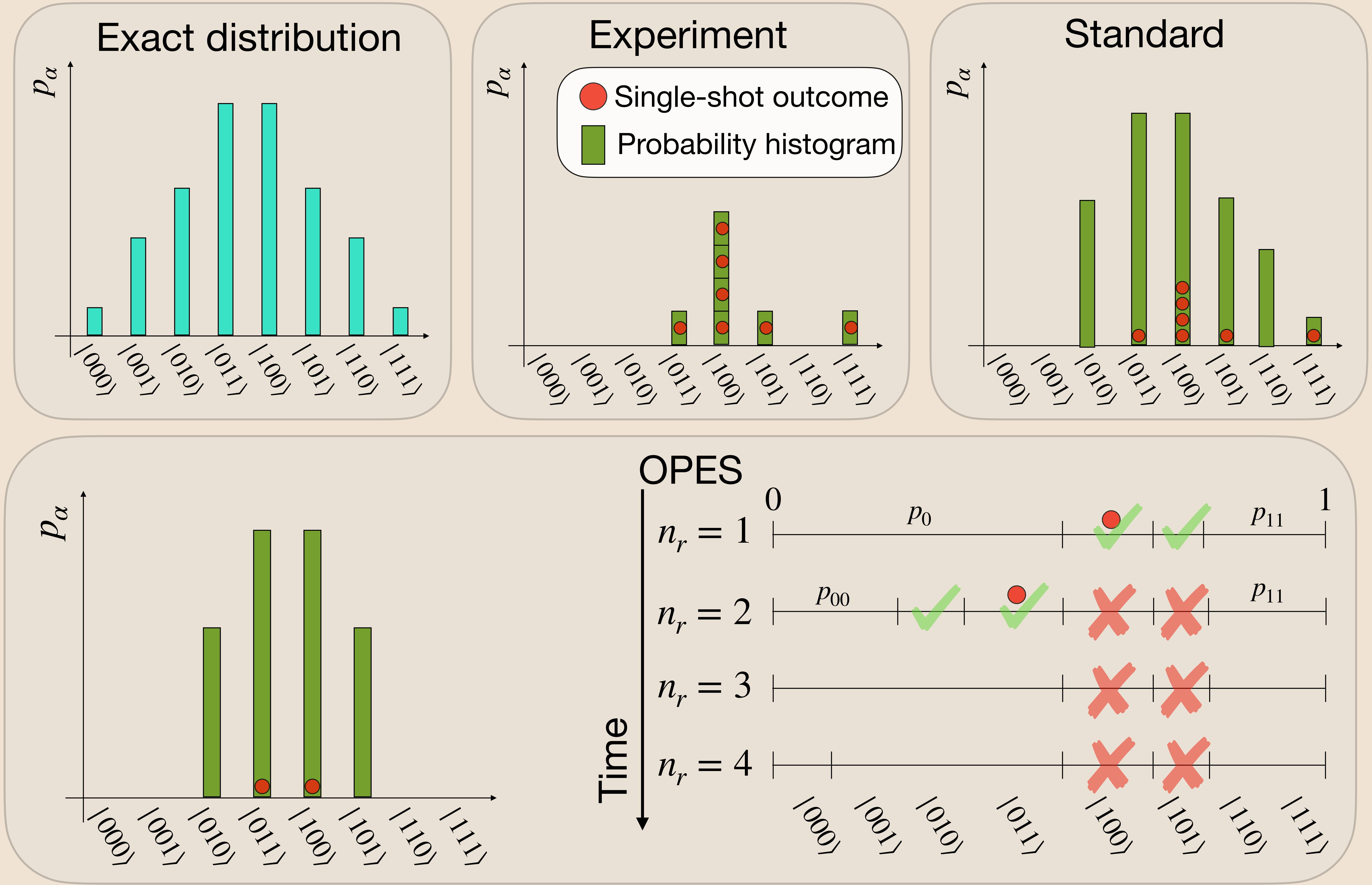

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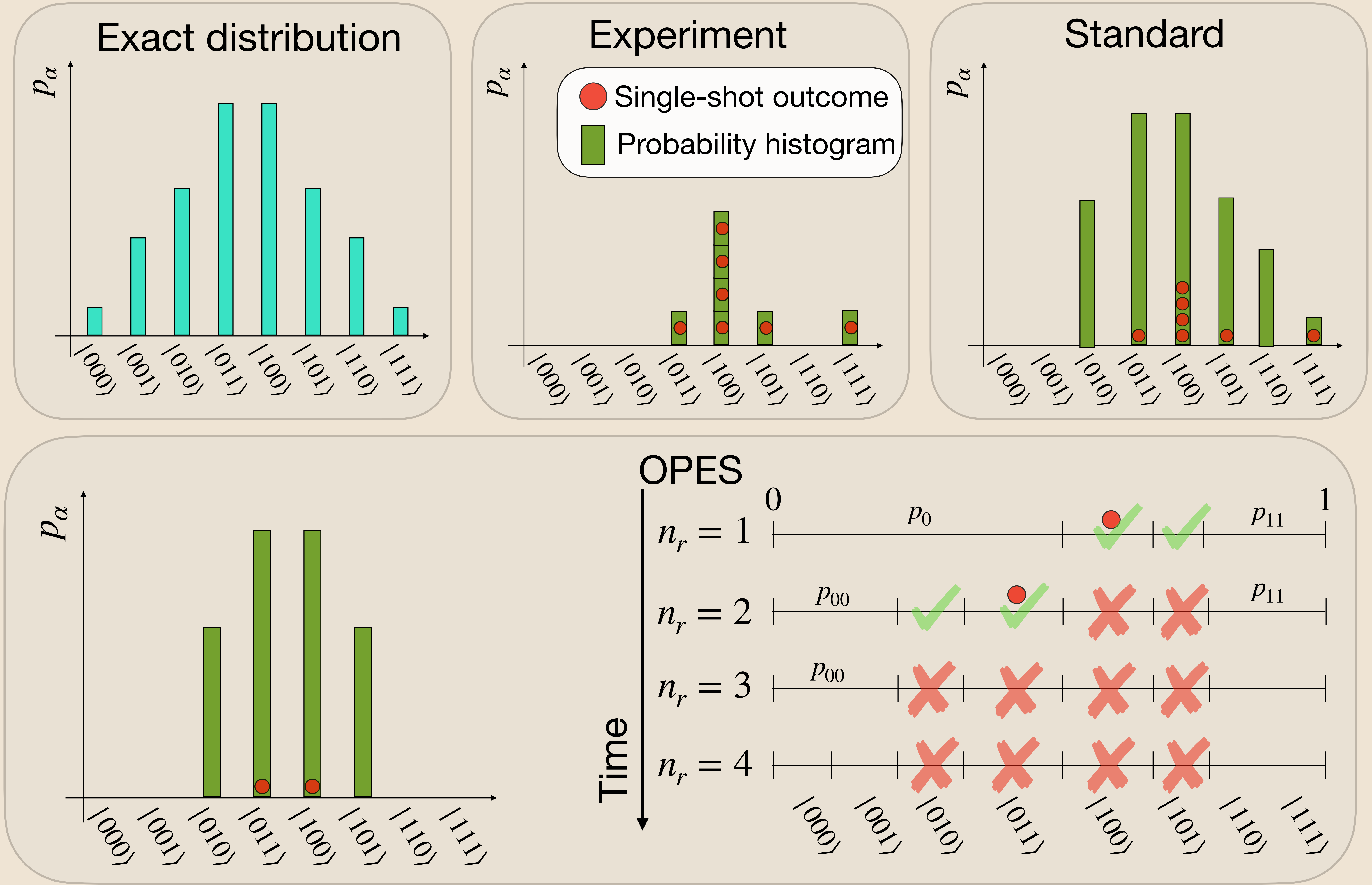

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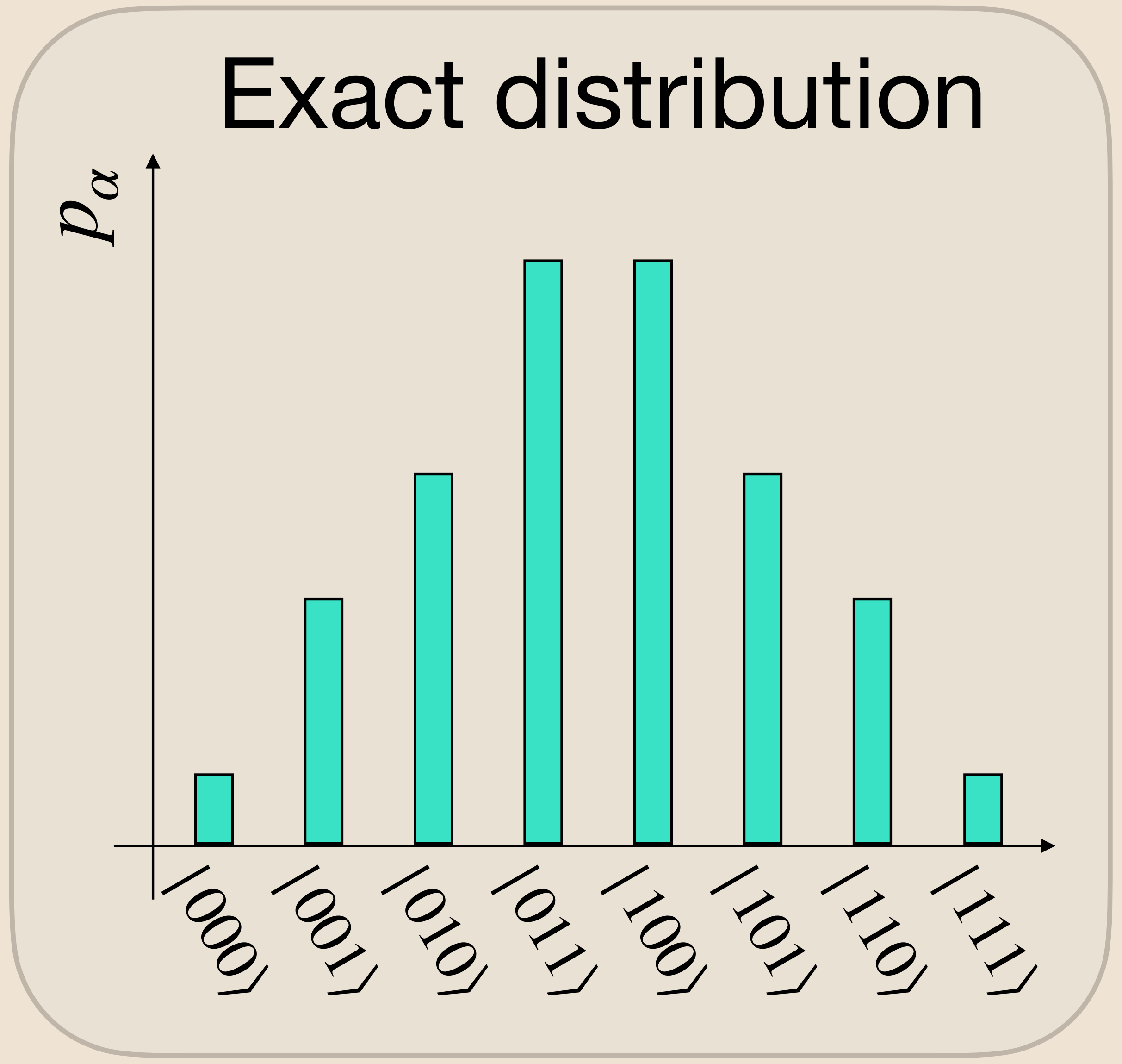

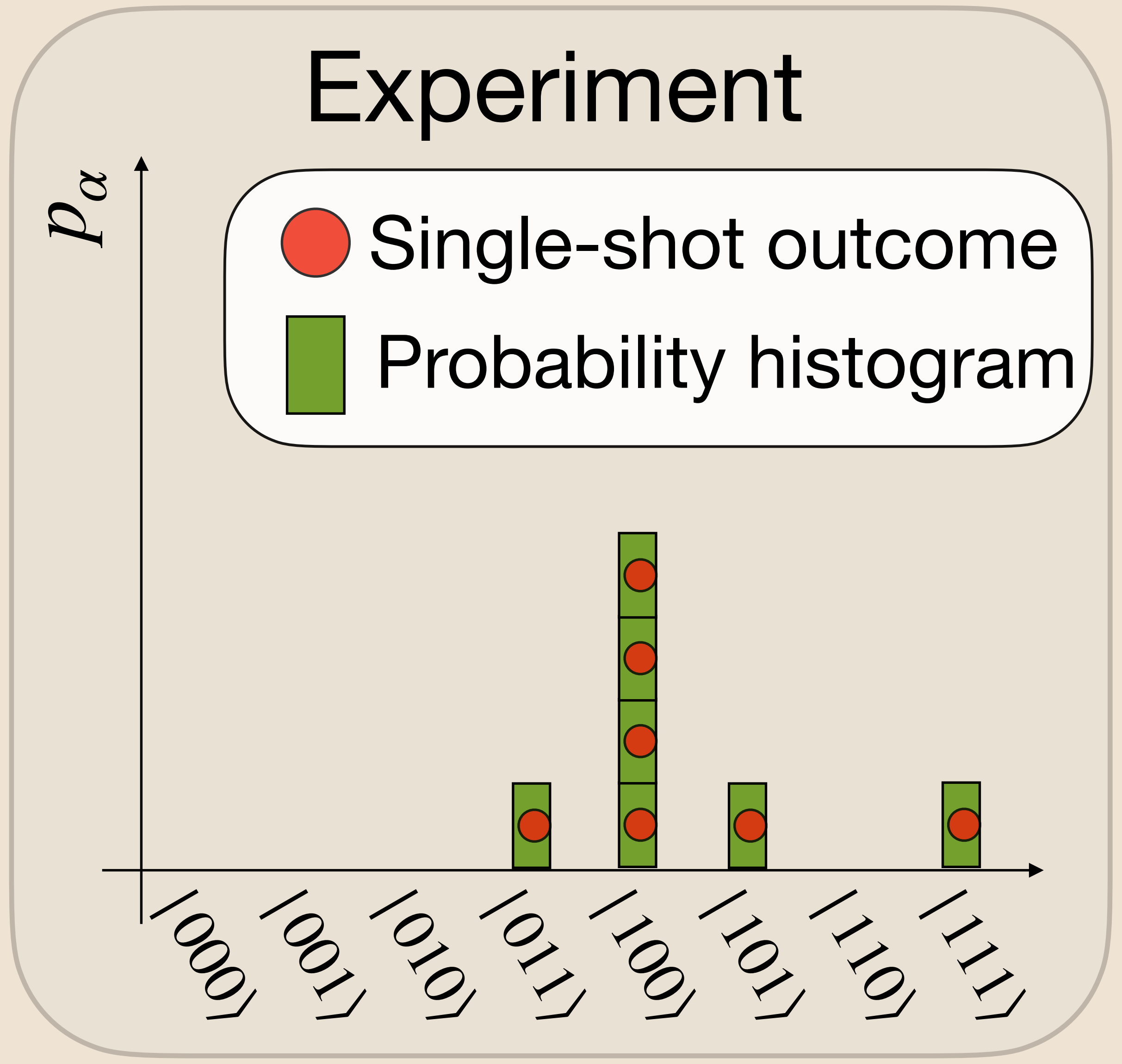

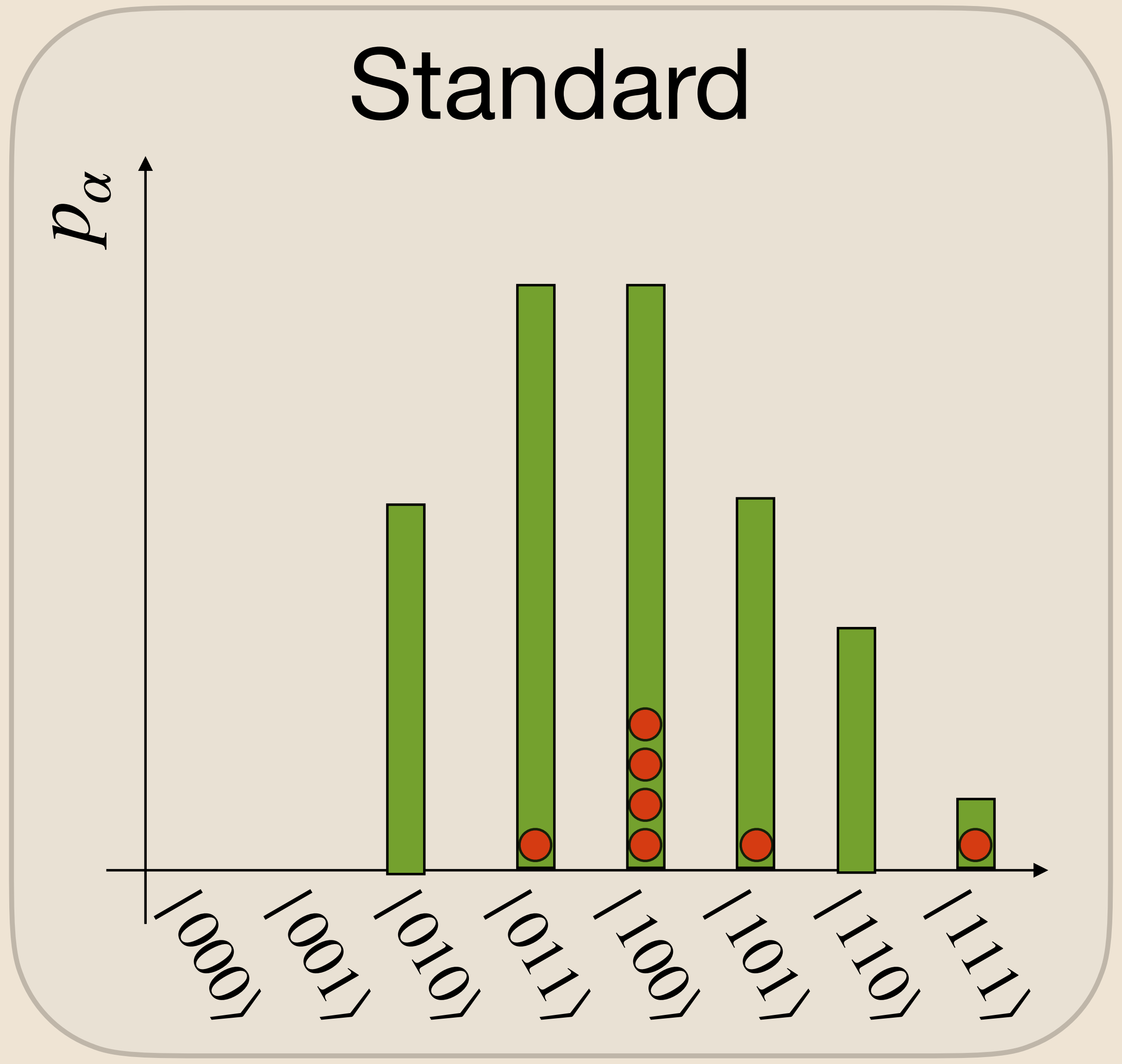

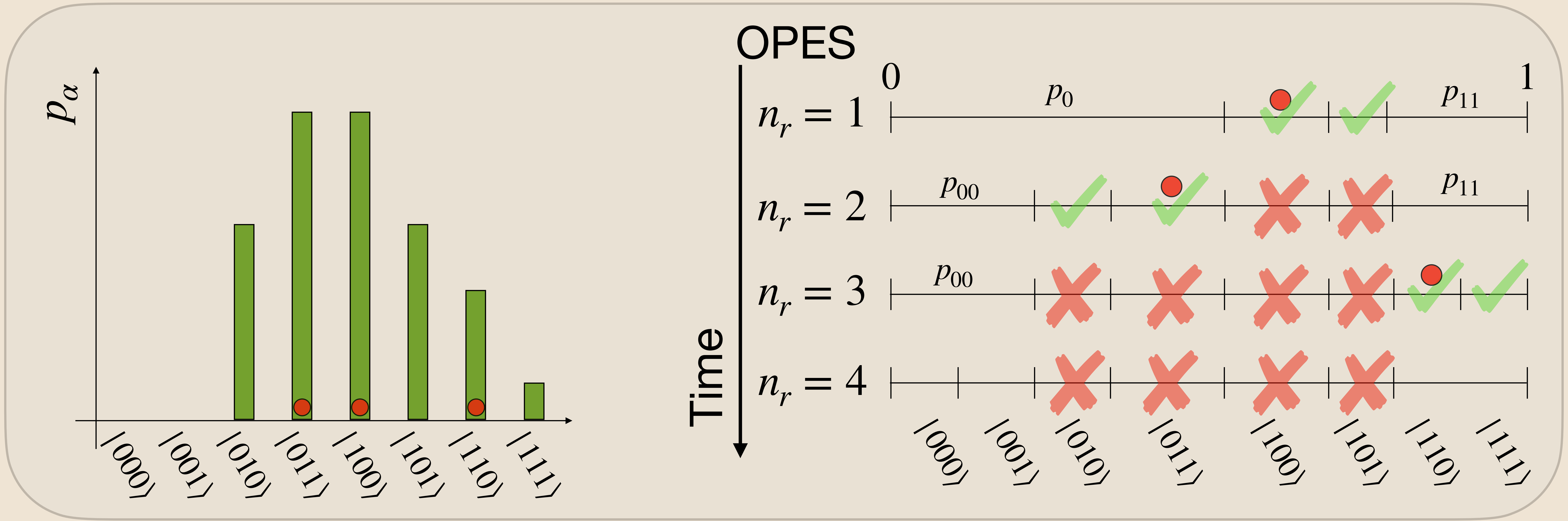

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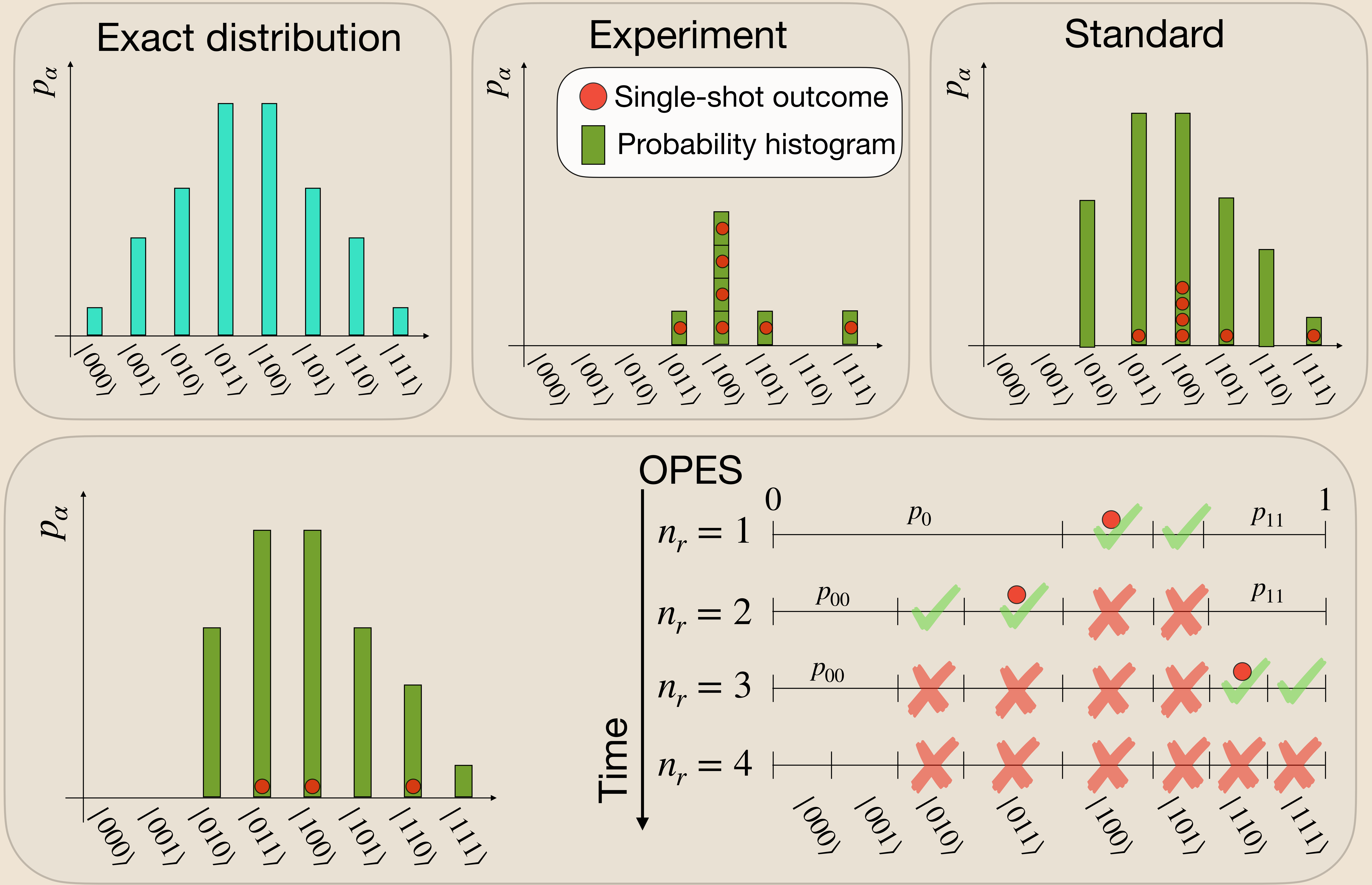

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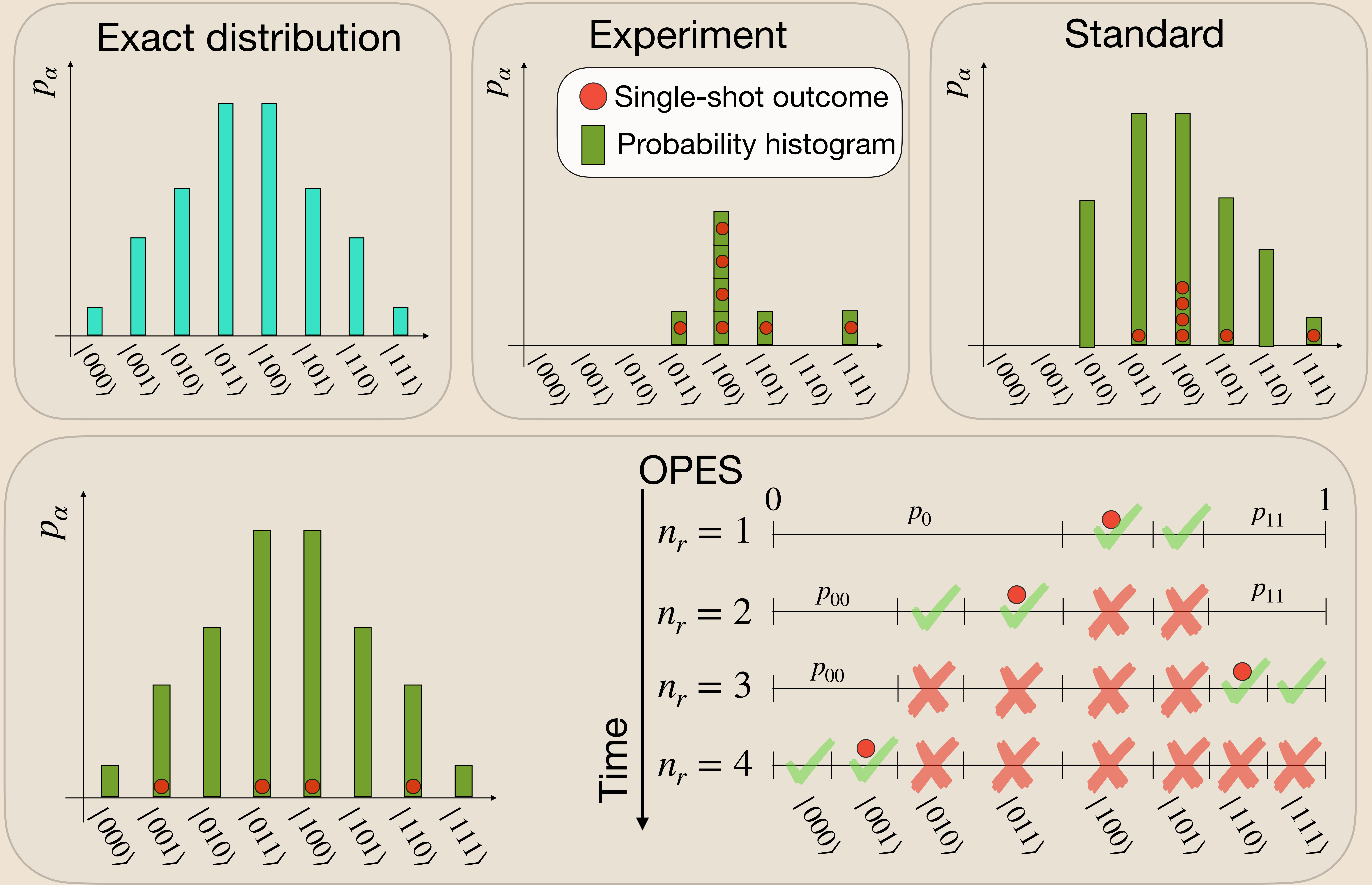
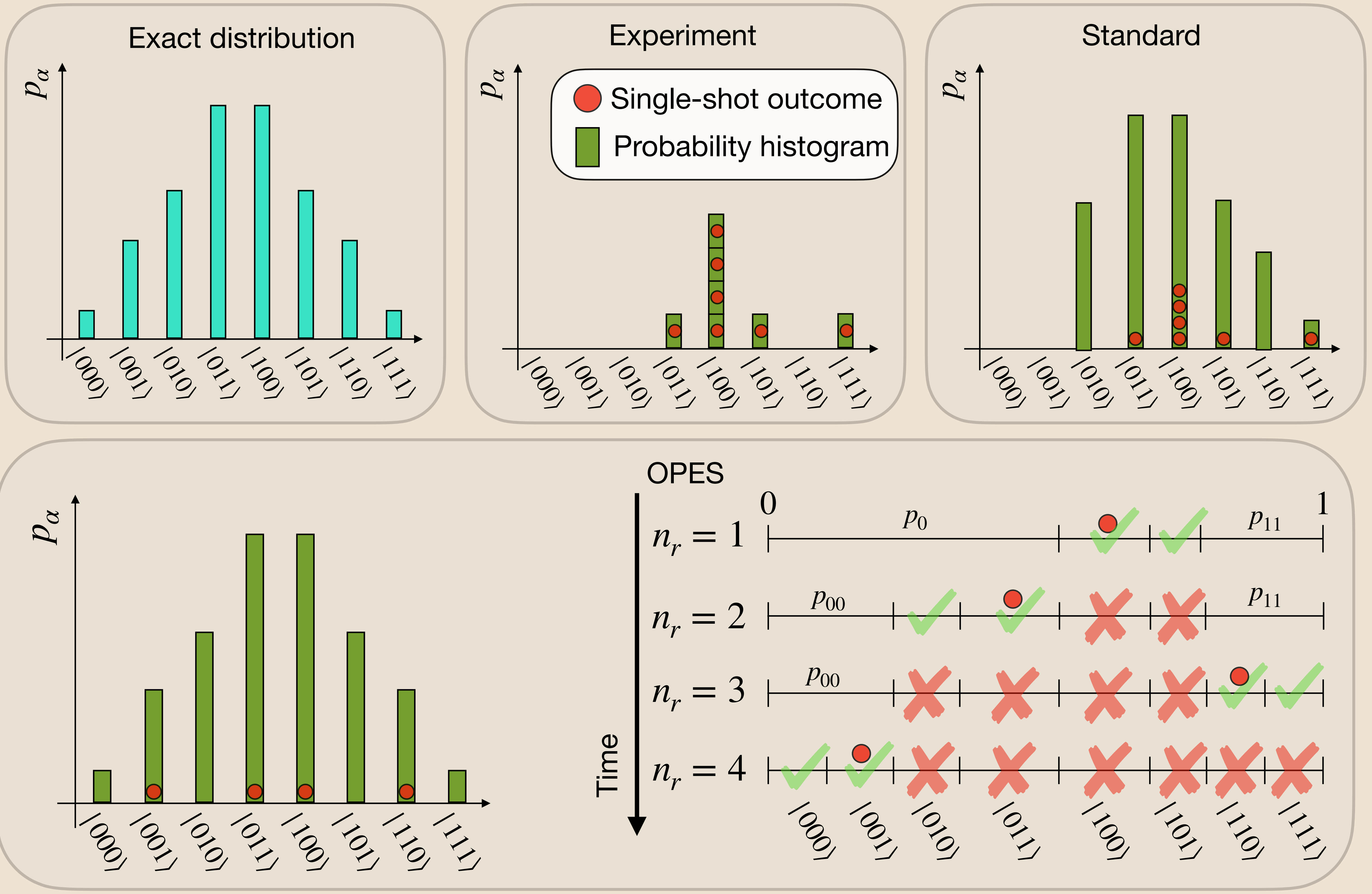
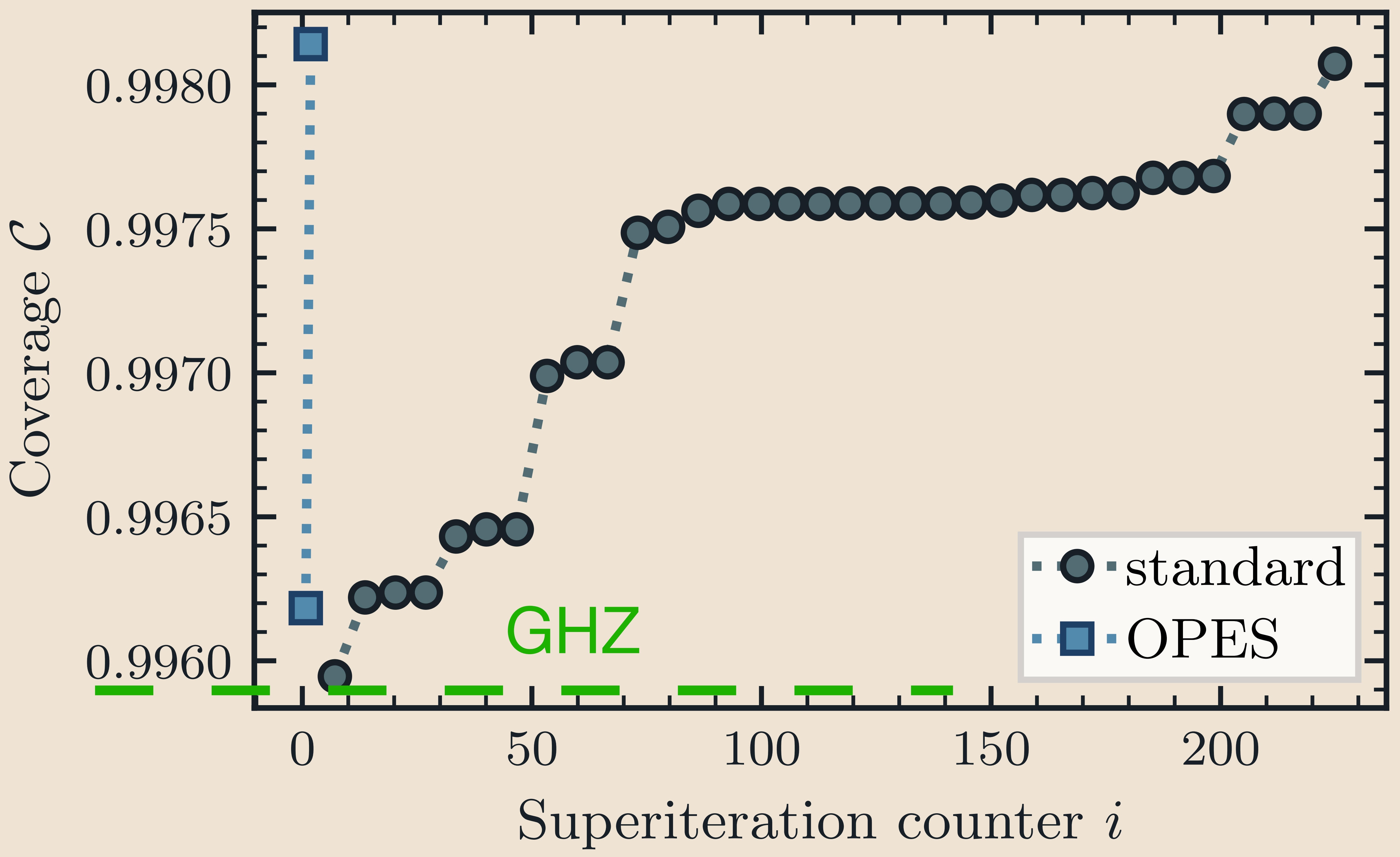


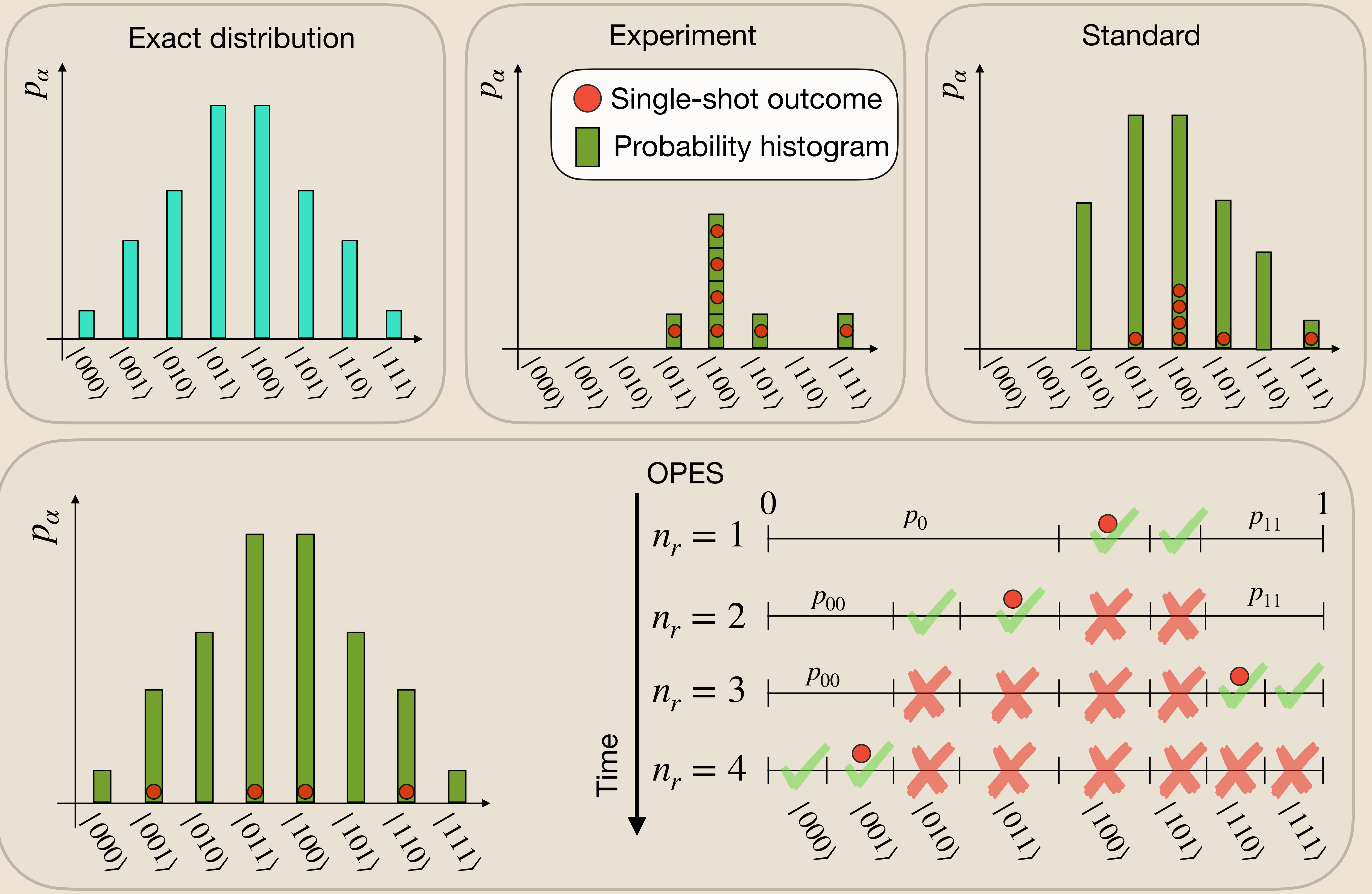
OPES on GHZ + errors

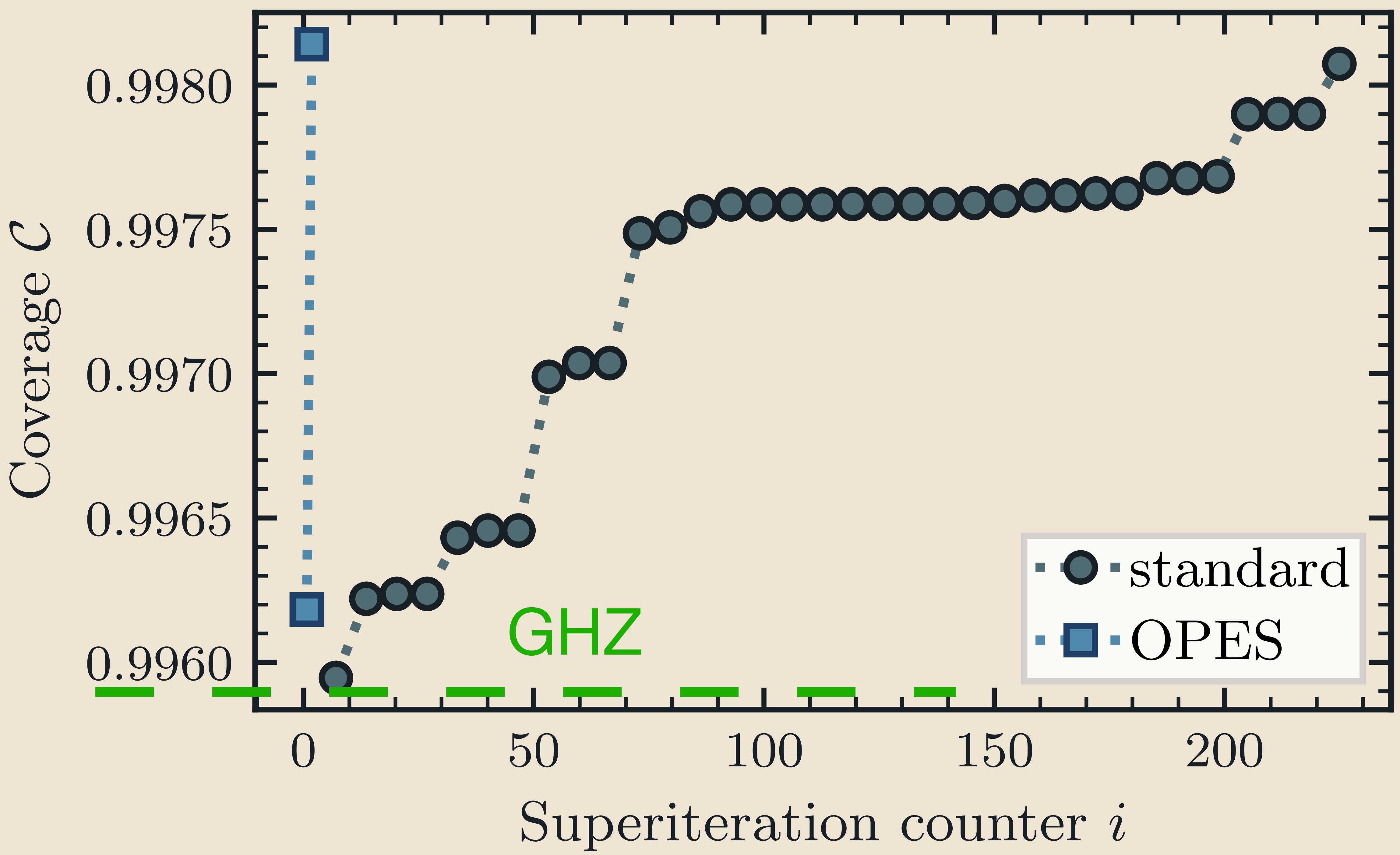




$$|\psi\rangle = \underbrace{\frac{1}{\sqrt{2+\epsilon}}\left(|00\ldots0\rangle + |11\ldots1\rangle\right)}_{\text{GHZ}} + \sqrt{\frac{\epsilon}{N(2+\epsilon)}}\sum_{\alpha=1}^{N}|\alpha\rangle$$

OPES on GHZ + errors





GHZ

Small contributions due to noise $\epsilon \ll 1$

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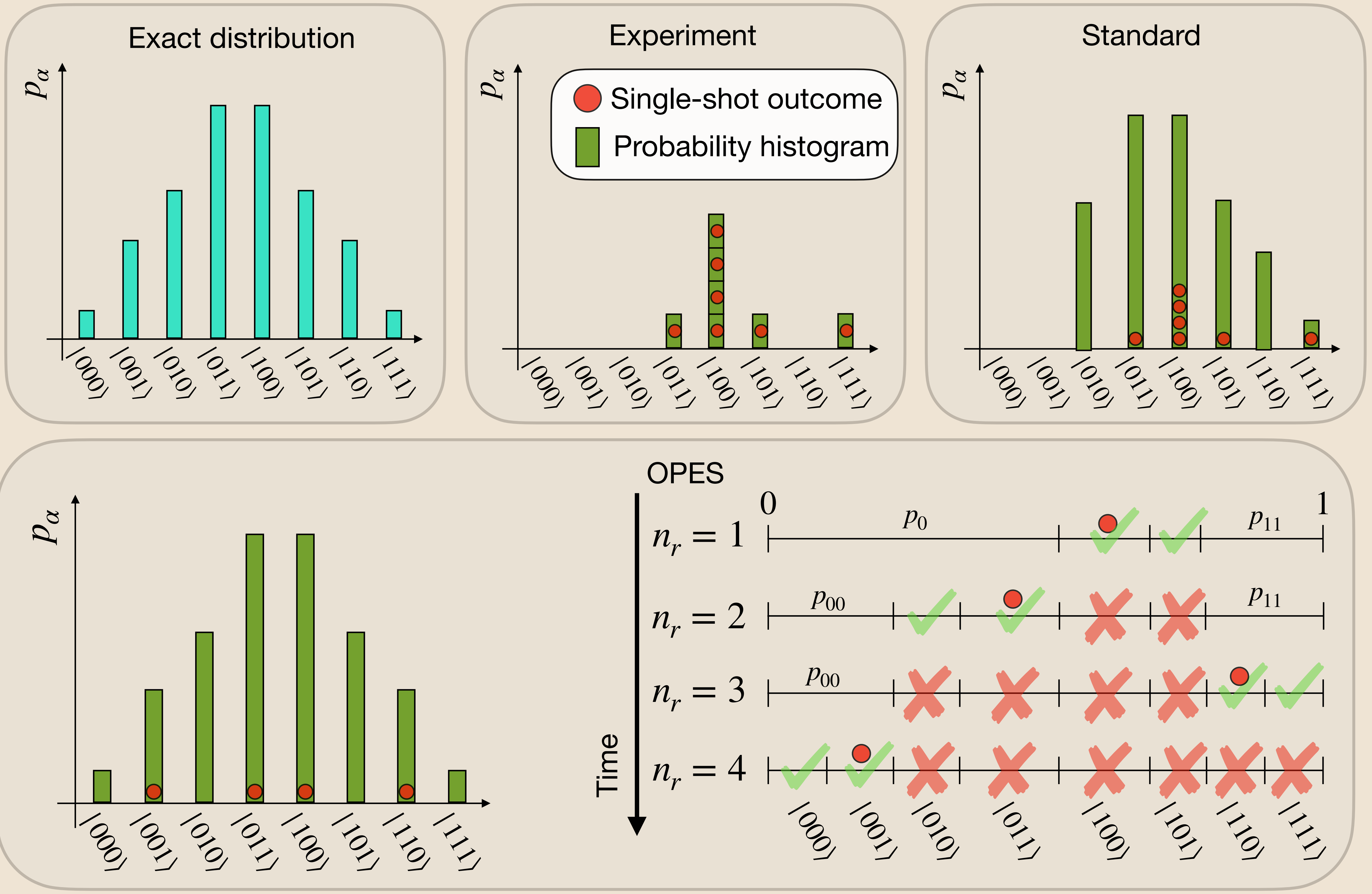
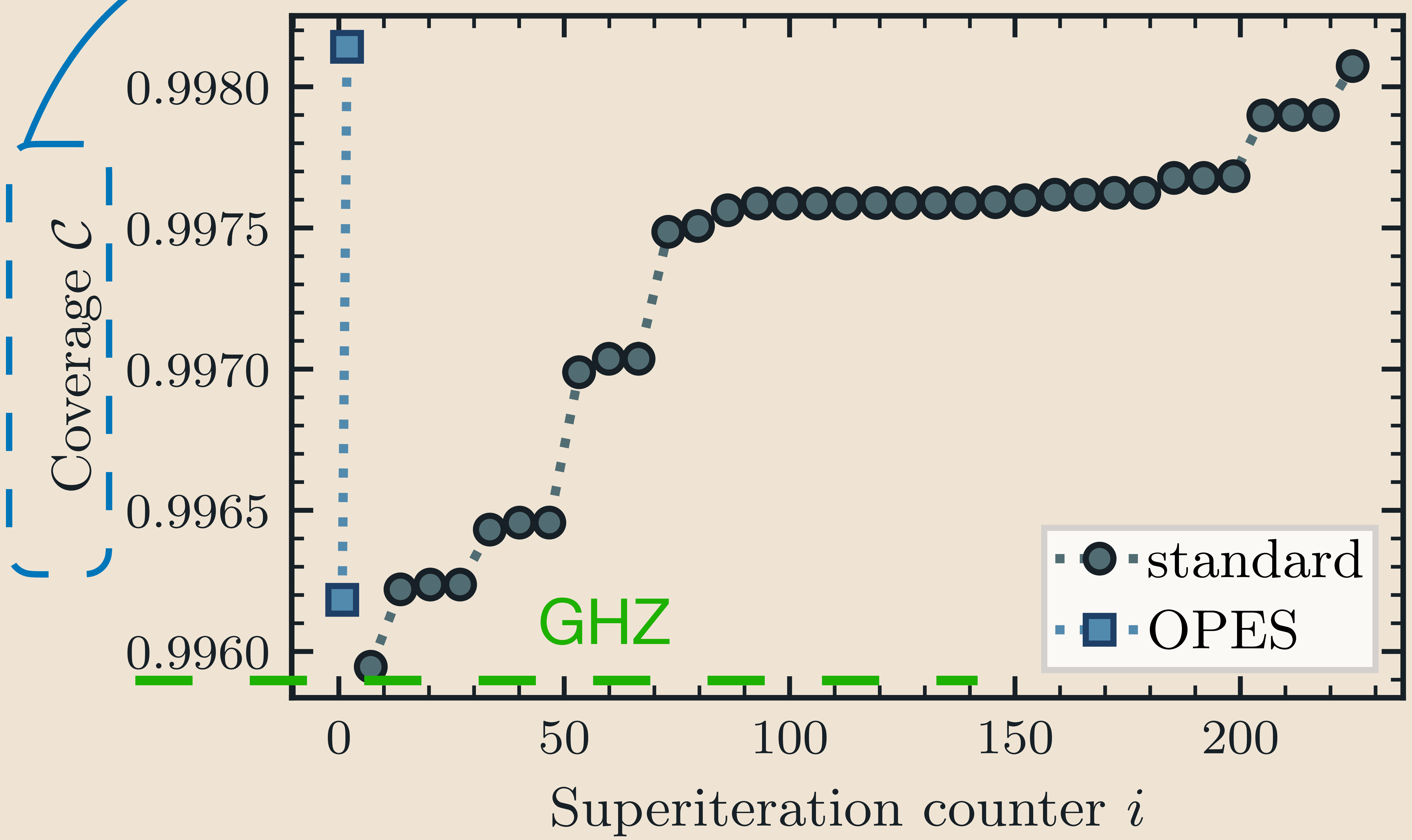


Sum of known probability.
The maximum is 1.



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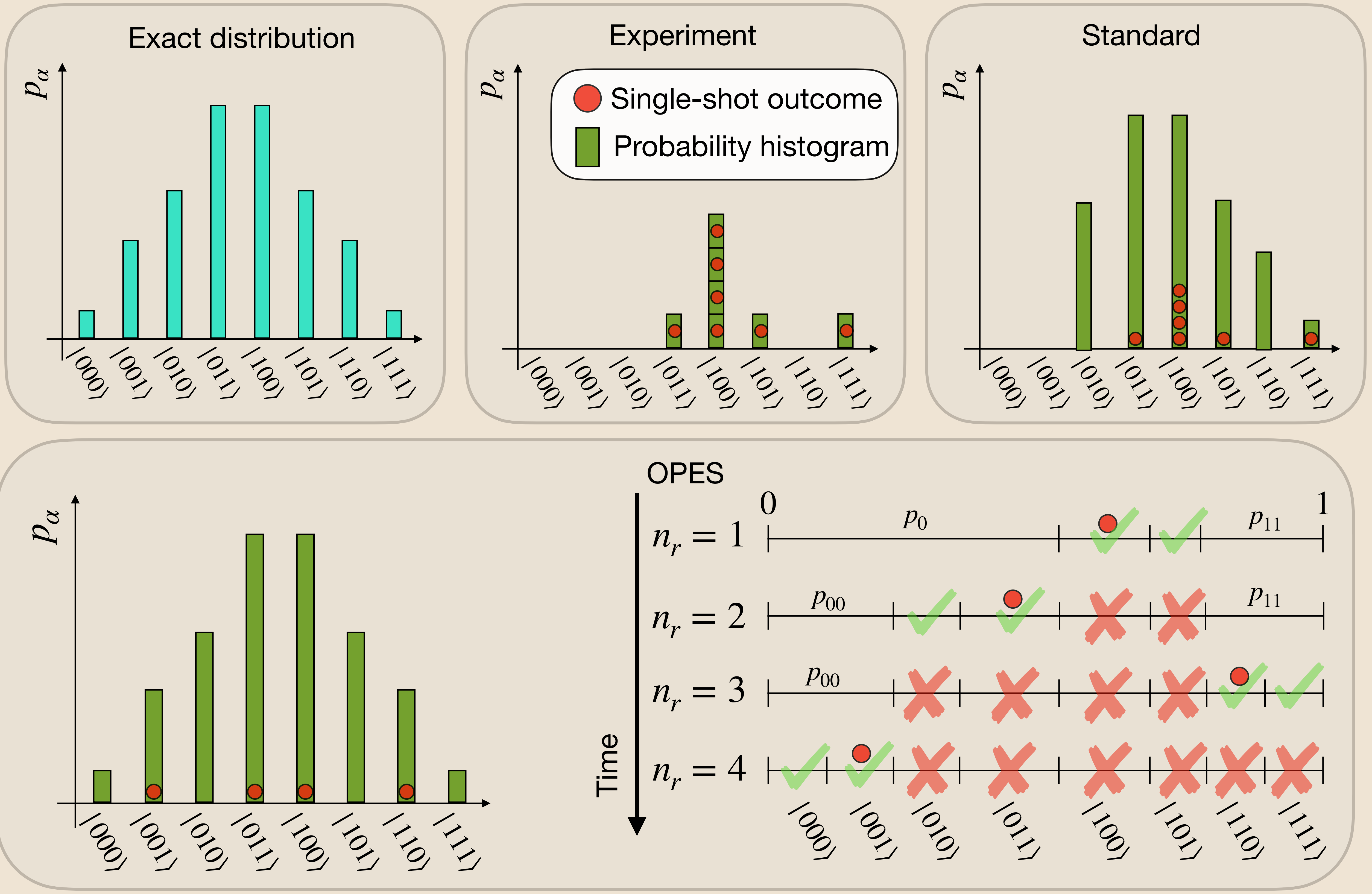

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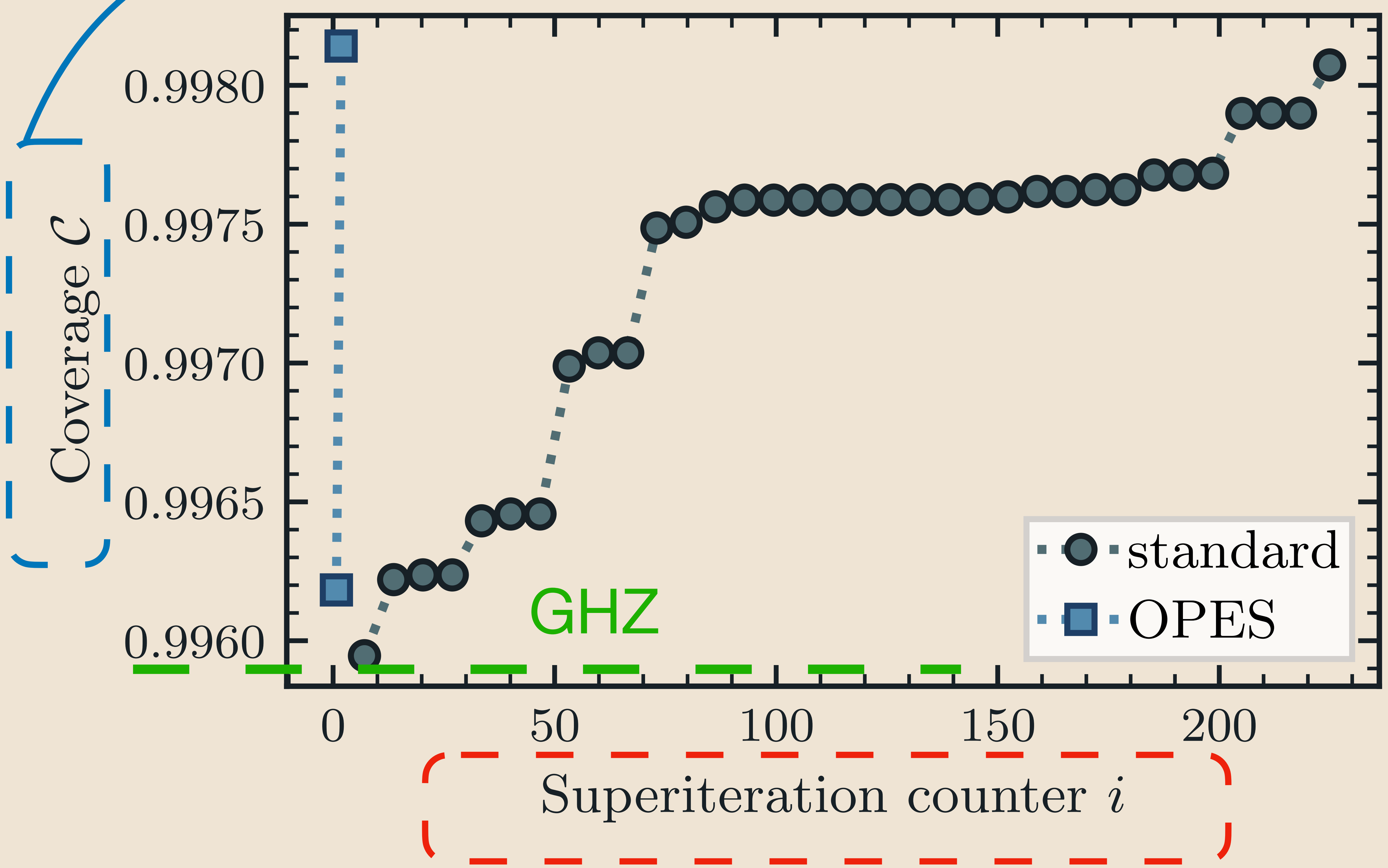

$\sim n_r$

Small contributions due to noise $\epsilon \ll 1$

GHZ

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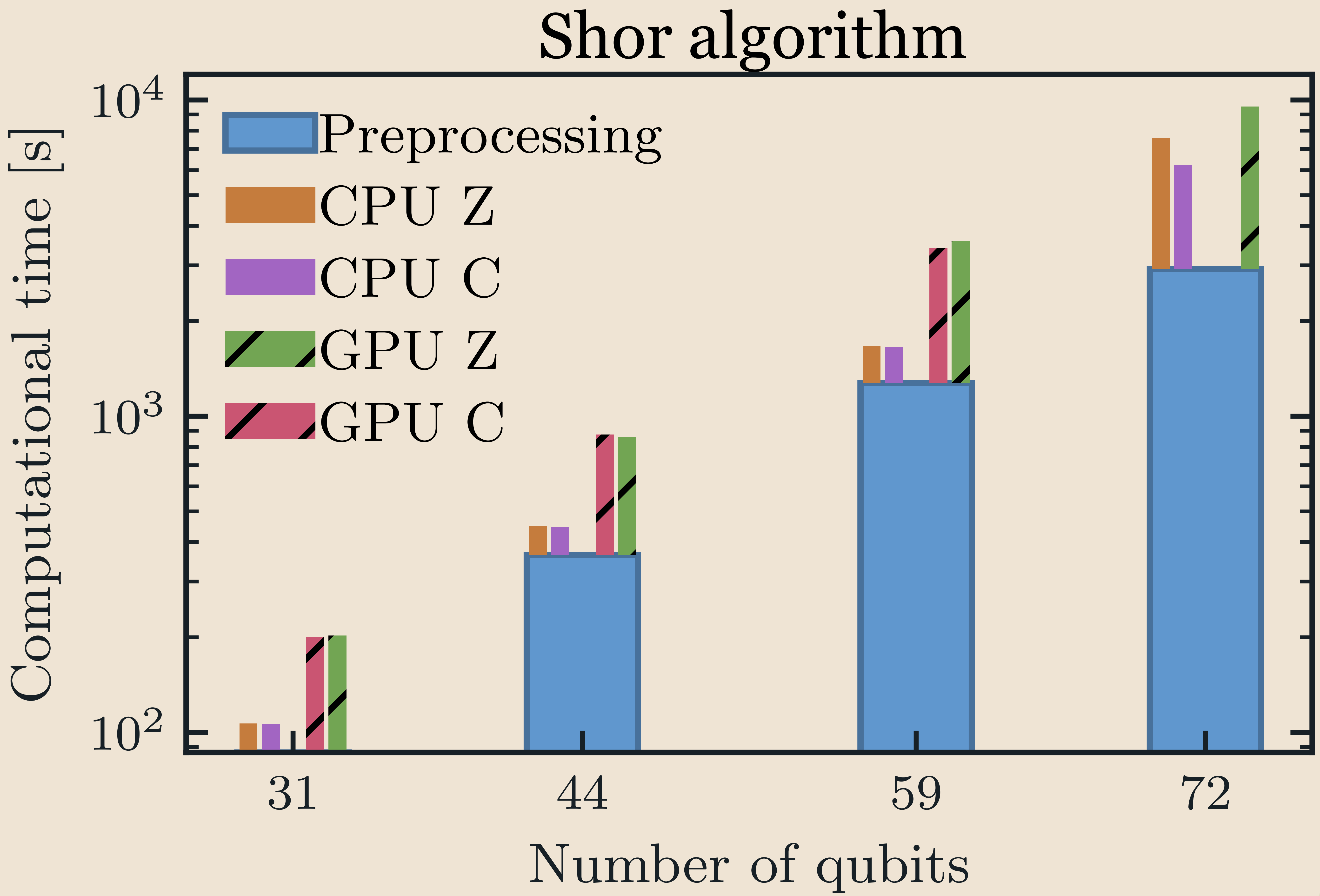
Conclusions

MPS simulations are not limited by the number of qubits but by the entanglement



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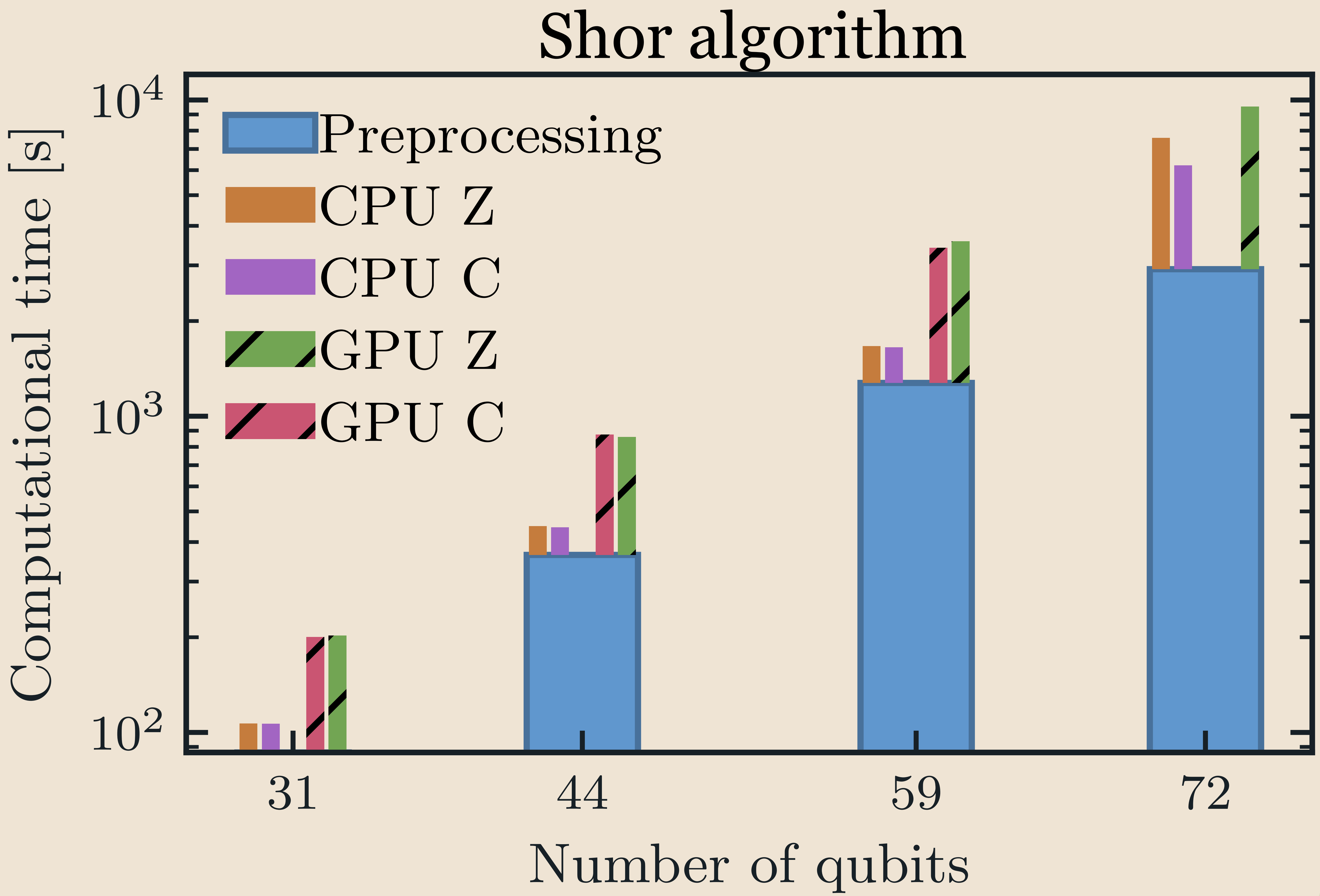
Easy-to-use python frontend and fast HPC-ready backend (Both GPU and CPU)



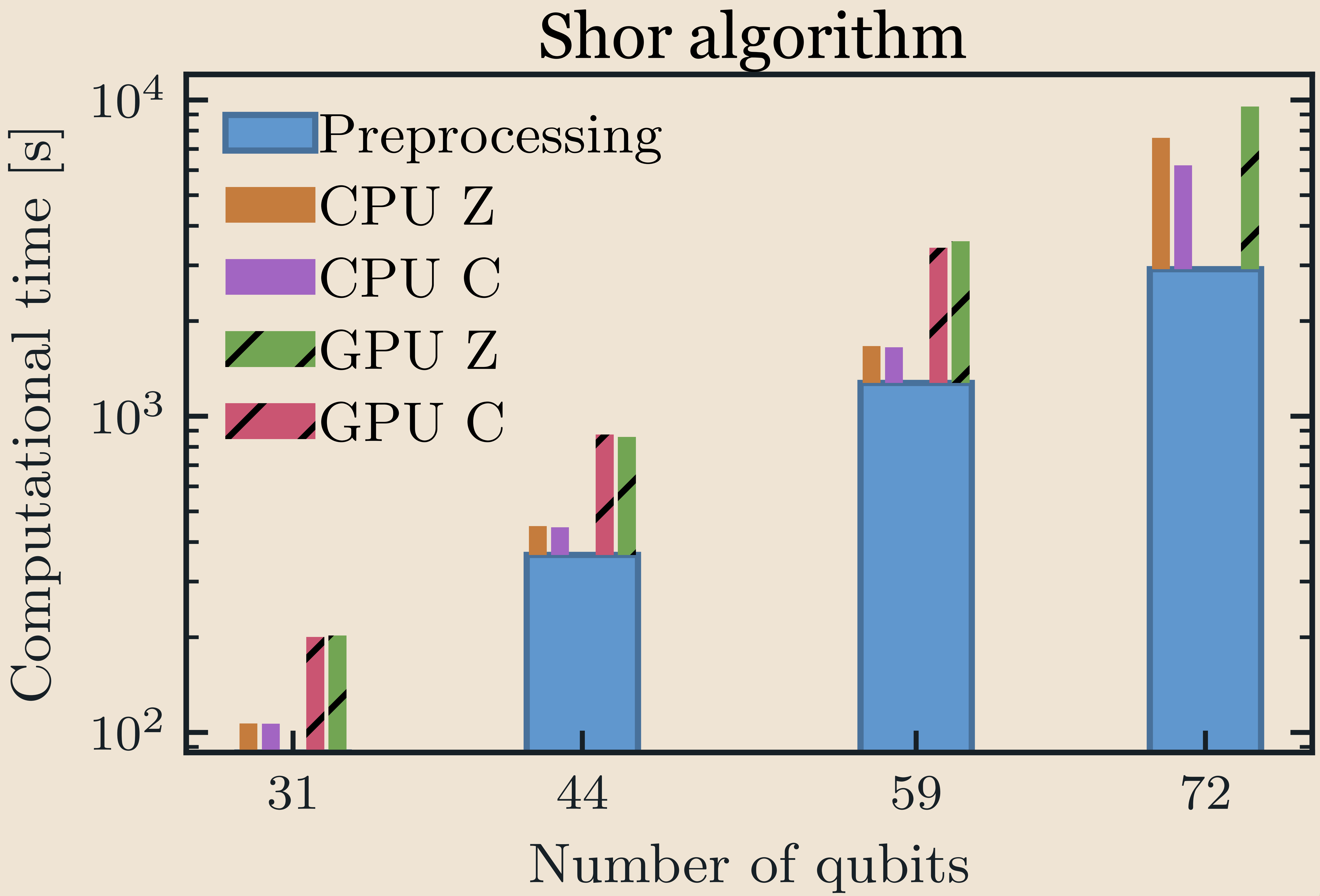
Conclusions

MPS simulations are not limited by the number of qubits but by the entanglement

Easy-to-use python frontend and fast HPC-ready backend (Both GPU and CPU)

Quantum Matcha TEA is available on Leonardo!

```
module load qmatcha_tea
```



Thanks for your attention

Dipartimento di Fisica e Astronomia Galileo Galilei

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universität uulm

QUANTUM COMPUTING AND SIMULATION CENTER

CINECA

https://baltig.infn.it/quantum_tea/quantum_tea

Simone Montangero

Daniel Jaschke

Riccardo Mengoni

Daniele Ottaviani

Sara Marzella

Gabriella Bettonte

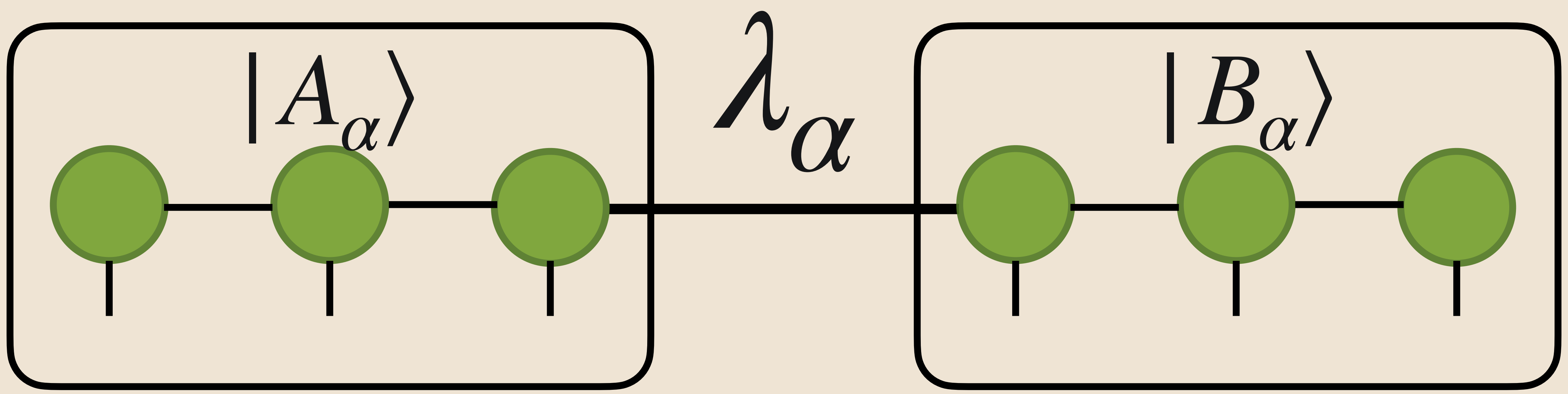
Additional slides

Convergence checks & error bound

$$|\psi\rangle = \sum_{\alpha=1}^{\chi_T^{i-1}} \boxed{|A_\alpha\rangle} \overset{\lambda_\alpha}{\text{———}} \boxed{|B_\alpha\rangle}$$

Convergence checks & error bound

$$|\psi\rangle = \sum_{\alpha=1}^{\chi_T^{i-1}}$$



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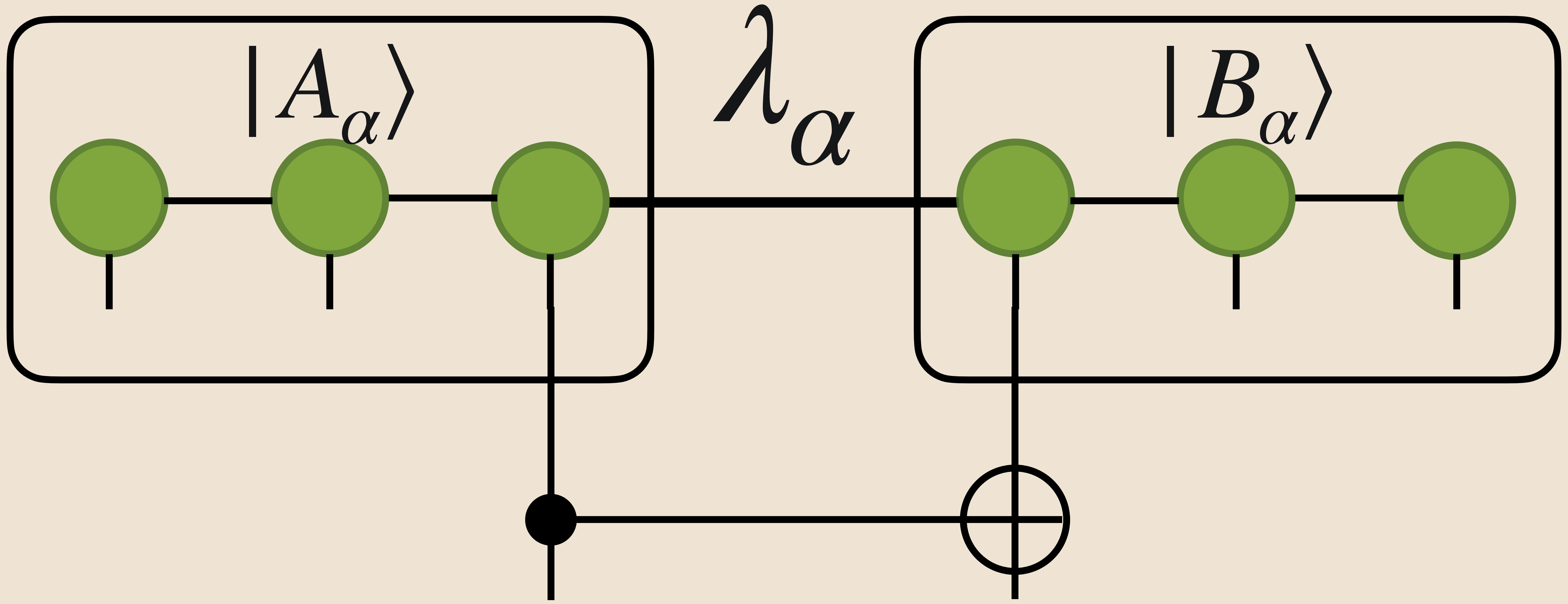

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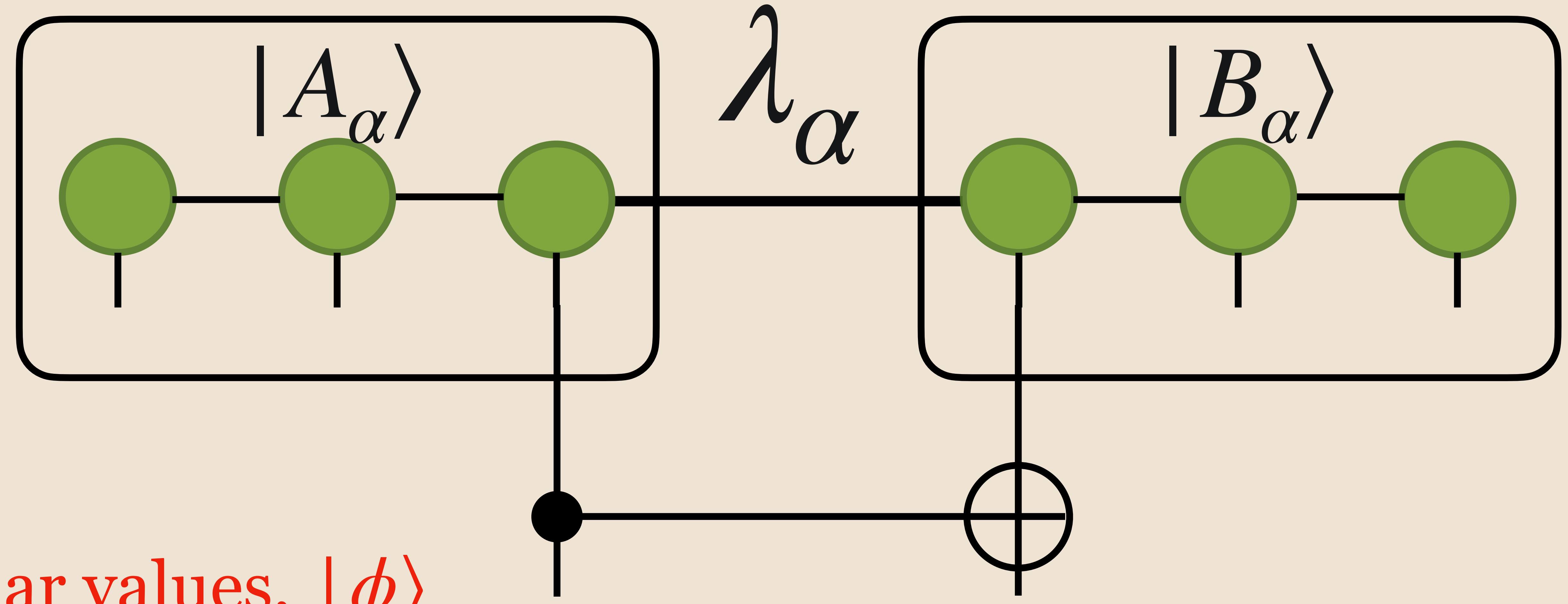

Only keep highest χ singular values, $|\phi\rangle$

Convergence checks & error bound

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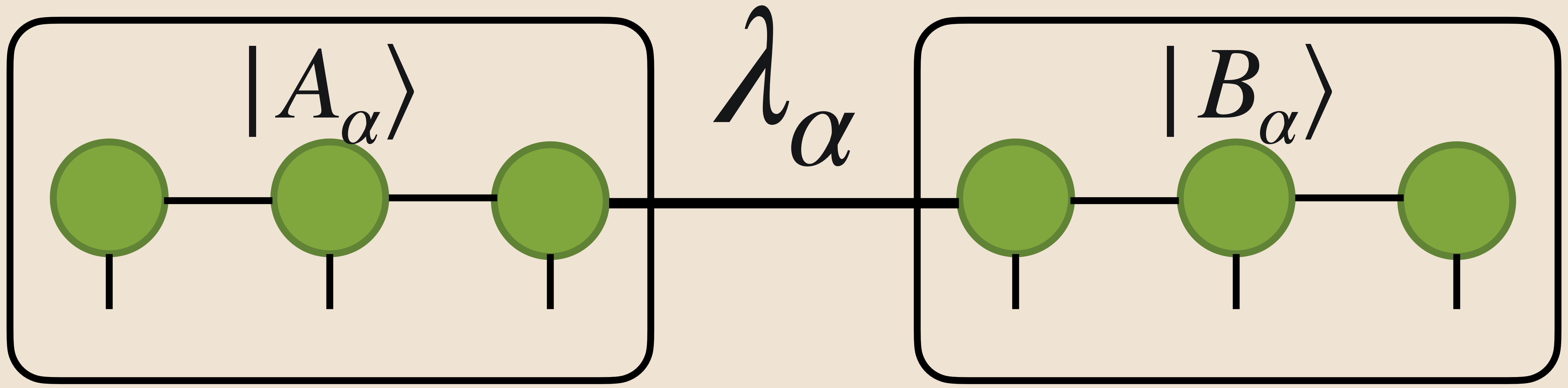

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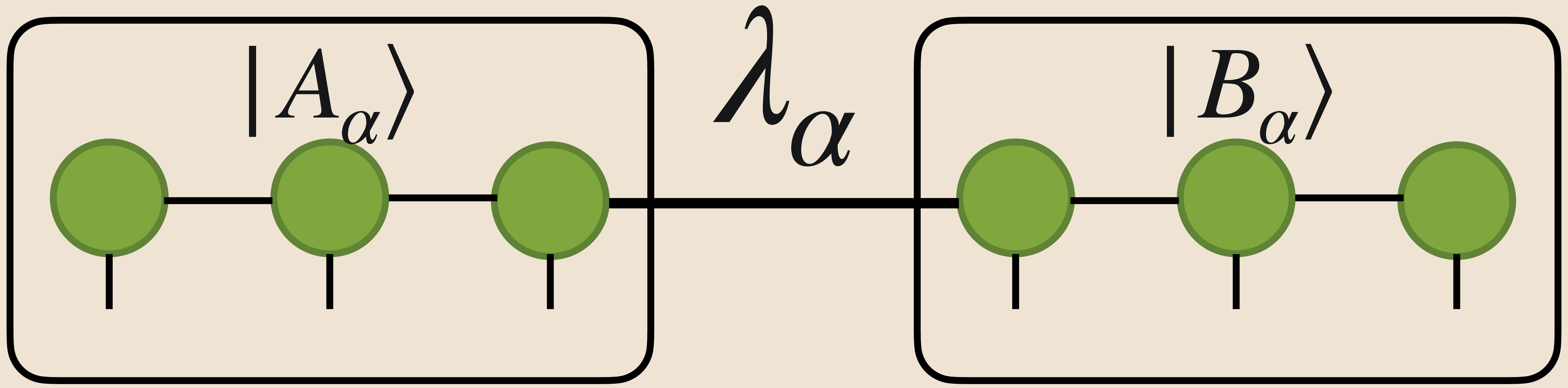

Only keep highest χ singular values, $|\phi\rangle$

Fidelity of the state

$$\mathcal{F}_i(\chi) = \left|\langle\psi|\phi\rangle\right|^2 = \left|1 - \sum_{\alpha=\chi+1}^{\chi_T^i} \lambda_\alpha^2\right|^2$$

Convergence checks & error bound

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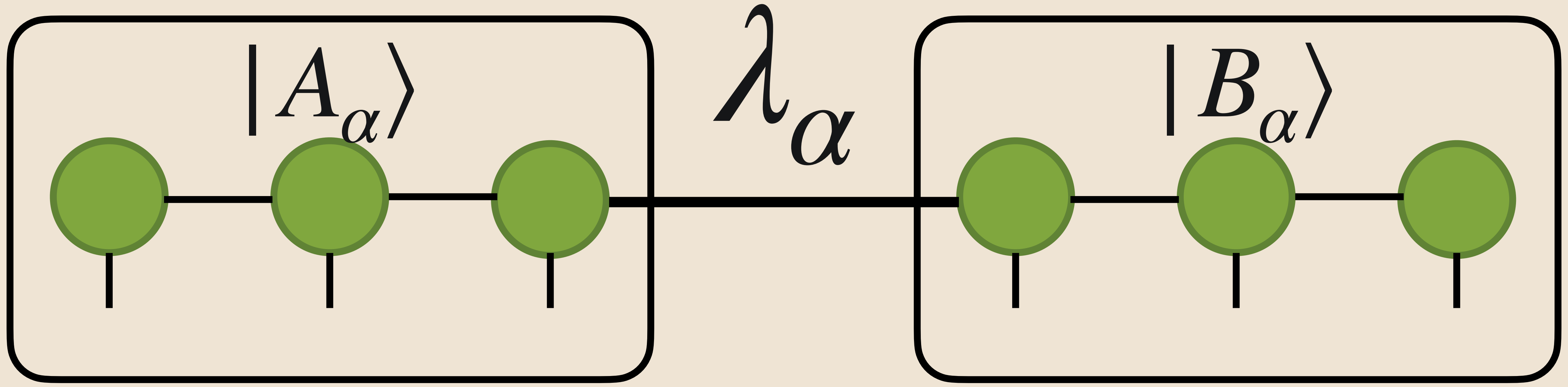

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Fidelity of the state

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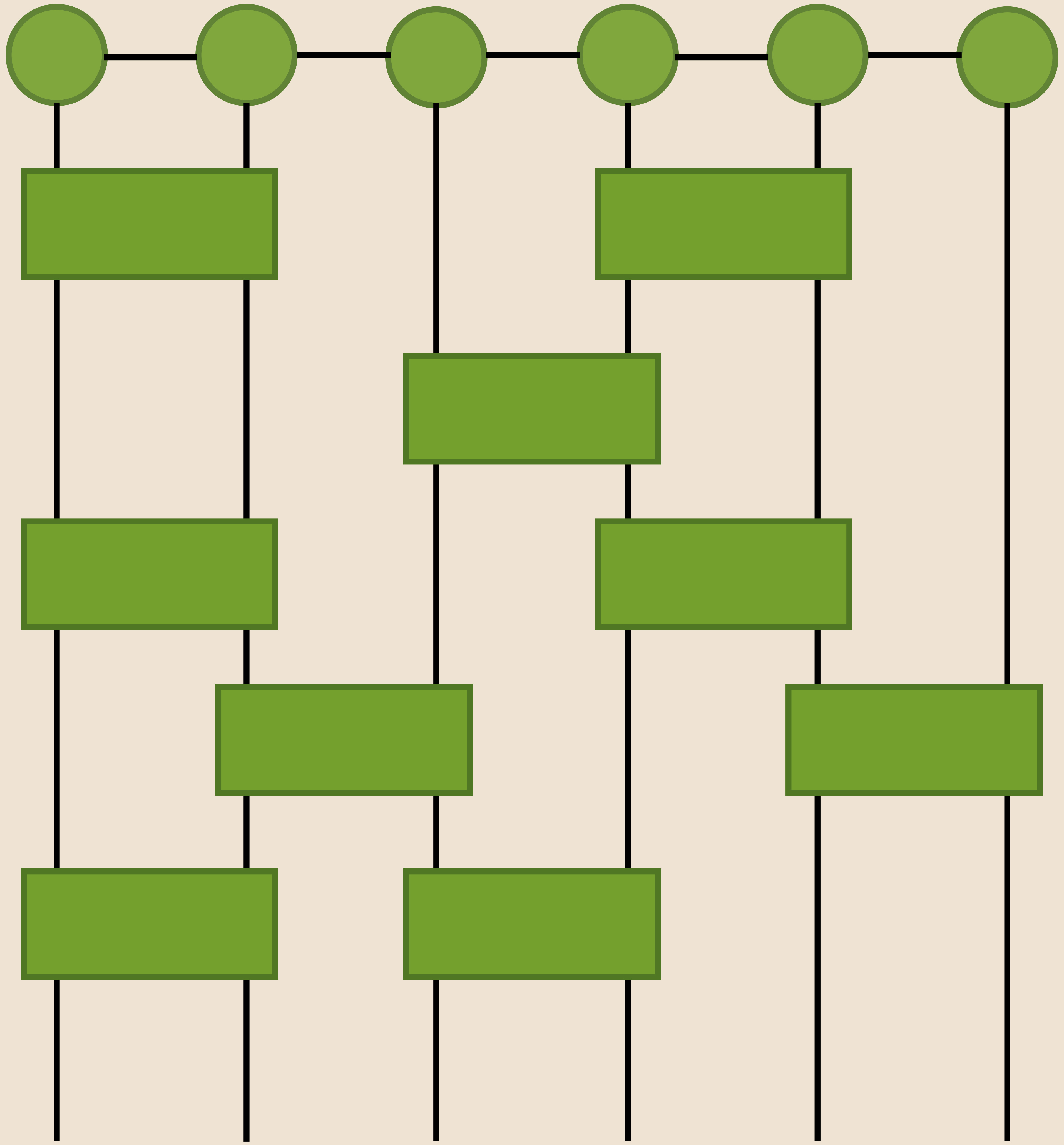
Computed during the simulation

Convergence and error checks



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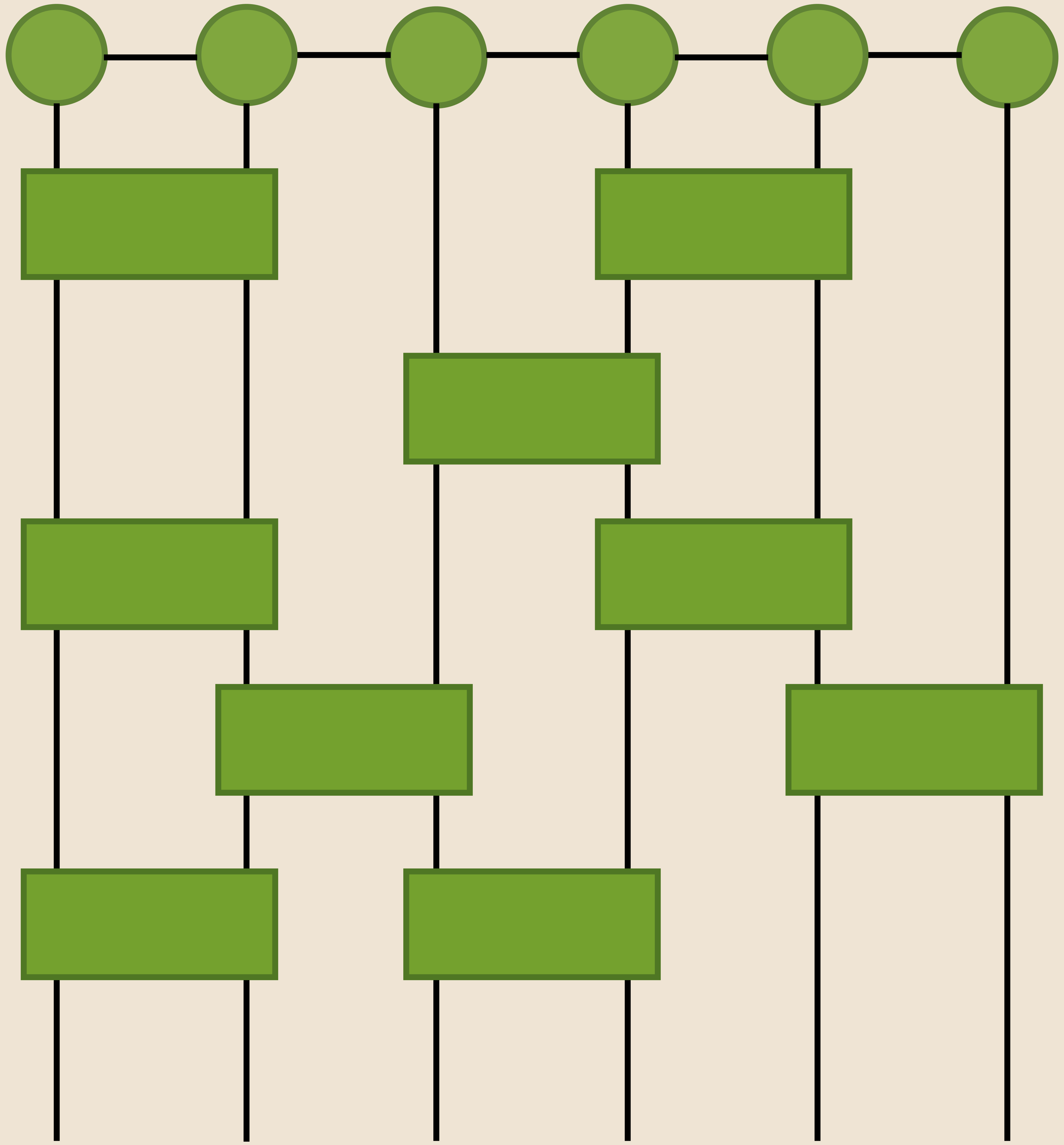

Fidelity of the state
after a **single** gate

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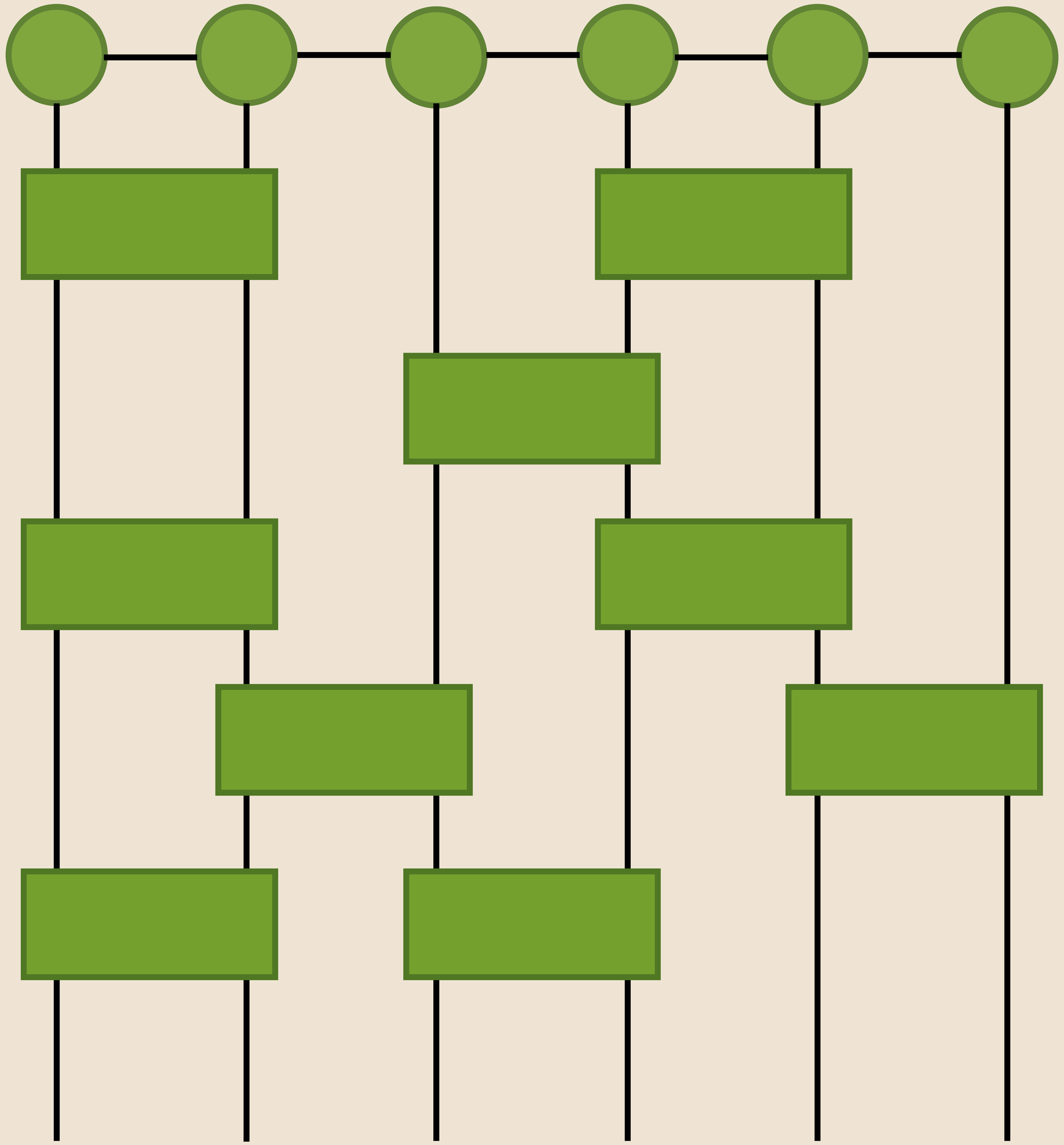
Convergence and error checks



Fidelity of the state after a **single** gate

$$\mathcal{F}_i(\chi) = \left| 1 - \sum_{\alpha=\chi+1}^{\chi_T^i} \lambda_\alpha^2 \right|^2$$

Fidelity at the end of the simulation

$$\boxed{\mathcal{F}^{tot}(\chi) \geq \prod_i \mathcal{F}_i(\chi)}$$