Classical simulation of qubits on DaVinci-1 HPC in the MBQC frame

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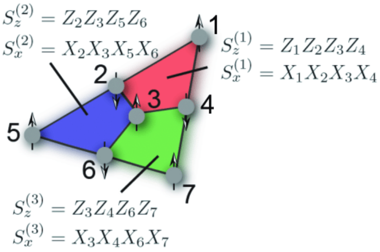
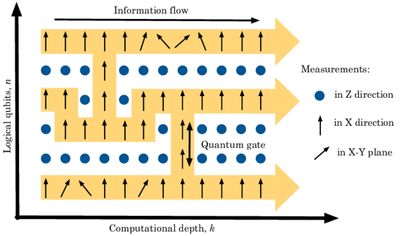
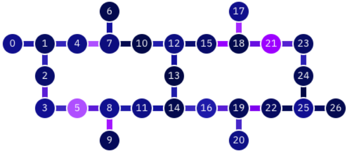
Graph Theory in Quantum Computing

Fields of interest

1. Error correction codes
2. Qubit connectivity
3. One-Way Quantum Computing

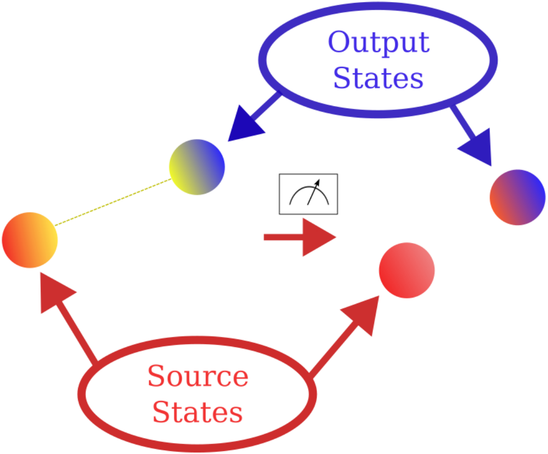






A. Bermudez et al. "Assessing the progress of trapped-ion processors towards fault-tolerant quantum computation." Physical Review X 7.4 (2017): 041061.

Measurement Based Quantum Computing (MBQC)



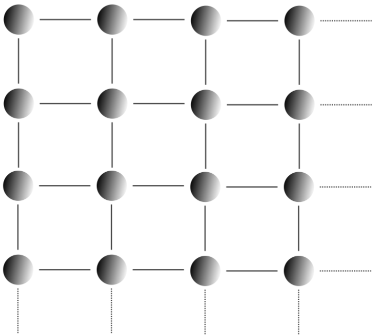
MBQC States

1. Source states
   - inputs
   - ancillae
2. Output states

MBQC Operators

- Entanglement
- Measurements
- Corrections

Graph States



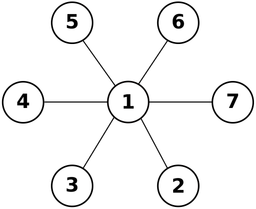
Graph

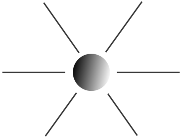
- Vertices:
  $V = \{1, 2, ..., n\}$
- Edges:
  $E = \{(1, 2), (2, 3), ..., (n-1, n)\}$
- Neighborhood of a:
  $N_a = \{b \in V | (a, b) \in E\}$

Cluster state

- Vertices: qubits
- Edges: entanglements

Stabilizers





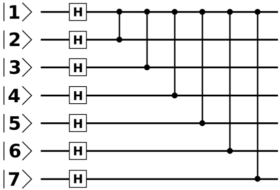
Graph Operators

- $\hat{X}$

| $\hat{Z}$

Stabilizer group

- $\mathcal{S}_n = \{\hat{K}_i, \hat{K}_j \in \mathcal{P}_n \,|\, [\hat{K}_i; \hat{K}_j] = 0\}$
- $\hat{K}_i \,|\psi\rangle = |\psi\rangle \;\forall i$



$$\begin{cases} \hat{K}_1 = \hat{X}_1 \hat{Z}_2 \hat{Z}_3 \hat{Z}_4 \hat{Z}_5 \hat{Z}_6 \hat{Z}_7 \\ \hat{K}_2 = \hat{Z}_1 \hat{X}_2 \end{cases}$$

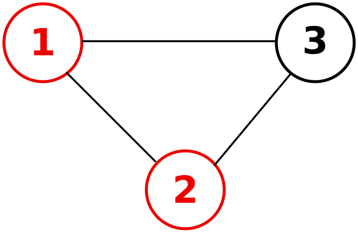
Hub code

Hubs set $\mathcal{B}$

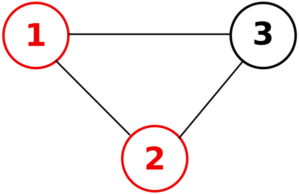
- Def: vertices covering all the edges $\cup_{a\in\mathcal{B}} N_a \supseteq E$

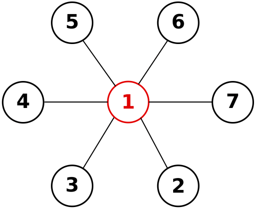
Algorithm Pseudo-code

$G = [V, E]$, $\mathcal{B}$
for $i \in V$ **do**
  if $i \in \mathcal{B}$ **then**
    $|i\rangle = |0\rangle$
  else
    $|i\rangle = |+\rangle$
for $i \in$ hubs **do**
  $|\Psi\rangle = \frac{1}{\sqrt{2}}[\hat{\mathbb{I}} + \hat{K}_i] |\Psi\rangle$



- $|\Psi\rangle = |00+\rangle$
- $\hat{G}_1 |\Psi\rangle = \frac{|00+\rangle + |10-\rangle}{\sqrt{2}}$
- $\hat{G}_2 |\Psi'\rangle = \frac{|00+\rangle + |10-\rangle + |01-\rangle - |11+\rangle}{2}$





Ring topology

- Number of hubs: $\lceil \frac{n}{2} \rceil$
- hub stabilizers: $\langle \hat{X}_1 \hat{Z}_2 \hat{Z}_3; \hat{Z}_1 \hat{X}_2 \hat{Z}_3 \rangle$
- state: $\frac{|00+\rangle + |10-\rangle + |01-\rangle - |11+\rangle}{2}$

Star topology

- Number of hubs: 1
- hub stabilizers: $\langle \hat{X}_1 \hat{Z}_2 \hat{Z}_3 \hat{Z}_4 \hat{Z}_5 \hat{Z}_6 \hat{Z}_7 \rangle$
- state: $\frac{|0{+}{+}{+}{+}{+}{+}\rangle + |1{-}{-}{-}{-}{-}{-}\rangle}{\sqrt{2}}$

Benchmarking Codes

Hub formalism

- Operations:
  - Preparing state: single qubit ($\hat{H}$ operator)
  - Entanglements: single qubit ($\hat{X}$, $\hat{Z}$ operators)
- Group generators: $\mathcal{S}'_n \subset \mathcal{S}_n$
- Scaling of operations:
  - Preparing state: $|\mathcal{B}|$ (number of hubs, $< n$)
  - Entanglements: $|\mathcal{B}| \cdot \sum_{i \in \mathcal{B}}(N_i + 1)$ (N_i number of neighboring vertices)

CZ formalism

- Operations:
  - Preparing state: single qubit ($\hat{H}$ operator)
  - Entanglements: two-qubits ($\hat{CZ}$ operator)
- Group generators: $\mathcal{S}_n$
- Scaling of operations:
  - Preparing state: n (number of qubits)
  - Entanglements: m (number of edges)

Classical simulation in MBQC frame

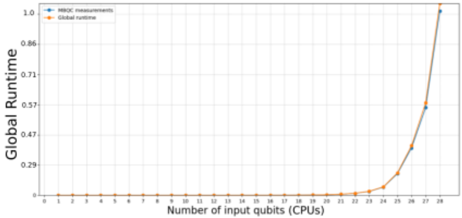
Implementation

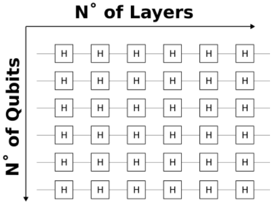
- Software: Paddle Quantum
- Hardware: DaVinci-1 HPC
- GPUs (high performances, low memory)
- CPUs (high memory, low performances)

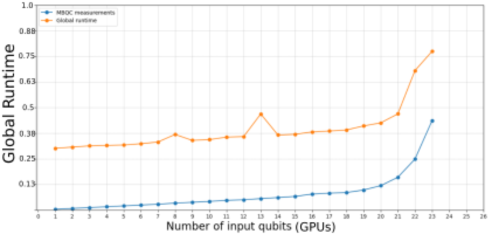
Our work

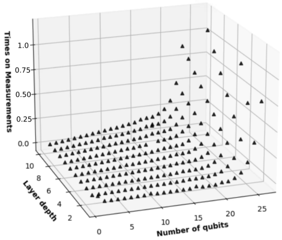
1. Stress Test: performances of Paddle Quantum on DaVinci-1
2.
3.

Stress Test: N-dimensional Hadamard Circuit









Classical simulation in MBQC frame

Implementation

- Software: Paddle Quantum
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Our work

1. Stress Test: performances of Paddle Quantum on DaVinci-1
2. Wrapper for Paddle Quantum
3.

Wrapper for Paddle Quantum

Cons of Paddle Quantum

- Lacking API
- Tricky calls to functions

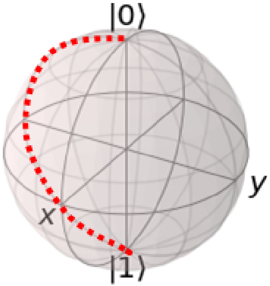
Wrapper

1. Better knowledge about the libraries
2. Better access to modules

```
##################################
## CLASS TEMPLATE FOR MCALCULUS ##
##################################
class Mbqc(Circuit):
    def __init__(self, angles, stabilizers, inputs, corrections, depth=1):
        super().__init__(len(inputs), depth)
        self.inputs = copy.copy(inputs)
        ## tuple are immutable type ##
        ## this turn to be useful   ##
        ## when defining __matmul__ ##
        self.save = tuple(inputs)
        self.angles = angles
        self.corrections = corrections
        ## Set the underlying graph ##
        self.mbqc = MBQC()
        self.mbqc.set_graph(stabilizers)
        ## Set the time intervals ##
        ## for entanglements
        self.dt_entangl = 0
        ## for measurements
        self.dt_meas = 0
        ## for corrections
        self.dt_corr = 0
```

Practical application

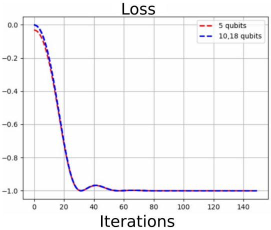
- Backpropagation for optimization
- Circuit: $[\hat{R}_x(\theta)\,|0\rangle]^{\otimes N}$
- Hyperparameter: θ

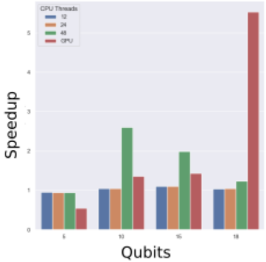


Numerical Results

Algorithm Pseudo-code

Set GPU/CPU threads
Set N
Set ITR
$|\Psi_0\rangle = |0\rangle^{\otimes N}$
$\hat{M} = \text{Diag}(-1, ..., -2^N)$
for $i \in ITR$ **do**
  $|\Psi_i(\theta)\rangle = \hat{R}_x(\theta)\,|\Psi_{i-1}\rangle$
  $Loss = -\langle\Psi_i(\theta)|\,\hat{M}\,|\Psi_i(\theta)\rangle$
  Backpropagation $\rightarrow \theta$

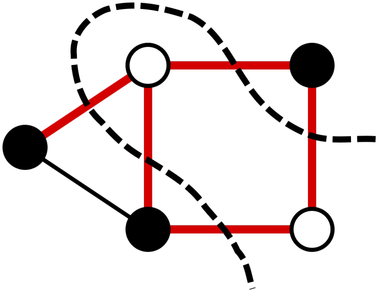




Highlights and Conclusions

Practical application

- New formalism in MBQC frame
- Numerical tests of Paddle Quantum libraries on DaVinci-1
- Wrapper development
- Backpropagation on Paddle Quantum libraries on DaVinci-1
- Future project: QAOA in MBQC frame for MaxCut problem



Pauli Matrices

$$\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{Y} = i \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_i \hat{\sigma}_j = \hat{\mathbb{I}} \delta_{ij} + i \epsilon_{ijk} \hat{\sigma}_k$$

Hadamard Gate

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

CZ gate

$$\hat{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$