Bayesian Adaptive Techniques for Quantum Optimization on NISQ devices

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Maximal Independent Set (MIS) problem:
find the set with the maximum number of vertices of a graph not linked to the others

ground state of the Hamiltonian

\[
C = - \sum_j n_j + P \sum_{<ij>} n_i n_j
\]

Solution: 011011

- Topology of the graph is encoded in the simulator through Rydberg blockade

- Assuming analog processing only (global pulses)

\[
\mathcal{H}(t) = \frac{\hbar}{2} \Omega(t) \sum_j \sigma_j^+ - \hbar \delta(t) \sum_j n_j + \sum_{i \neq j} \frac{C_6}{r_{ij}^6} n_i n_j
\]
Hybrid classical-quantum protocol: QAOA (quantum approximate optimisation algorithm)

- Objective function to be minimised
- Quantum evolution (circuit) with variational parameters
- Measurement scheme for average Hamiltonian to get objective function

- Optimisation algorithm to get the best values of the parameters

✓ All these steps have theoretical issues to be kept under control
✓ All these steps might become easily very resource demanding: NISQ device?
- The quantum circuit

\[ H_m = \frac{\hbar \Omega}{2} \sum_i \sigma^x + \sum_{i<j} U_{ij} n_i n_j \]

\[ H_c = \frac{\hbar \Omega}{2} \sum_i \sigma^x - \frac{\hbar \delta}{2} \sum_i \sigma^z + \sum_{i<j} U_{ij} n_i n_j \]

Pulser emulators: signals for P=3

Time: \( T \sim 2 \mu s \)
- Measurement of the cost function

- On the final state, we measure the value of the objective function

\[ C = - \sum_j n_j + P \sum_{\langle ij \rangle} n_i n_j \quad (P = 2) \]

- We repeat the evolution and the measurement \( N_{\text{shot}} \) times to get an histogram of the probability to find each computational state, from which one gets the average value of \( C \)

On real computer: \( \text{TIME} = 0.2 \ s \times N_{\text{shot}} \)
Classical optimisation: Bayesian approach

- Use Bayesian inference to reduce the number $N_{\text{shot}}$ of experimental acquisitions
- The optimization problem shifts from finding the min in the energy landscape to the max in the acquisition function landscape, which - on the basis of previous measurements - gives us the best point where to perform the new measurement
- Genetic (differential evolution) algorithm: a starting set of points are “mutated” into new one, keeping only the best ones - up to convergence

Convergence parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD ENERGY</td>
<td>$&lt; 10E-5$</td>
<td>ensures every point tends to same value</td>
</tr>
<tr>
<td>AVERAGE SQUARED DISTANCE</td>
<td>$&lt; 10E-2$</td>
<td>ensures convergence to the same spot in flat landscapes</td>
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</tbody>
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Kernel: $K(x_1, x_2) = \sigma_f^2 \exp\left[ - \frac{|x_1 - x_2|^2}{2l^2} \right]$
- BOUNDS for ANGLES: $\beta, \gamma \in [100,3000]\]$
- $N_{\text{shot}} = 128$

- DEPTH P=1,2,...,9
- # initial points: 10-20
- # steps: ~ 200

- max # iteration: 100*P
- STD ENERGY CONVERGENCE: 10E-5
- AVG DISTANCE CONVERGENCE: 10E-2
- kernel corr. length bounds: [10e-2, 10e2]
- kernel constant bounds: [10E-5, 10E5]
• Analysis of results
• Summary: best energy/fidelity vs. depth P
• Analysis of final state

The ratio of counts between the solution and second most likely state

<table>
<thead>
<tr>
<th>p</th>
<th>Ratio</th>
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<tbody>
<tr>
<td>1</td>
<td>2.1</td>
</tr>
<tr>
<td>3</td>
<td>11.3</td>
</tr>
<tr>
<td>4</td>
<td>9.1</td>
</tr>
</tbody>
</table>
- Differential evolution is slow, about 90% of computation time
- Total # quantum circuit evolutions: $N_{\text{shot}} \times N_{\text{step}}$
WHAT’S NEXT?

- Optimise our software and run on Galileo 100

- Benchmark Bayesian approach with other classical optimisation strategies

- Run simulations on Pulser with (different kinds of) noise

- Run on real atomic platform