Near-Optimal Graph Coloring on Neutral Atoms Quantum Computer

CHIARA VERCELLINO

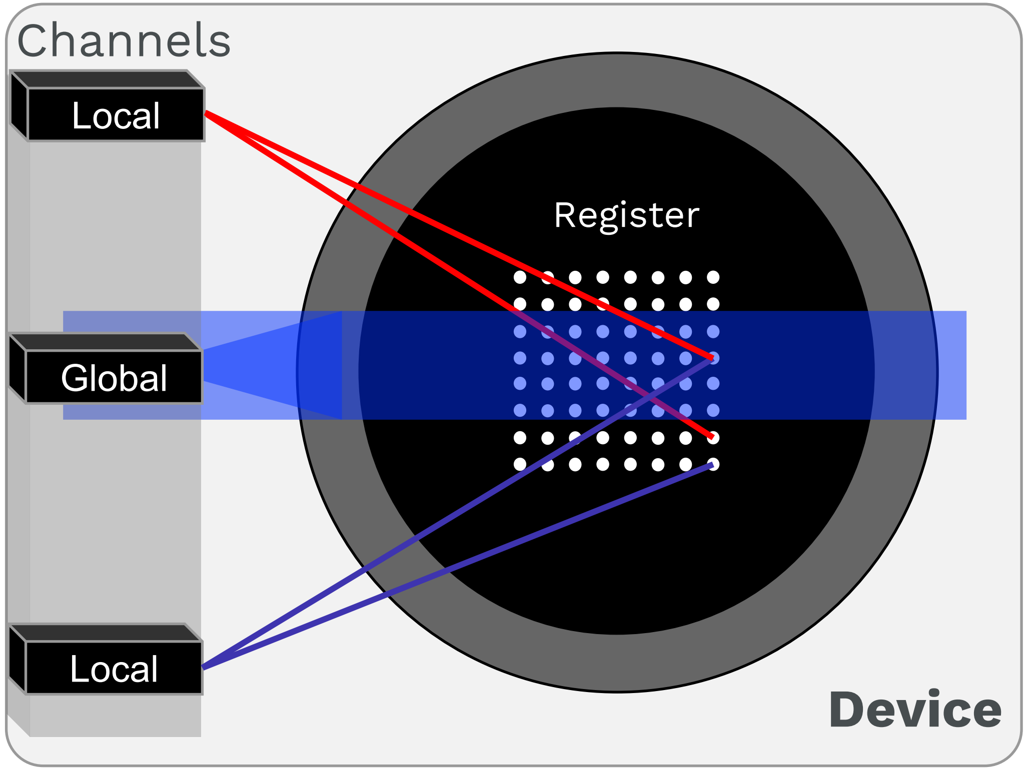
WORKSHOP QUANTUM COMPUTING AND HIGH PERFORMANCE COMPUTING 4TH EDITION

CINECA, DEC 16TH 2021

Neutral atoms quantum computer

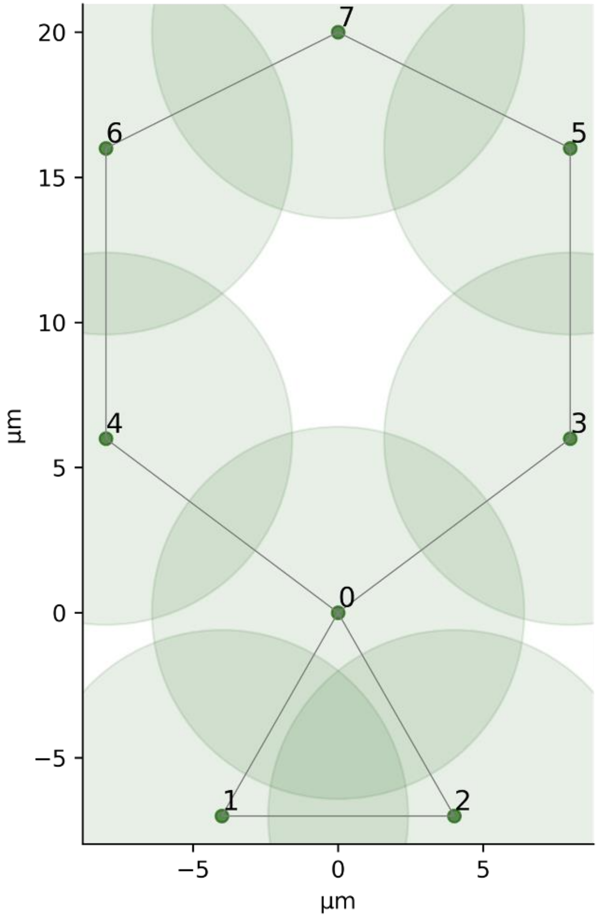
The device features

- This work involved the collaboration with **CINECA** and **Pasqal**, this last one provides quantum hardware based on Rydberg atoms array
- In this system, qubits are represented by rubidium atoms arranged by means of optical tweezers in a 2D or 3D array (register)
- Rubidium atoms can be **excited by means of laser pulses into the Rydberg state**, that encodes the $|1\rangle$ state in the computational basis



Neutral atoms quantum computer

The atoms interactions



- The interaction between atoms at the same Rydberg level is described by the **Van der Waals force**, which can be used to create **Rydberg Blockade effect**
- Rydberg blockade prevents two atoms within radius R (correlated to laser frequency) to be both in the excited state
- This creates quantum **superposition and entanglement between atoms that are "connected"** (i.e. closer than R to each other). The register is in fact a so-called **Unit-Disk graph** (UD)
- The behaviour of the qubit register is defined by the following **hamiltonian**

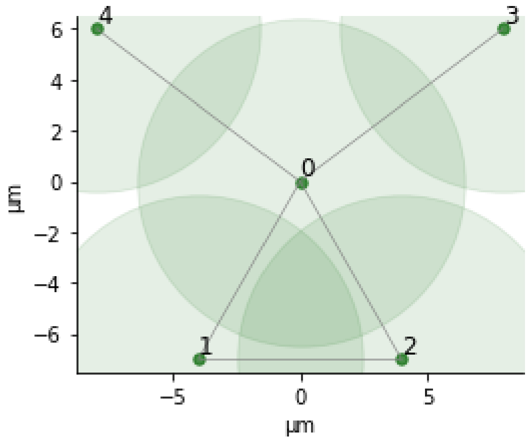
$$\mathcal{H} = \sum_{i=1}^{N} \frac{\hbar\Omega}{2}\sigma_i^x - \sum_{i=1}^{N} \frac{\hbar\delta}{2}\sigma_i^z + \sum_{j<i} \frac{C_6}{|r_i - r_j|^6} n_i n_j$$

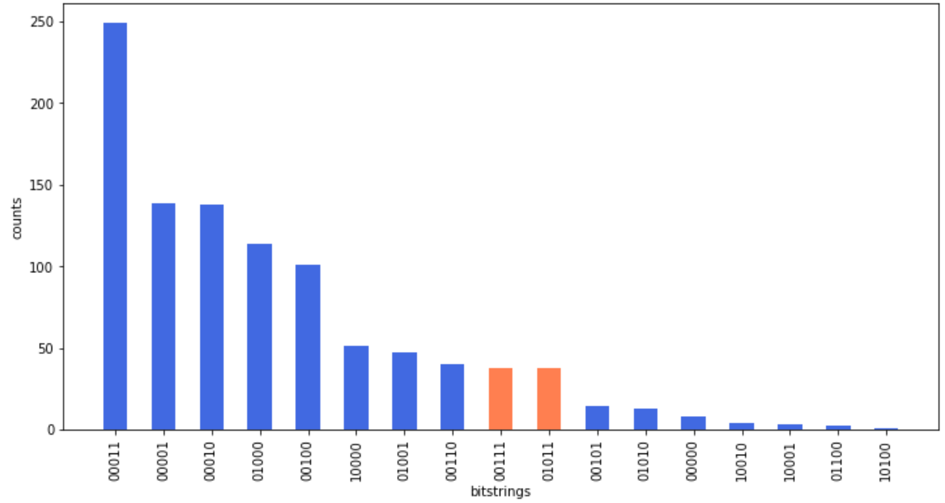
Building Block: QAOA for UD-MIS

Atoms as binary variables

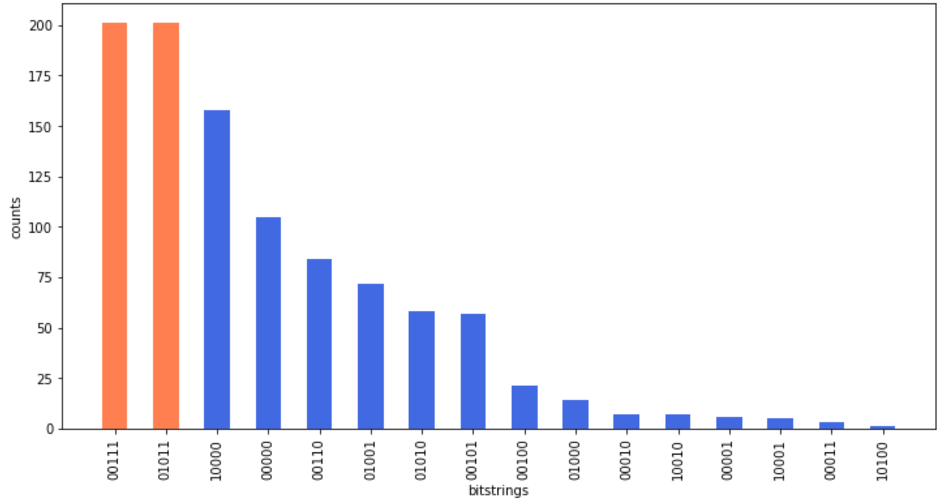
The Hamiltonian of the Rydberg atoms array can be directly mapped to a Maximal Independent Set (MIS) problem, i.e.,

> laser pulses can be optimized by means of a classic optimizer, to obtain a configuration of the register where the $|1\rangle$ qubits represent the members of a MIS





Optimizer



Industrial use case

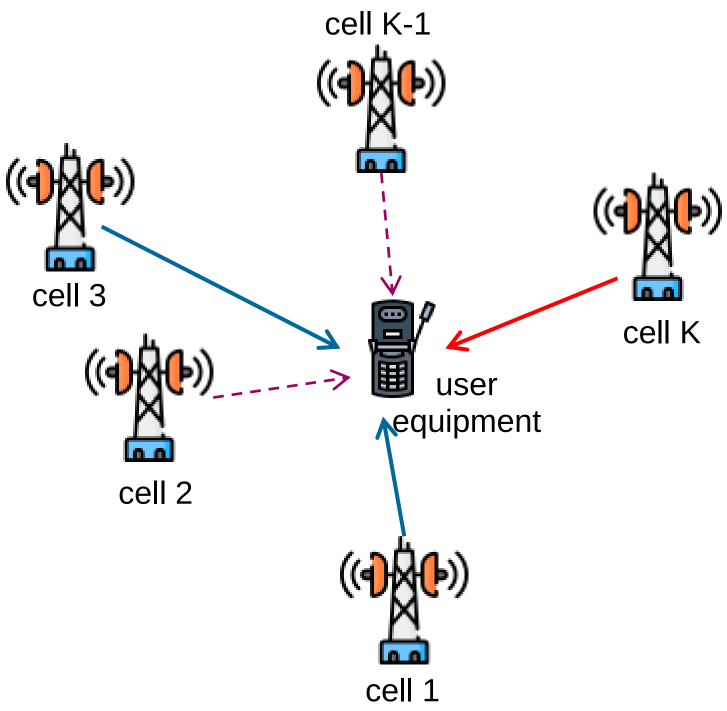
The optimization problem

Physical Cell Identifiers (PCIs) problem

Assign PCIs to the nodes of a cellular communication network

- **Goal:** minimize the needed PCIs
- **Constraints:** good quality communication leverages ‘conflicts’ reduction

This problem translates quite well to one or more instances of **Graph Coloring (GC) problem**



From Graph Coloring back to Neutral Atoms

How do we use Rydberg atoms to solve graph coloring?

A first approach could be to solve GC problems in the standard QUBO formulation:

- ✓ Binary variables through one-hot-encoding ➔ qubits
- ✓ QUBO matrix values ➔ global pulses modelling
- X QUBO matrix structure ➔ arbitrary graph embedding into a UD graph harder than the original problem, and rarely feasible - https://arxiv.org/abs/2012.14859
- X Number of optimization variables ➔ Strong limit on the instances' size that can be solved

However, the PCI problem is not about arbitrary graphs:

- ✓ Cellular antennas can be placed on a 2D map and their **interference pattern can be represented as a UD graph**.
- ✓ Maximal Independent Set (**MIS**) **based heuristics can** both **provide feasible solutions** and **reduce the number of variables**.

PCI topological solution: Iterative MIS

GREEDY APPROACH (Greedy-itMIS)

➔ MIS implementation through Neutral Atoms is straightforward given the antennas positions (qubits positions) and the radius which determines conflicts (Rydberg radius)

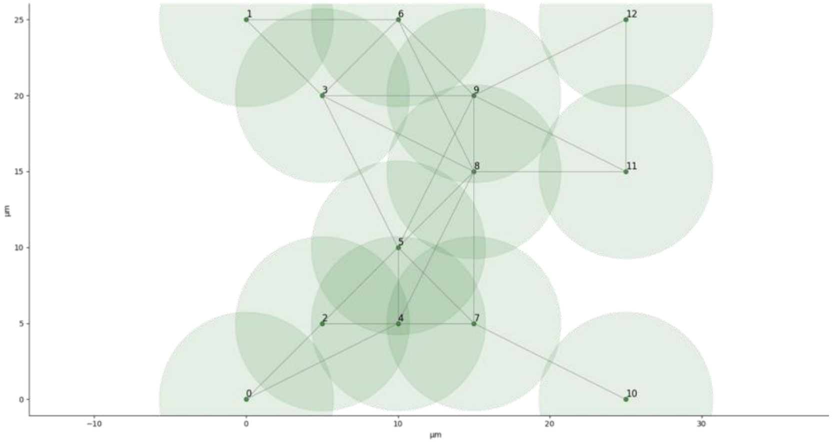
Graph coloring problem is solved by iteratively solving MIS problems:

- Find a MIS solution (with analog QAOA)
- Assign a color to the subset of graph’s nodes and remove them → induced graph
- Iterate on the induced graph until all conflicts are solved → no edges remain in the induced subgraph

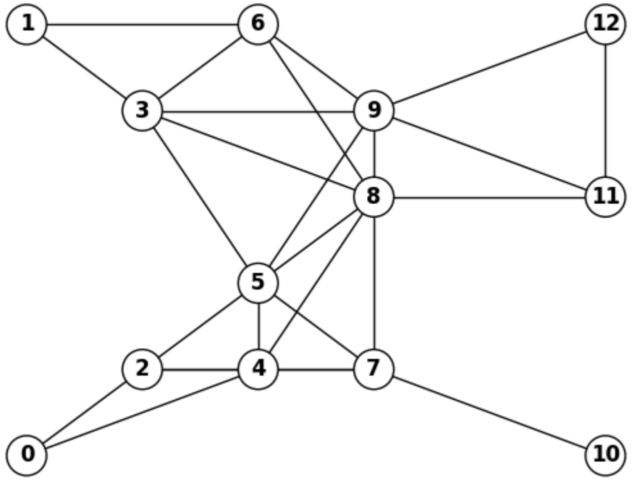
The solution in this case is typically feasible, but suboptimal

PCI topological solution: Iterative MIS

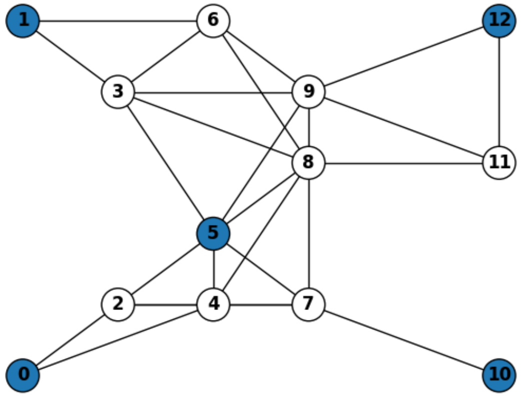
GREEDY APPROACH (Greedy-itMIS)



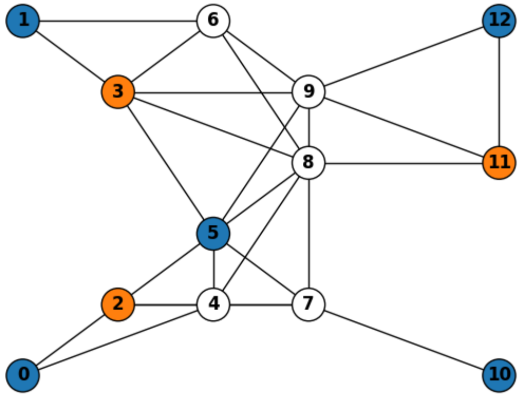
Iteration 0



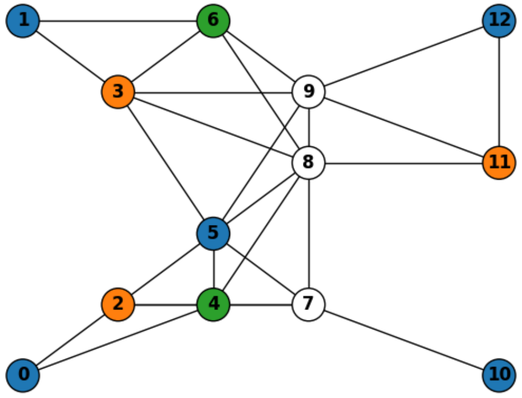
Iteration 1



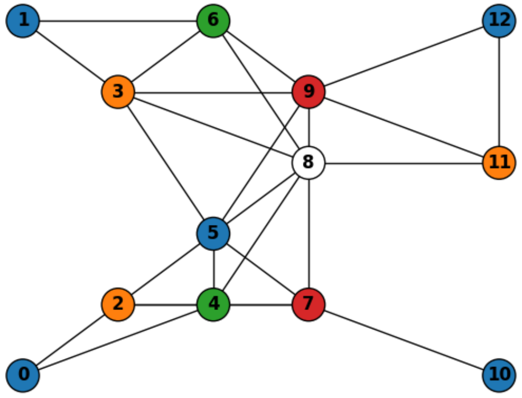
Iteration 2



Iteration 3



Iteration 4



PCI topological solution: Iterative MIS

BRANCH&BOUND APPROACH (BB-itMIS)

Greedy-itMIS issue: consider just one MIS solution at each iteration ➔ **BB-itMIS improvement**: outer optimization loop that considers all MIS solutions at each iteration

Theorem: Every graph G has an optimal coloring in which (at least) one of the colors is a MIS.

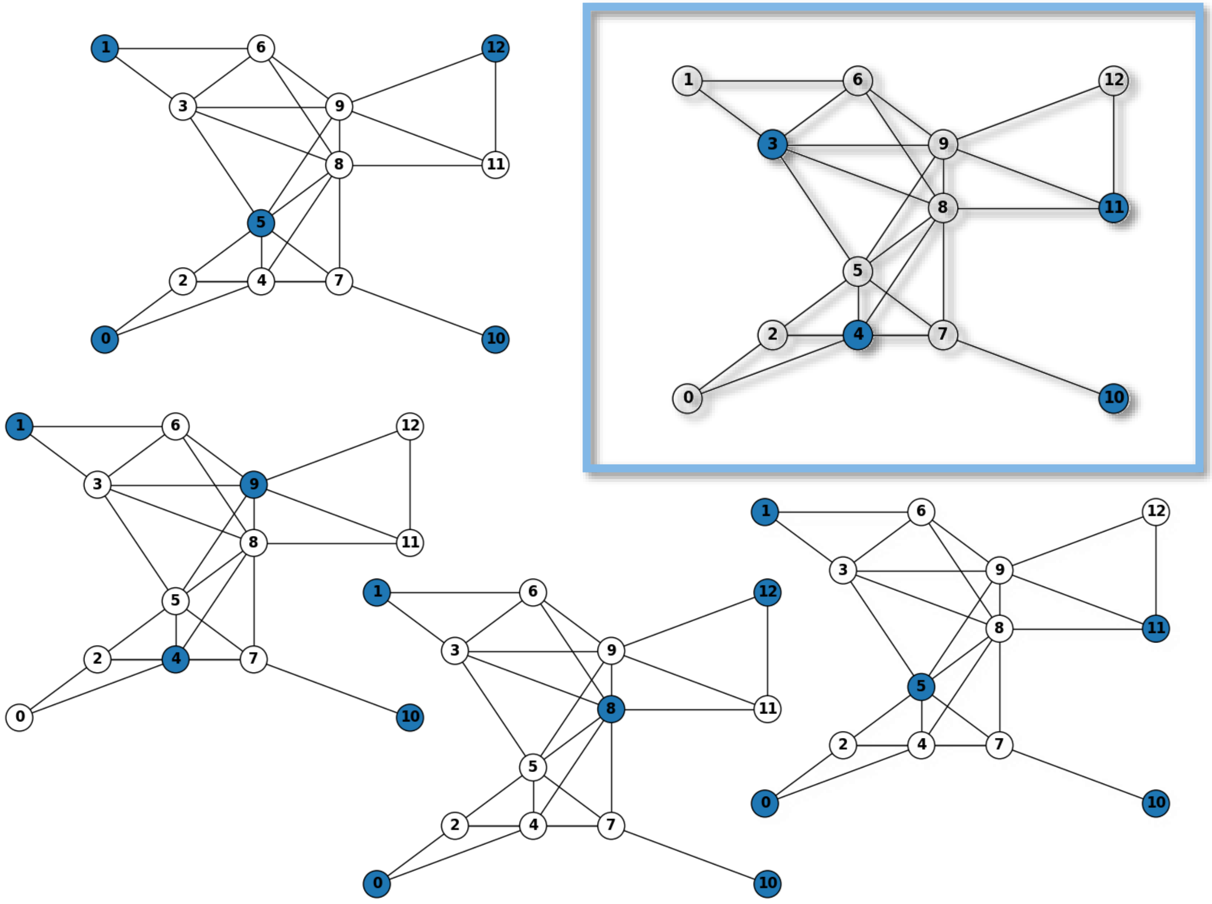
BB-itMIS algorithm:

- Start with the whole graph G in the root node of the BB tree
- For each BB node find a set of MIS solutions and generate one branch for each solution
- Each BB node is associated with a subgraph of G obtained by removing all the vertexes of one MIS solution of the parent node

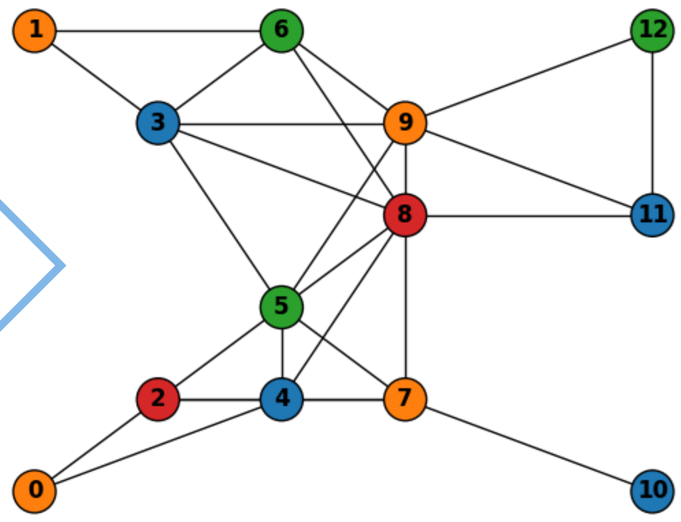
PCI topological solution: Iterative MIS

BRANCH&BOUND APPROACH (BB-itMIS)

MIS solutions at BB root node



BB-itMIS solution



4 colors: 1 less than the Greedy-itMIS solution

Results and next steps

From emulation to real device

itMIS algorithms comparison

Use SoA MIS solver to validate the approach on larger graph

Vertices	Greedy-itMIS	BB-itMIS	SoA GC solver
25	4 colors	3 colors	3 colors
35	5 colors	5 colors	5 colors
45	8 colors	6 colors	6 colors
55	7 colors	7 colors	7 colors
65	8 colors	8 colors	7 colors
75	10 colors	9 colors	8 colors
85	10 colors	9 colors	8 colors
95	11 colors	9 colors	8 colors

Future works

- Evaluation of quantum background noise on solution quality
- Further analysis on optimization parameters:
  - Outer optimization loop: BB hyper-parameters
  - Inner optimization loop: pulses modelling and optimizer hyper-parameters
- Real quantum machine exploitation

Questions?

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