INTRODUCTION TO QUANTUM COMPUTING

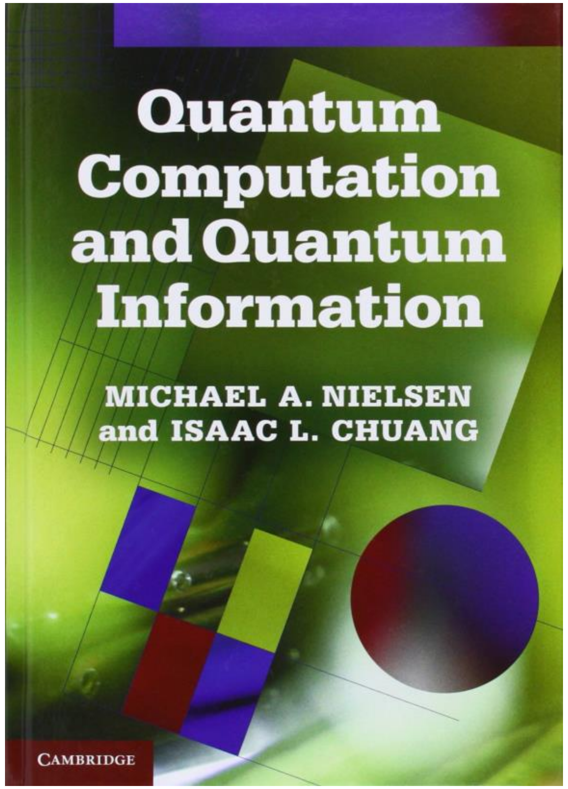
Quantum computing and Quantum computers

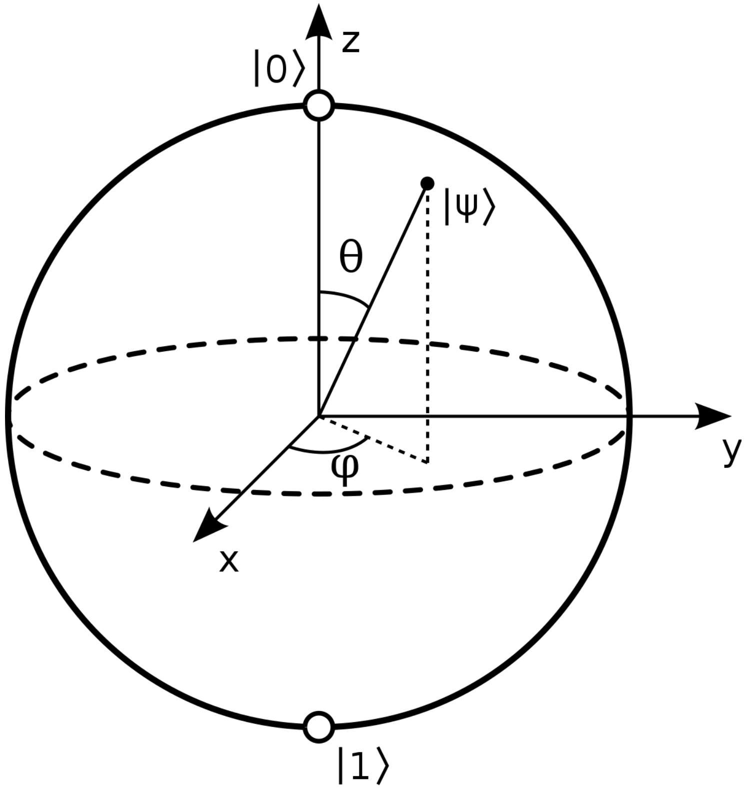
Daniele Ottaviani

CINECA QUANTUM COMPUTING LAB

Introduction to Quantum Computing

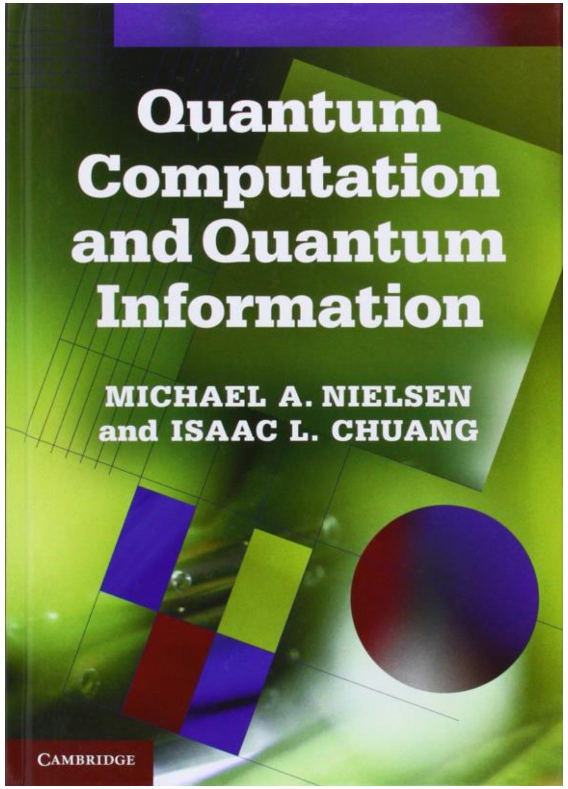
- Quantum computing is a science based on principles of information theory and quantum mechanics

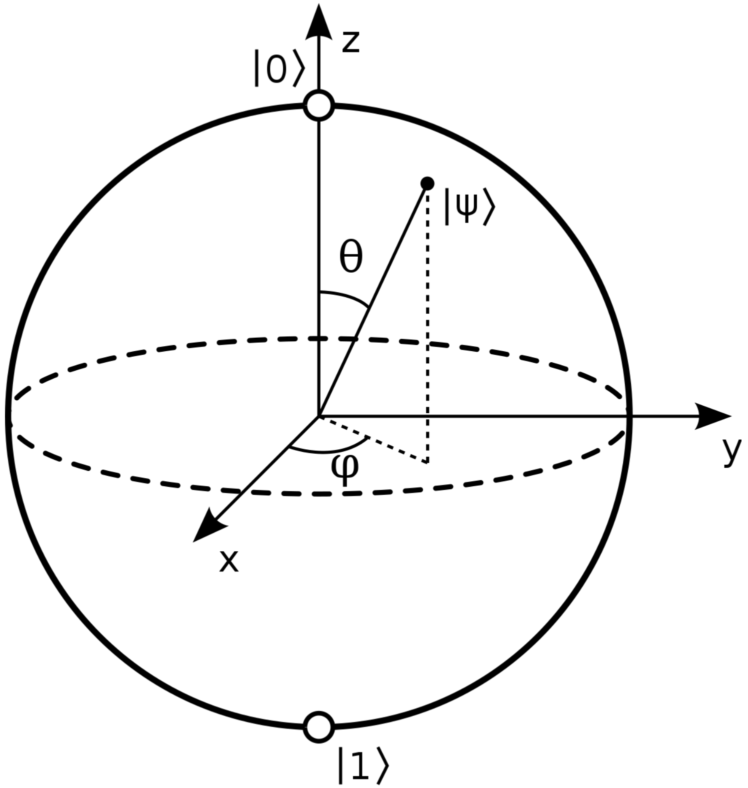




Introduction to Quantum Computing

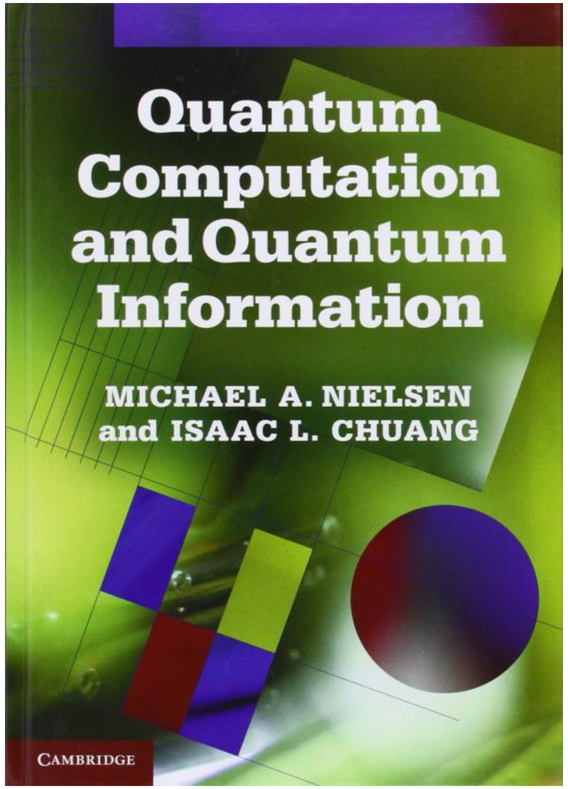
- Quantum computing is a science based on principles of information theory and quantum mechanics
- In its structure it defines the universal language to efficiently write quantum algorithms

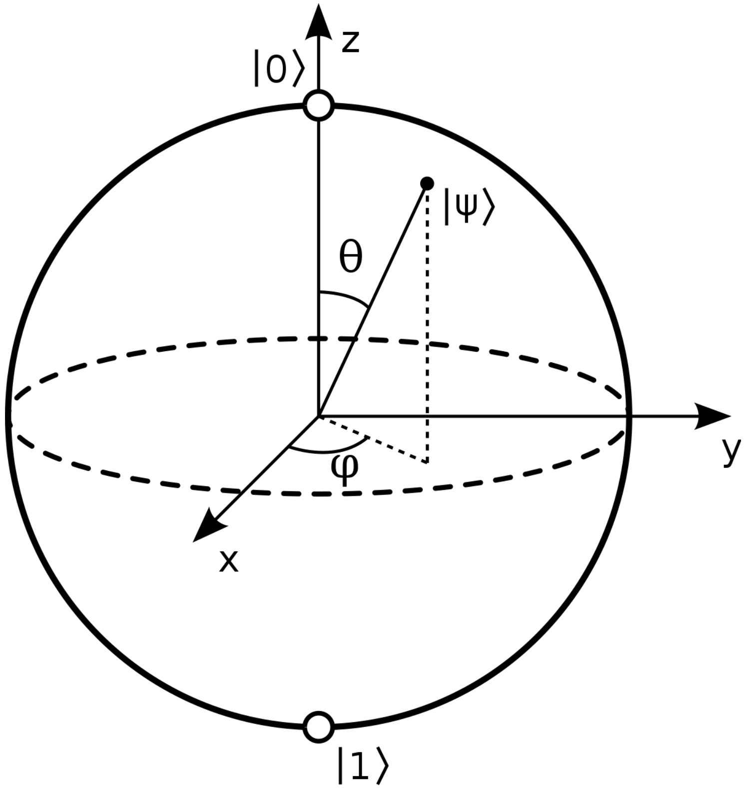




Introduction to Quantum Computing

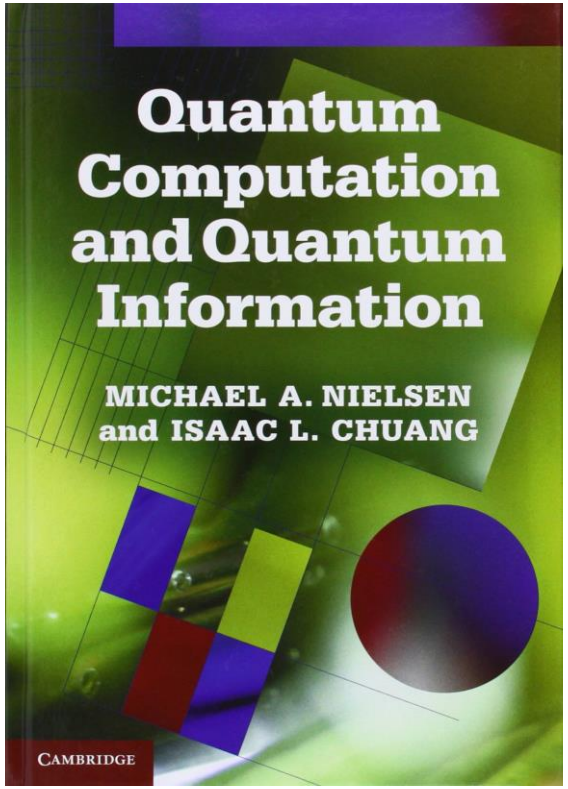
- Quantum computing is a science based on principles of information theory and quantum mechanics
- In its structure it defines the universal language to efficiently write quantum algorithms
- Quantum algorithms are particular algorithms that exploit some properties deriving from quantum mechanics

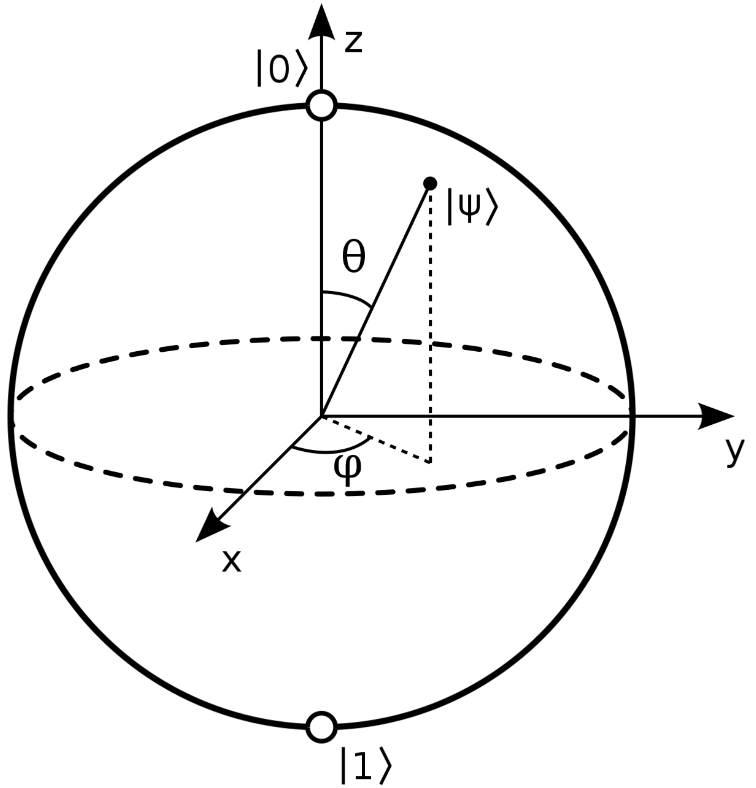




Introduction to Quantum Computing

- Quantum computing is a science based on principles of information theory and quantum mechanics
- In its structure it defines the universal language to efficiently write quantum algorithms
- Quantum algorithms are particular algorithms that exploit some properties deriving from quantum mechanics
- In order to work, quantum algorithms therefore need to operate through computers capable of manipulating objects in which the quantum component is sufficiently manifest

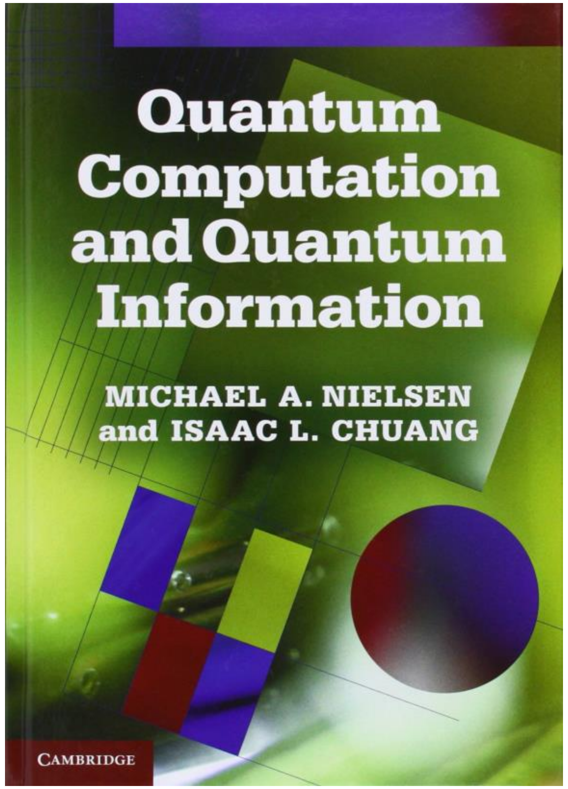


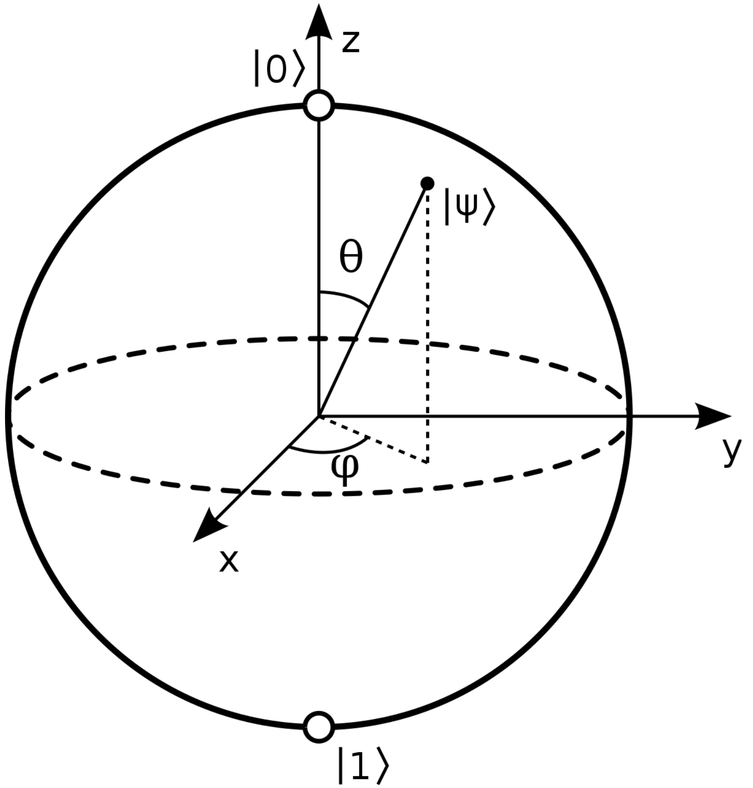


CINECA QUANTUM COMPUTING LAB

Introduction to Quantum Computing

- Quantum computing is a science based on principles of information theory and quantum mechanics
- In its structure it defines the universal language to efficiently write quantum algorithms
- Quantum algorithms are particular algorithms that exploit some properties deriving from quantum mechanics
- In order to work, quantum algorithms therefore need to operate through computers capable of manipulating objects in which the quantum component is sufficiently manifest
- Such computers are called **Quantum Computers**





Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics

Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics
- To understand at least the idea behind it, however, it is good to know some fundamental principles

Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics
- To understand at least the idea behind it, however, it is good to know some fundamental principles
- The first principle we need to know is undoubtedly the superposition principle

Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics
- To understand at least the idea behind it, however, it is good to know some fundamental principles
- The first principle we need to know is undoubtedly the superposition principle

Quantum Superposition Principle

In the quantum mechanical regime, objects do not manifest only a single state, but all the possible states that they can assume at the same time (superposition state). If significantly disturbed (for example, with a measurement), the superposition state collapses and the object is forced to assume a classical state.

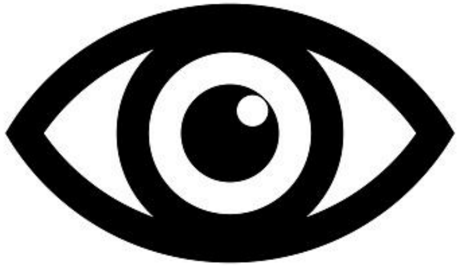
Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics
- To understand at least the idea behind it, however, it is good to know some fundamental principles
- The first principle we need to know is undoubtedly the superposition principle

Measurement

With the term *observation* (or *measurement*) of a quantum state, we mean that process which, through special instruments, quantifies a certain quantity belonging to a microscopic object.

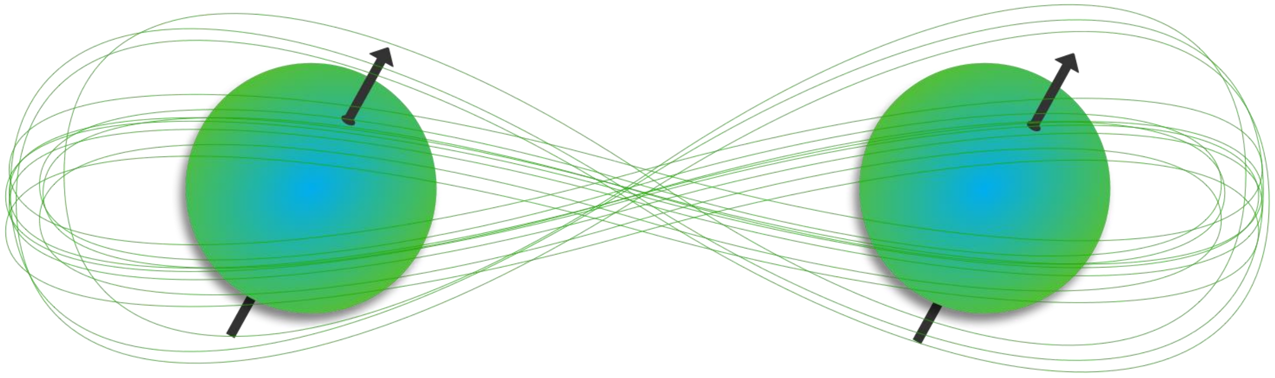
This measurement can be seen as a sort of *intrusion* of the macroscopic world into the microscopic world: consequently, it inevitably perturbs the quantum state of matter, causing the state to collapse into a classical state, the only category of states that we, humans of the macroscopic world, can recognize.



Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics
- To understand at least the idea behind it, however, it is good to know some fundamental principles
- The first principle we need to know is undoubtedly the **superposition principle**
- Second principle is **quantum entanglement**

Quantum Entanglement
Phenomenon closely related to the superposition principle and the collapse of the quantum state into a classical state after a measurement, this principle states that there is a particular condition that can bind two quantum particles regardless of distance.
This link is such that the observation of one of the two particles is also reflected on the other, conditioning the classical state in which it collapses (which can be the same or diametrically opposite)



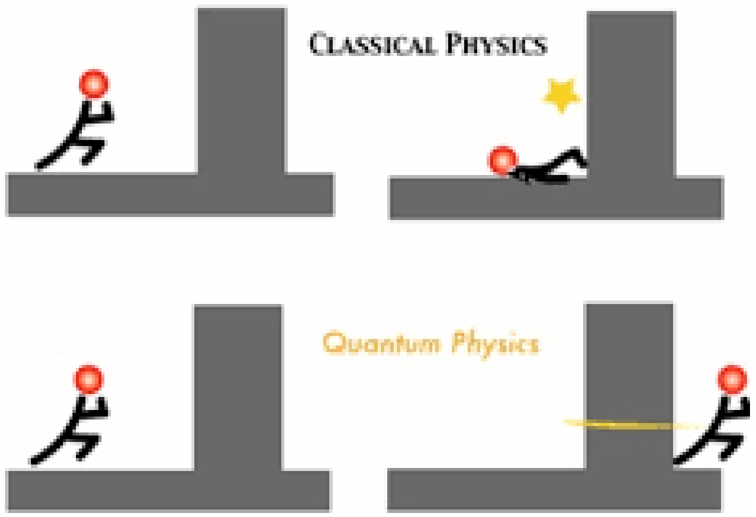
Quantum Mechanics

- To understand quantum computing, we don't necessarily have to be an expert in quantum mechanics
- To understand at least the idea behind it, however, it is good to know some fundamental principles
- The first principle we need to know is undoubtedly the **superposition principle**
- Second principle is **quantum entanglement**
- Third one is the **tunneling effect**

Tunnelling Effect

By definition, we speak of tunneling effect when a particle manages to make a transition that is not normally possible according to the rules of classical mechanics.

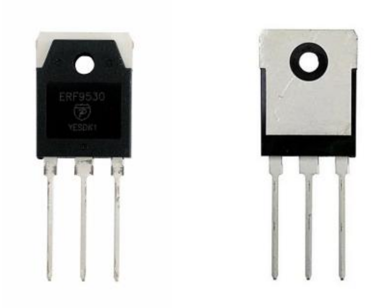
In simple words, this principle states that, under certain circumstances, a quantum particle can cross energy barriers, as if a ball could be able to pass through a wall instead of bouncing against it!



The First Quantum Revolution

- Despite their strange nature, over the years more and more experiments have validated the theories of quantum mechanics

Laser



Transistor

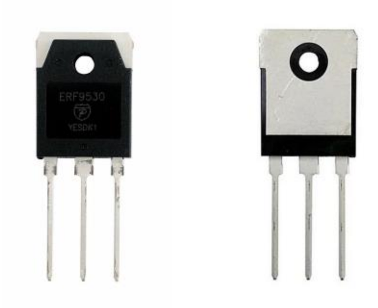
GPS

Touch Screen

The First Quantum Revolution

- Despite their strange nature, over the years more and more experiments have validated the theories of quantum mechanics
- As often happens in science, therefore, after confirming the theories with numerous experiments (exploration phase), we moved on to the exploitation phase, i.e. the concretization of the results obtained by the theory in order to improve everyone's lives.

Laser



Transistor

GPS

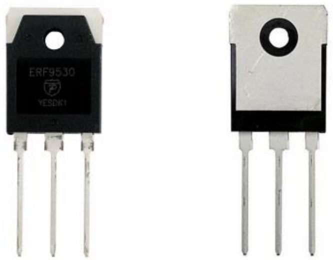
Touch Screen

CINECA QUANTUM COMPUTING LAB

The First Quantum Revolution

- Despite their strange nature, over the years more and more experiments have validated the theories of quantum mechanics
- As often happens in science, therefore, after confirming the theories with numerous experiments (exploration phase), we moved on to the exploitation phase, i.e. the concretization of the results obtained by the theory in order to improve everyone's lives.
- The exploitation phase of quantum physics starts between 1960 and 1970, in the period that we could define as the First Quantum Revolution, and continues to this day.

Laser



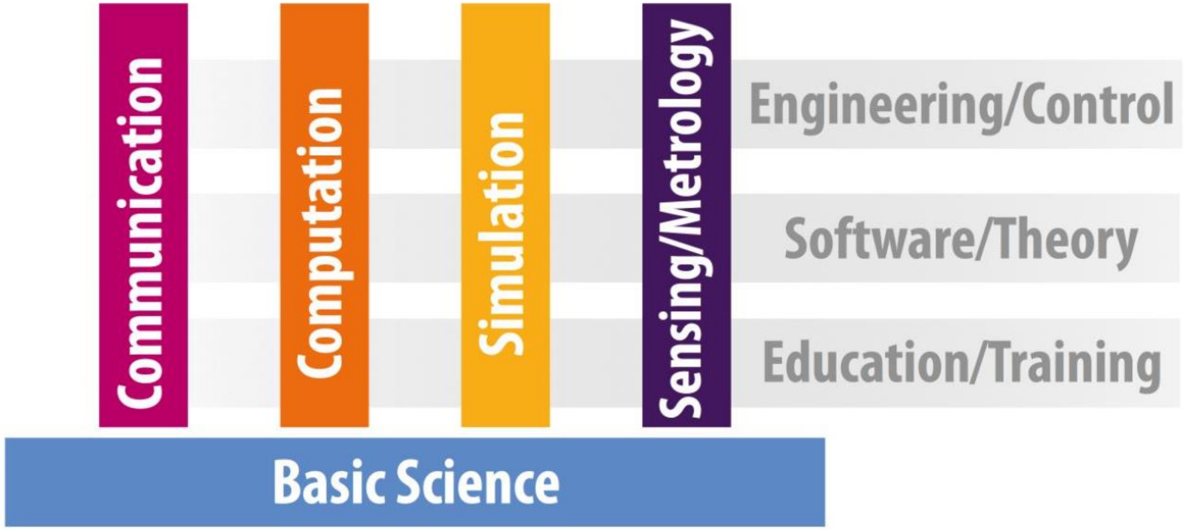
Transistor

GPS

Touch Screen

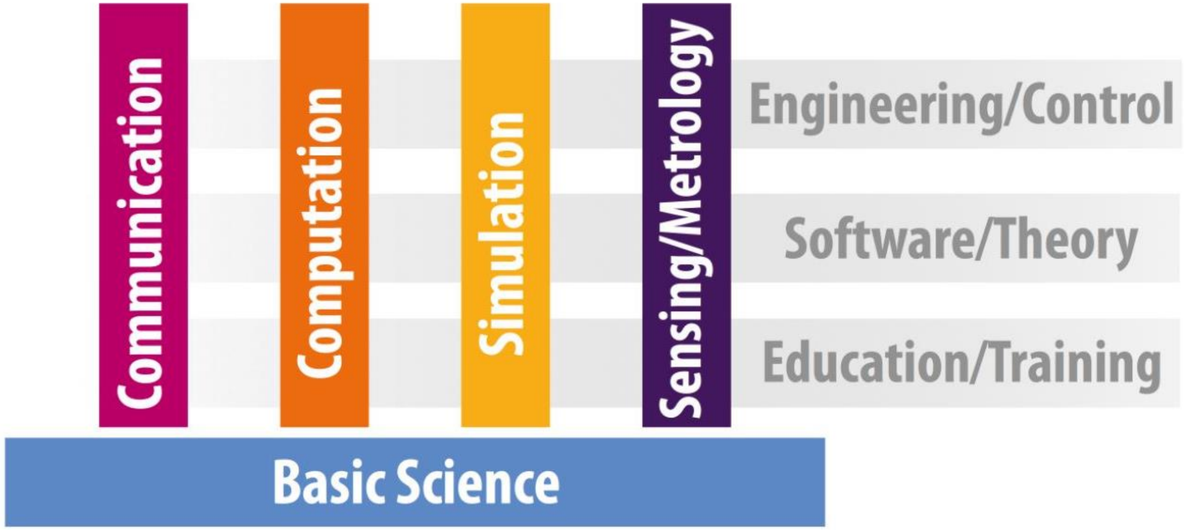
The Second Quantum Revolution

- While in the first quantum revolution the basic concept was:
  We are able to exploit quantum mechanics to dominate the behavior of a ***system*** *of atoms*



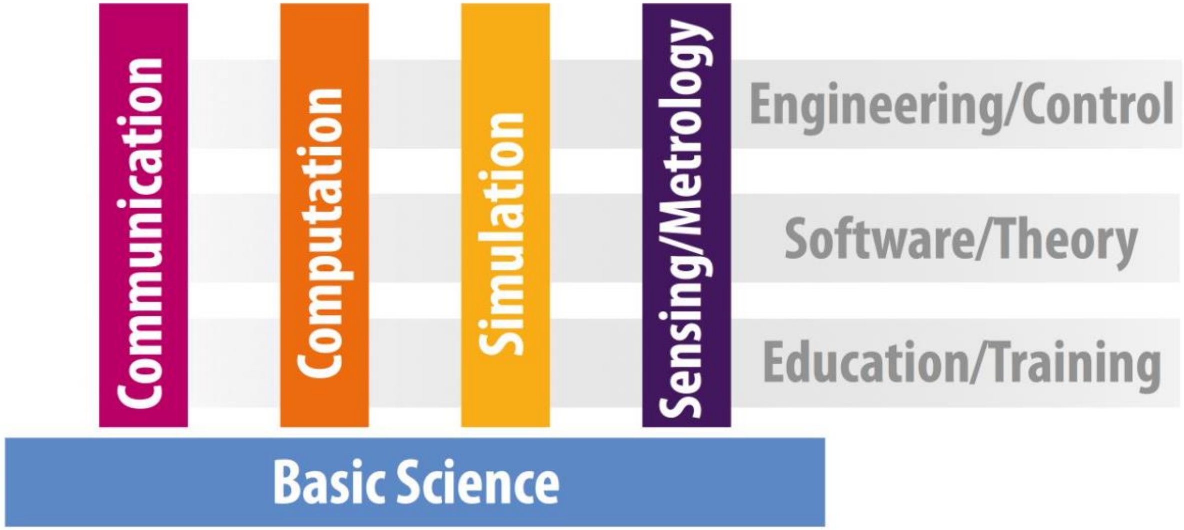
The Second Quantum Revolution

- While in the first quantum revolution the basic concept was:
  We are able to exploit quantum mechanics to dominate the behavior of a ***system*** *of atoms*
- In the context of the second quantum revolution, however, the key concept becomes:
  We are able to exploit quantum mechanics to dominate the behavior of a ***single*** *quantum object*



The Second Quantum Revolution

- While in the first quantum revolution the basic concept was:
  We are able to exploit quantum mechanics to dominate the behavior of a ***system*** *of atoms*
- In the context of the second quantum revolution, however, the key concept becomes:
  We are able to exploit quantum mechanics to dominate the behavior of a ***single*** *quantum object*
- Quantum Technologies are born: Quantum Computing is one of them



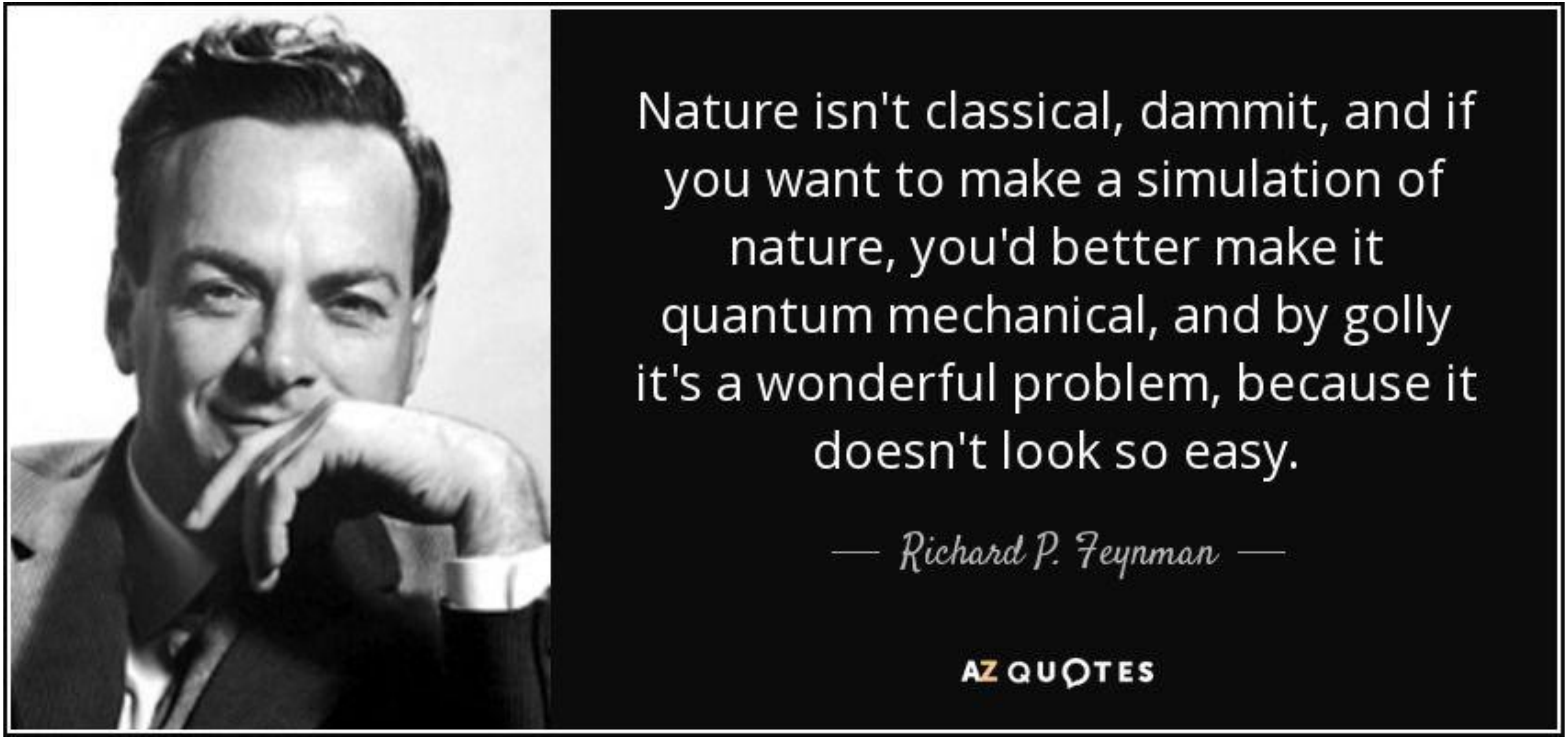
The Idea of a Quantum Computer

- The authorship of the quantum computer idea is historically attributed to the famous physicist Richard P. Feynman

The Idea of a Quantum Computer

- The authorship of the quantum computer idea is historically attributed to the famous physicist Richard P. Feynman
- Feynman was one of the first to think of an alternative computer, albeit in the midst of the development of classical computers (1982). His idea was basically that of using computers that had as a basis for calculation not classical objects, such as the bits crossed by the current, but some other object in which quantum behaviors could manifest themselves visibly.

The Idea of a Quantum Computer



The Idea of a Quantum Computer

- The authorship of the quantum computer idea is historically attributed to the famous physicist Richard P. Feynman
- Feynman was one of the first to think of an alternative computer, albeit in the midst of the development of classical computers (1982). His idea was basically that of using computers that had as a basis for calculation not classical objects, such as the bits crossed by the current, but some other object in which quantum behaviors could manifest themselves visibly.
- With the sentence reported in the previous slide, Feynman has attracted the attention of the scientific community to a non-trivial problem

The Idea of a Quantum Computer

- The authorship of the quantum computer idea is historically attributed to the famous physicist Richard P. Feynman
- Feynman was one of the first to think of an alternative computer, albeit in the midst of the development of classical computers (1982). His idea was basically that of using computers that had as a basis for calculation not classical objects, such as the bits crossed by the current, but some other object in which quantum behaviors could manifest themselves visibly.
- With the sentence reported in the previous slide, Feynman has attracted the attention of the scientific community to a non-trivial problem
- In parallel, many other scientists (Deutsch, Benioff) also began to develop the concept of quantum computer: quantum computing was born

The Qubit

- In classical computing, the basic logical unit for a computer is the BIT

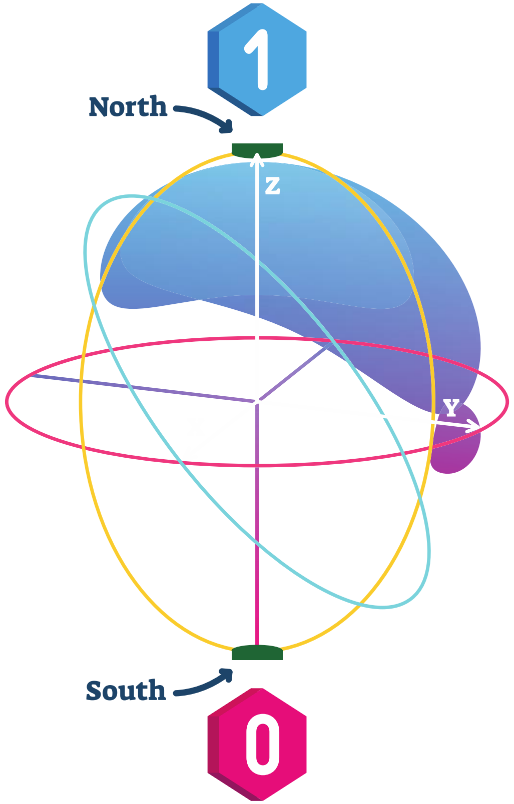
The Qubit

- In classical computing, the basic logical unit for a computer is the **BIT**
- The BIT is an object capable of assuming **only two states**, on and off (commonly 0 and 1)

The Qubit

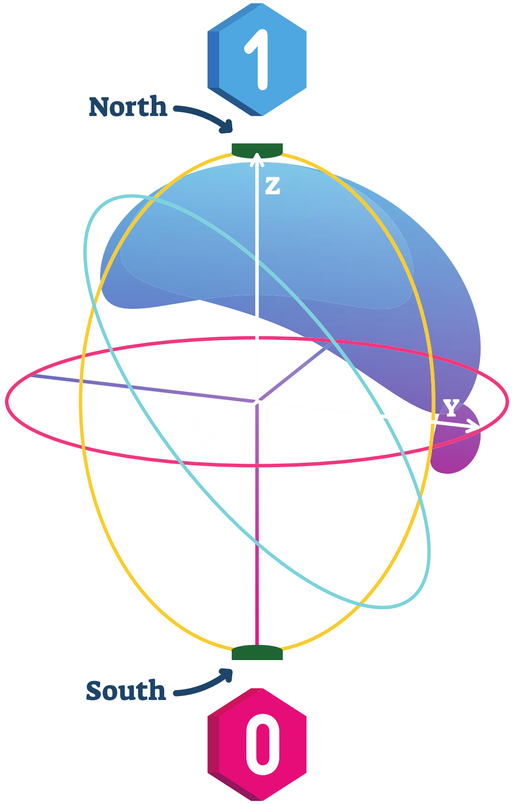
- In classical computing, the basic logical unit for a computer is the BIT
- The BIT is an object capable of assuming **only two states**, on and off (commonly 0 and 1)
- The bit is considered the **basic logical unit of a computer** since every problem, regardless of its complexity, must be translated into binary language understandable to a computer through the manipulation of its bits.

The Qubit



- In classical computing, the basic logical unit for a computer is the BIT
- The BIT is an object capable of assuming **only two states**, on and off (commonly 0 and 1)
- The bit is considered the **basic logical unit of a computer** since every problem, regardless of its complexity, must be translated into binary language understandable to a computer through the manipulation of its bits.
- The idea behind quantum computing is to **replace the bit with something that is capable of manifesting evident quantum behaviors**, first of all **superposition** (between the 0 and 1 state)

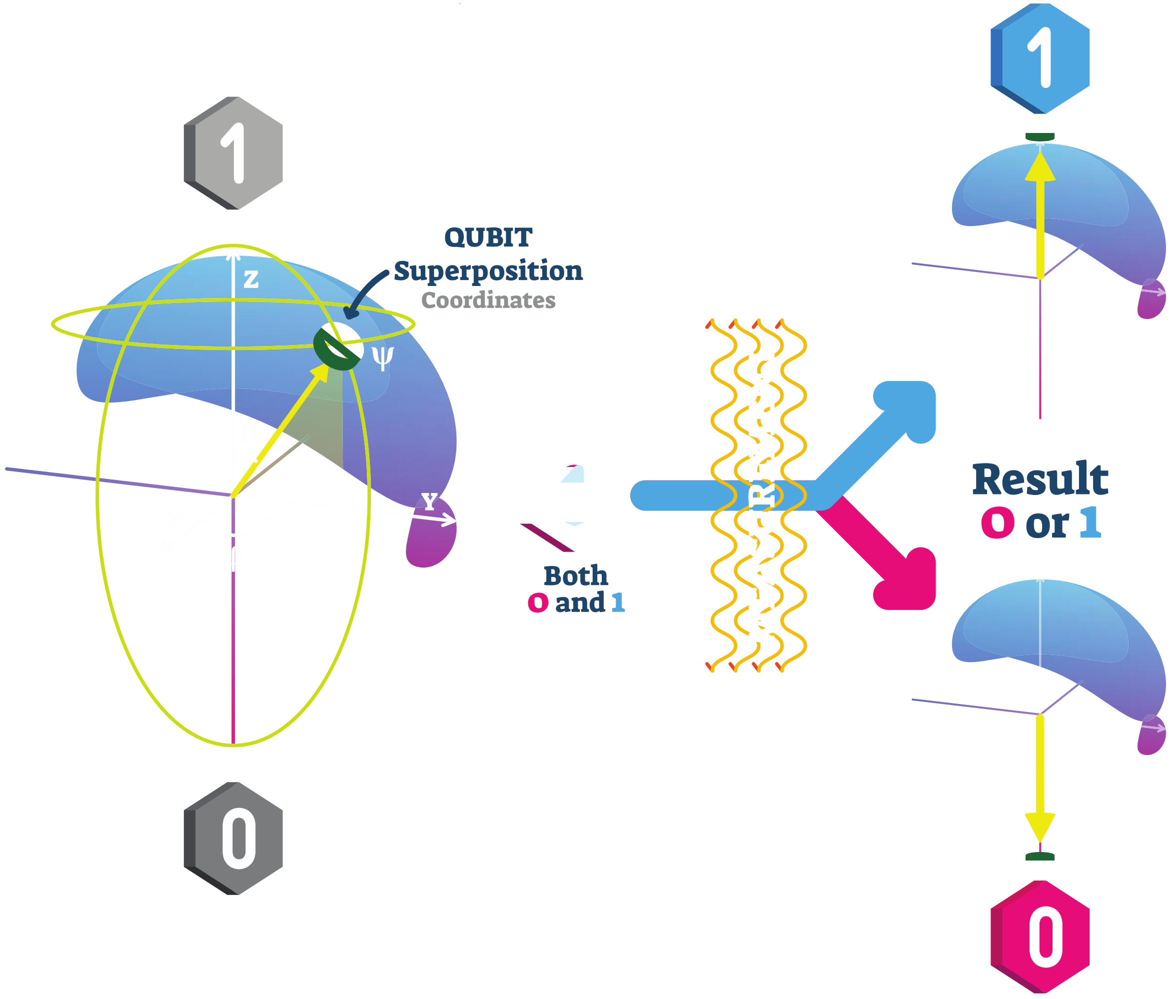
The Qubit



- In classical computing, the basic logical unit for a computer is the **BIT**
- The BIT is an object capable of assuming **only two states**, on and off (commonly 0 and 1)
- The bit is considered the **basic logical unit of a computer** since every problem, regardless of its complexity, must be translated into binary language understandable to a computer through the manipulation of its bits.
- The idea behind quantum computing is to **replace the bit with something that is capable of manifesting evident quantum behaviors**, first of all **superposition** (between the 0 and 1 state)
- This *something* is called **Qubit**

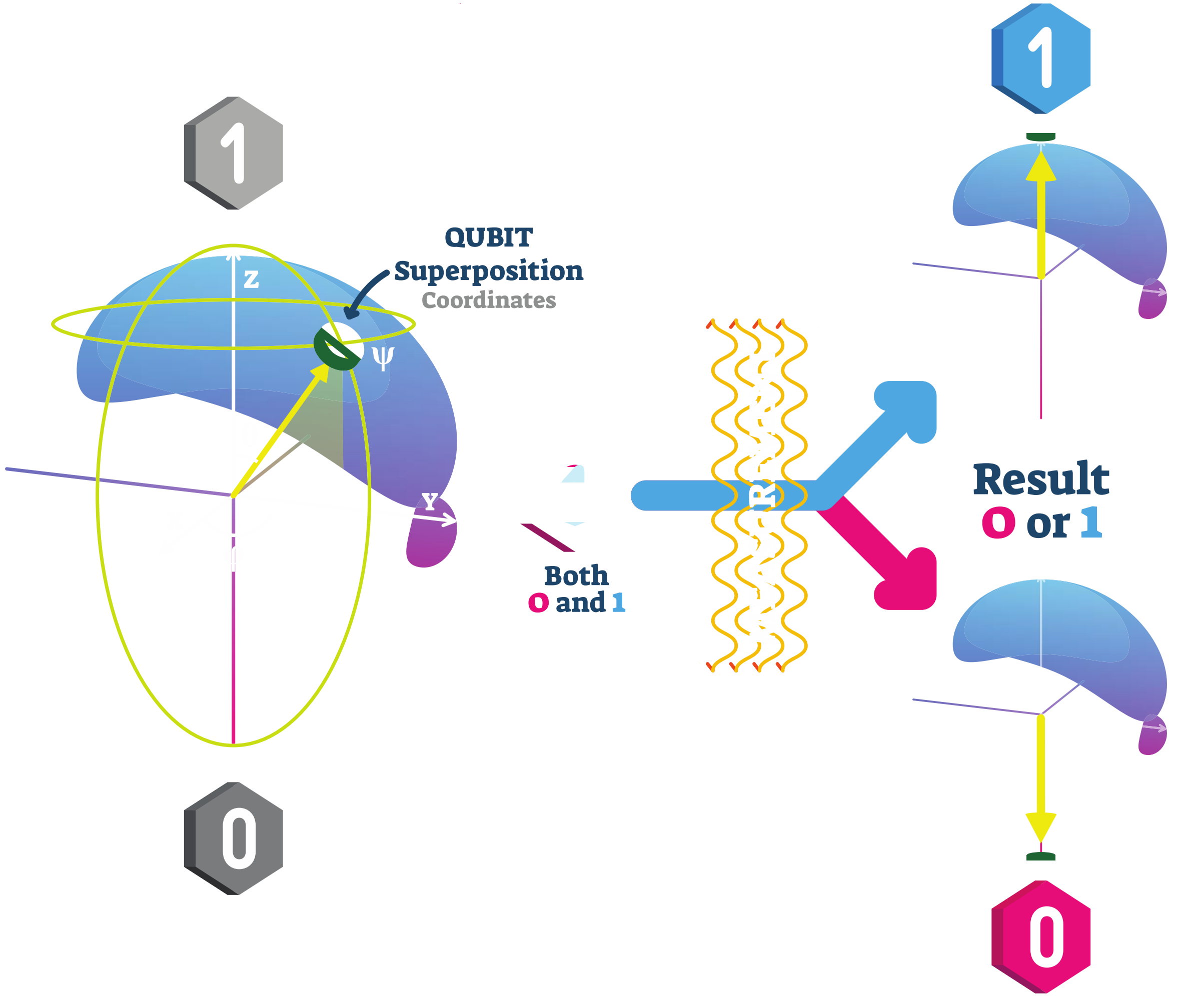
The Qubit

- A **QUBIT** is the quantum equivalent of a bit: it can be represented by an **atom**, a trapped ion or by macroscopic objects which, under particular conditions, can exhibit quantum behaviors



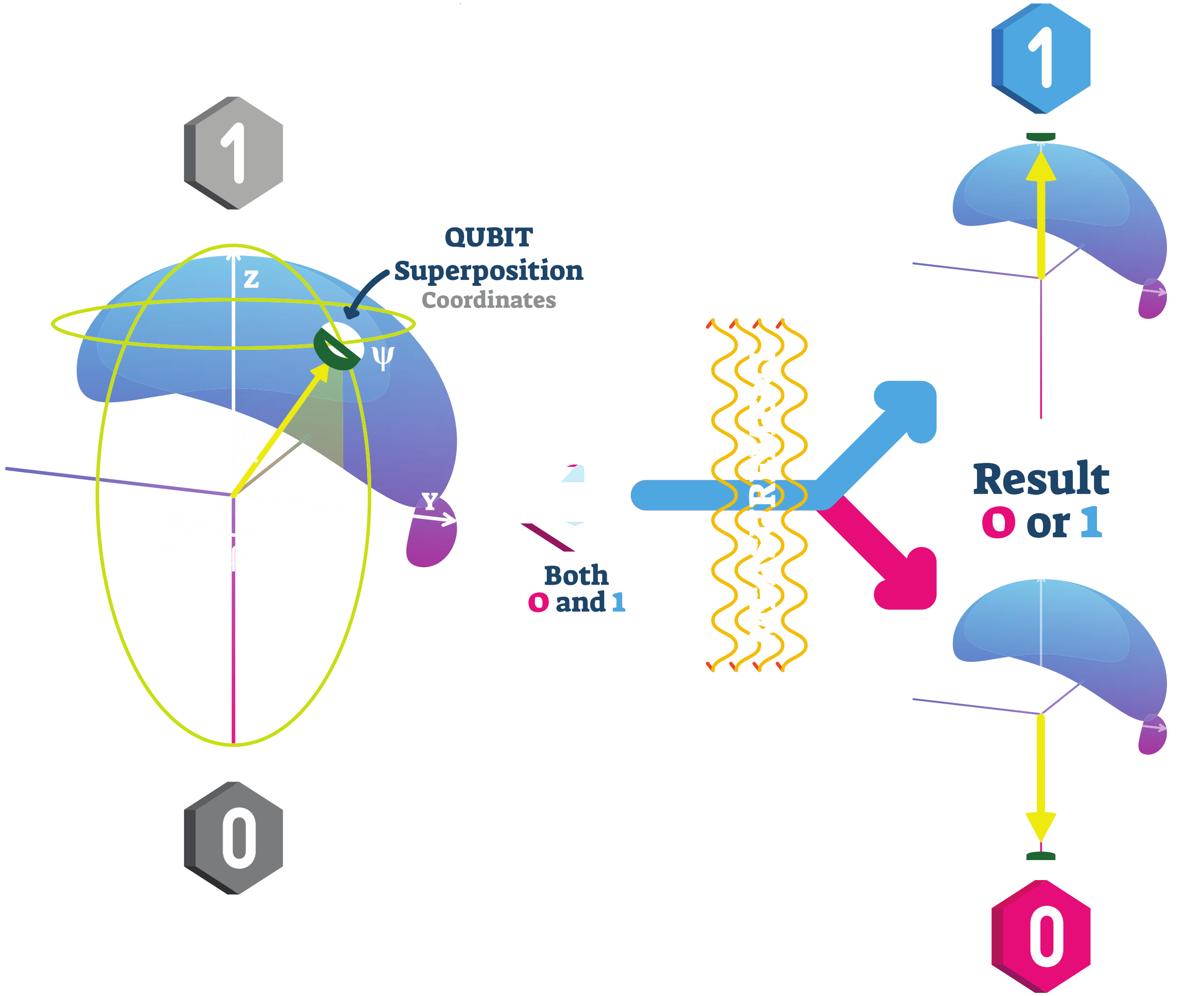
The Qubit

- A **QUBIT** is the quantum equivalent of a bit: it can be represented by an **atom**, a trapped ion or by macroscopic objects which, under particular conditions, can exhibit quantum behaviors
- On the basis of the qubit chosen, it is determined **what state is the 0-state and what state is the 1-state** (it can be, for example, the direction of the current that runs through the superconducting qubit or the spin for an atom)



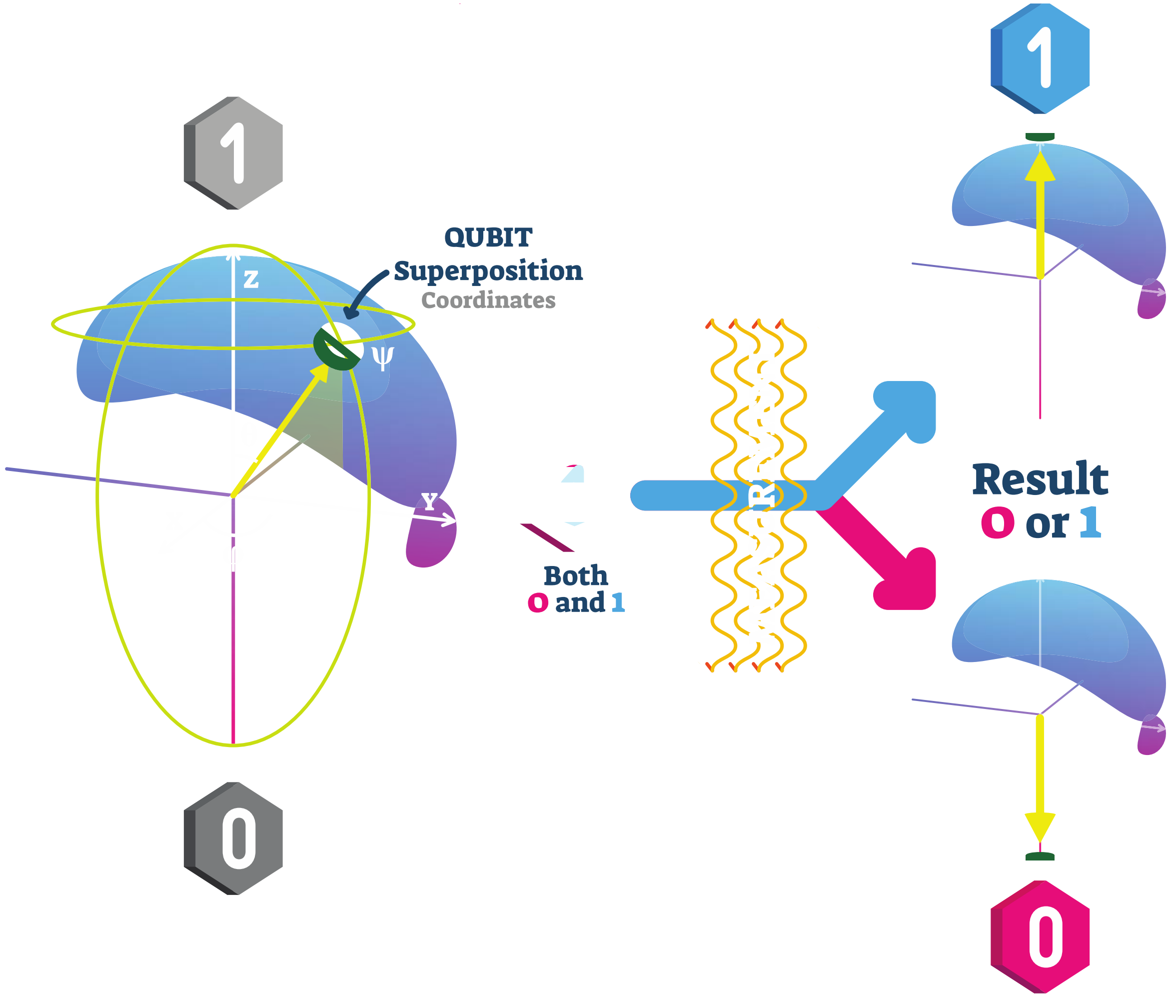
The Qubit

- A **QUBIT** is the quantum equivalent of a bit: it can be represented by an **atom**, a trapped ion or by macroscopic objects which, under particular conditions, can exhibit quantum behaviors
- On the basis of the qubit chosen, it is determined **what state is the 0-state and what state is the 1-state** (it can be, for example, the direction of the current that runs through the superconducting qubit or the spin for an atom)
- In this way, the qubit allows us to take quantum effects into account in its manipulation



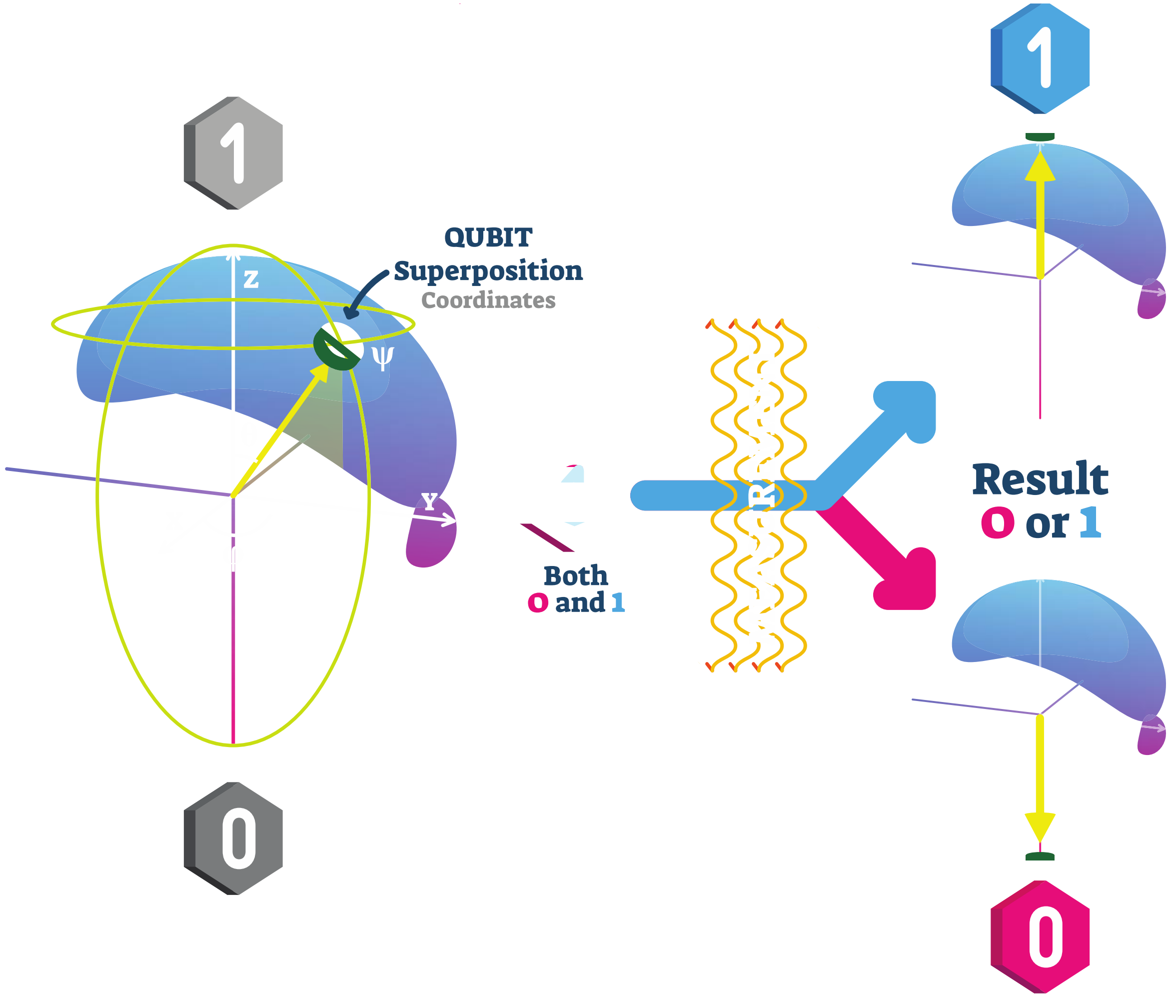
The Qubit

- But how can we manipulate qubits while taking into account the effects induced by quantum mechanics? To do this, we need to start introducing the **concept of quantum computing**



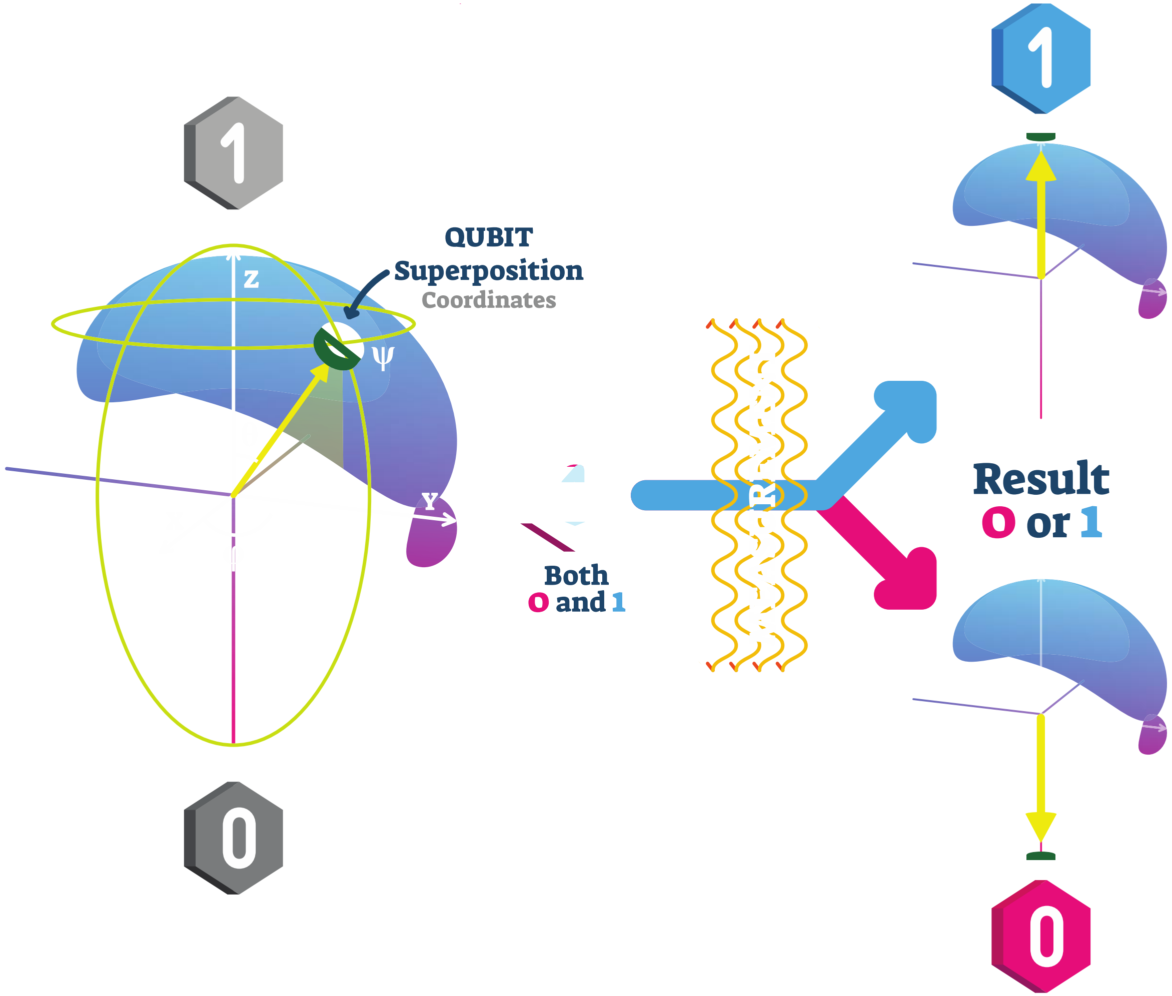
The Qubit

- But how can we manipulate qubits while taking into account the effects induced by quantum mechanics? To do this, we need to start introducing the **concept of quantum computing**
- Quantum computing is a science that combines the mathematical formalism of quantum mechanics with the creation of computing circuits of classical computation



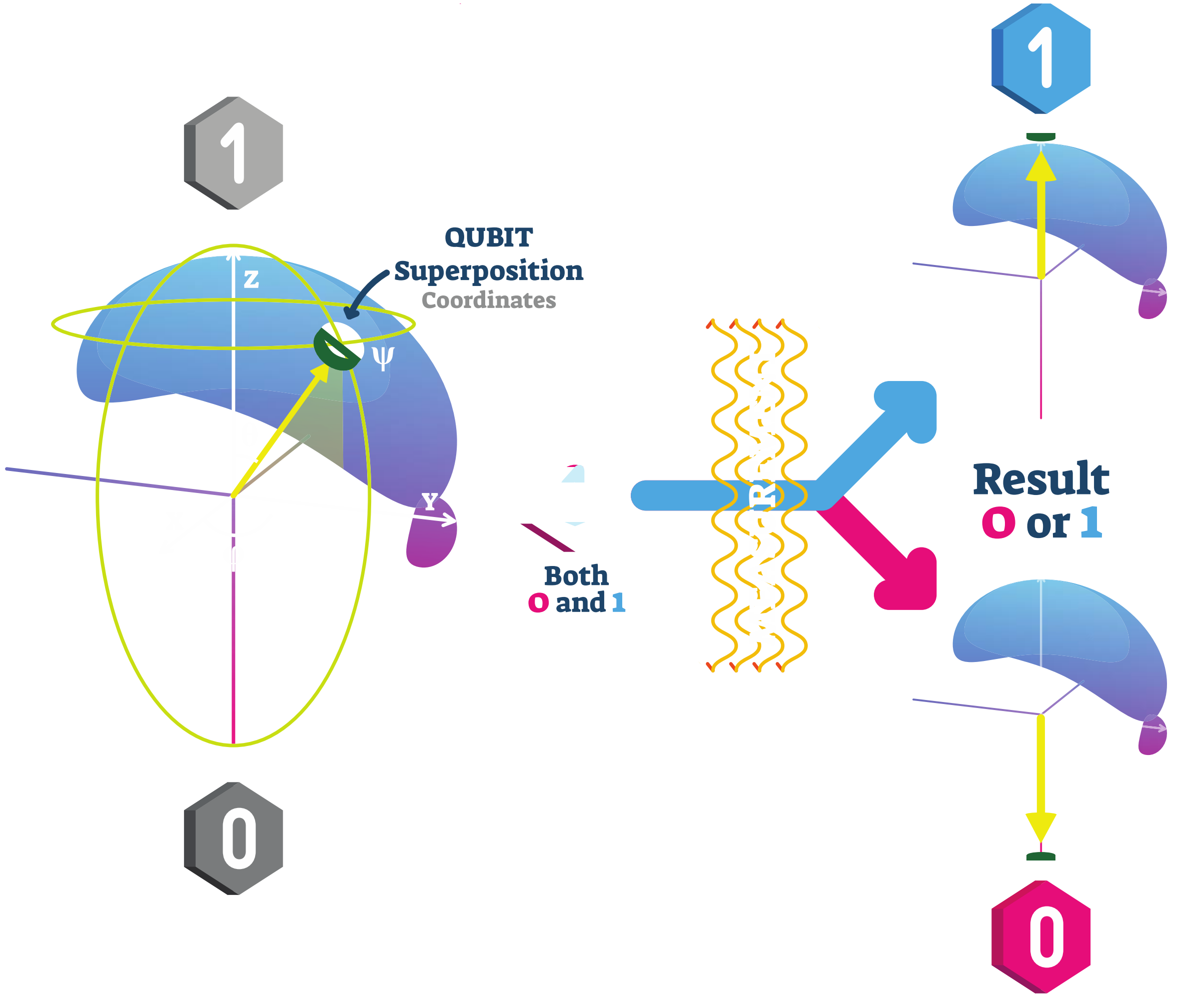
The Qubit

- But how can we manipulate qubits while taking into account the effects induced by quantum mechanics? To do this, we need to start introducing the **concept of quantum computing**
- Quantum computing is a science that combines the mathematical formalism of quantum mechanics with the creation of computing circuits of classical computation
- To fully understand it, it is good to first do a little review of the tools that we will use during this lesson



The Qubit

- But how can we manipulate qubits while taking into account the effects induced by quantum mechanics? To do this, we need to start introducing the concept of quantum computing
- Quantum computing is a science that combines the mathematical formalism of quantum mechanics with the creation of computing circuits of classical computation
- To fully understand it, it is good to first do a little review of the tools that we will use during this lesson
- Let's start with a quick review of linear algebra and quantum mechanics mathematical formalism



Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements

$$|\psi\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}, \qquad \langle\rho| = (\rho_0, \rho_1, \dots, \rho_N)$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product

$$|\psi\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}, \qquad \langle\rho| = (\rho_0, \rho_1, \dots, \rho_N)$$

$$\langle\rho|\psi\rangle = \sum_{i=1}^{N} \rho_i \cdot \psi_i \in \mathbb{C}$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product

$$|\psi\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}, \quad \langle\rho| = (\rho_0, \rho_1, \dots, \rho_N)$$

$$\langle\rho|\psi\rangle = \sum_{i=1}^{N} \rho_i \cdot \psi_i \in \mathbb{C}$$

$$|\rho\rangle\langle\psi| = \rho \otimes \psi = \{\rho_i \psi_j\}_{ij} \in \mathbb{C}^{N\times N}$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \cdot x + b \cdot y \\ c \cdot x + d \cdot y \end{pmatrix}$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \cdot x + b \cdot y \\ c \cdot x + d \cdot y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 2 + 3 \cdot (-1) \\ 2 \cdot 2 + 1 \cdot (-1) \end{pmatrix}$$

$$= \begin{pmatrix} 2 - 3 \\ 4 - 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication
- Tensor product between matrices

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} =$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication
- Tensor product between matrices

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} =$$

$$= \begin{bmatrix} a_{1,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{1,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \\ a_{2,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{2,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \end{bmatrix} =$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication
- Tensor product between matrices

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} =$$

$$= \begin{bmatrix} a_{1,1}b_{1,1} & a_{1,1}b_{1,2} & a_{1,2}b_{1,1} & a_{1,2}b_{1,2} \\ a_{1,1}b_{2,1} & a_{1,1}b_{2,2} & a_{1,2}b_{2,1} & a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} & a_{2,1}b_{1,2} & a_{2,2}b_{1,1} & a_{2,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,1}b_{2,2} & a_{2,2}b_{2,1} & a_{2,2}b_{2,2} \end{bmatrix}.$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication
- Tensor product between matrices
- Unitary Matrices

a complex square matrix U is unitary if its conjugate transpose U* is also its inverse, that is, if

$$UU^* = U^*U = I$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication
- Tensor product between matrices
- Unitary Matrices

a complex square matrix U is unitary if its conjugate transpose U* is also its inverse, that is, if

$$UU^* = U^*U = I$$

Unitary matrices have significant importance in quantum mechanics because they preserve norms.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ z \end{pmatrix}$$

$$|(x, y)| = C \Rightarrow |(t, z)| = C$$

Quantum Mechanics and Linear Algebra

- To understand the basics of quantum computing it is not absolutely necessary to be an expert in quantum mechanics. However, it can be convenient to know a minimum of formalism
- Dirac Notation: a way to indicate a vector of elements
- Inner (dot) Product
- Outer (tensor) Product
- Matrix-Vector Multiplication
- Tensor product between matrices
- Unitary Matrices

a complex square matrix U is unitary if its conjugate transpose U* is also its inverse, that is, if

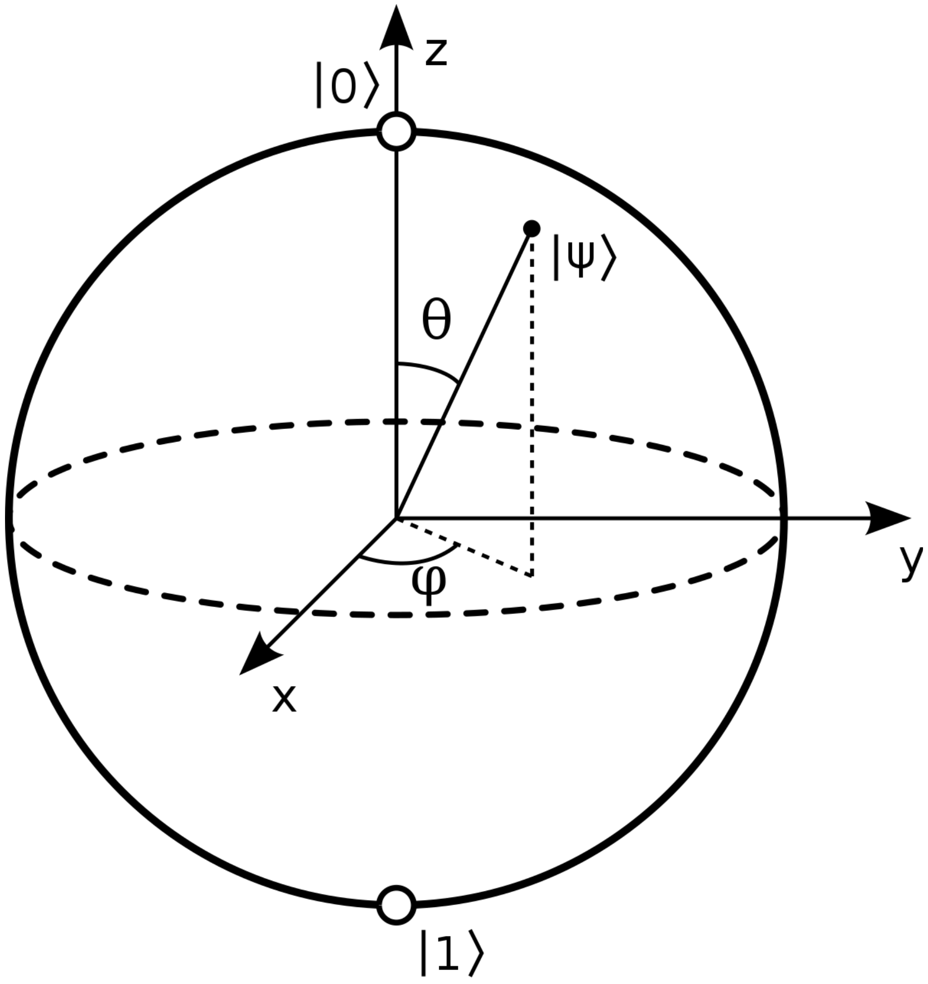
$$UU^* = U^*U = I$$

Unitary matrices have significant importance in quantum mechanics because they preserve norms.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ z \end{pmatrix}$$

$$|(x, y)| = 1 \Rightarrow |(t, z)| = 1$$

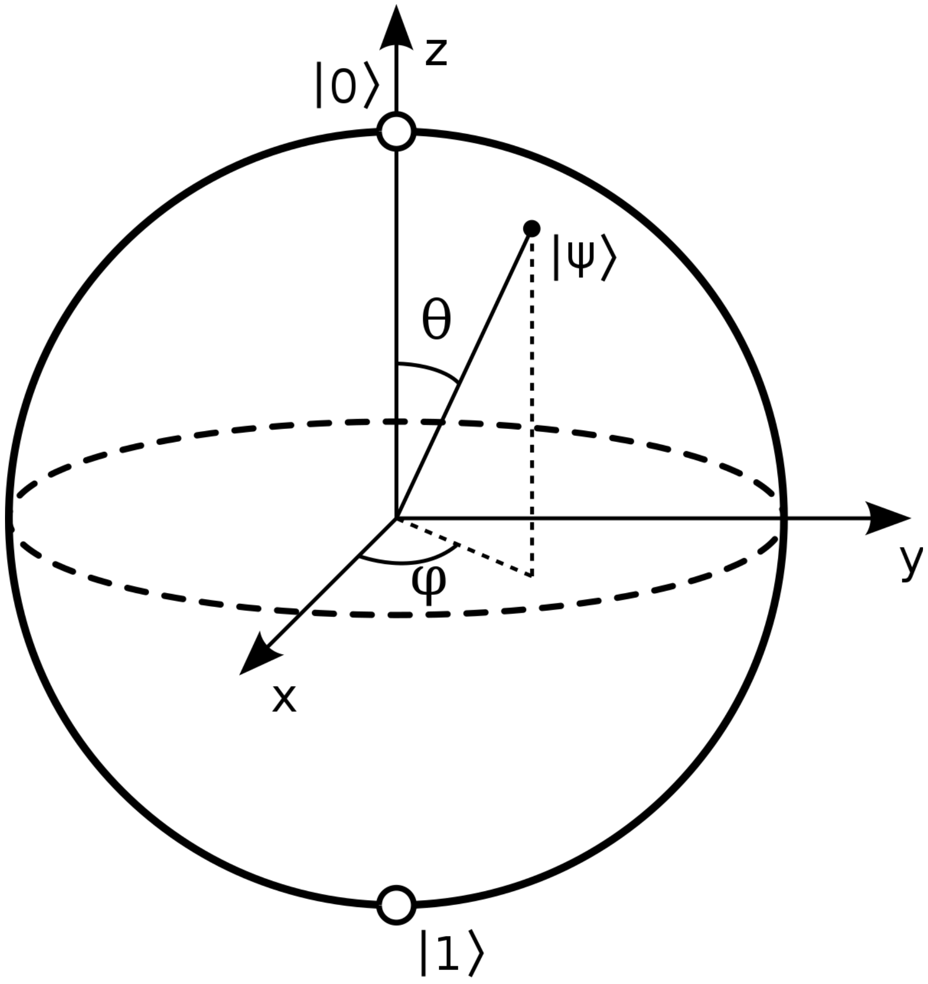
Qubits and Bloch Sphere



- A qubit is defined as a ray of the Bloch sphere.

CINECA QUANTUM COMPUTING LAB

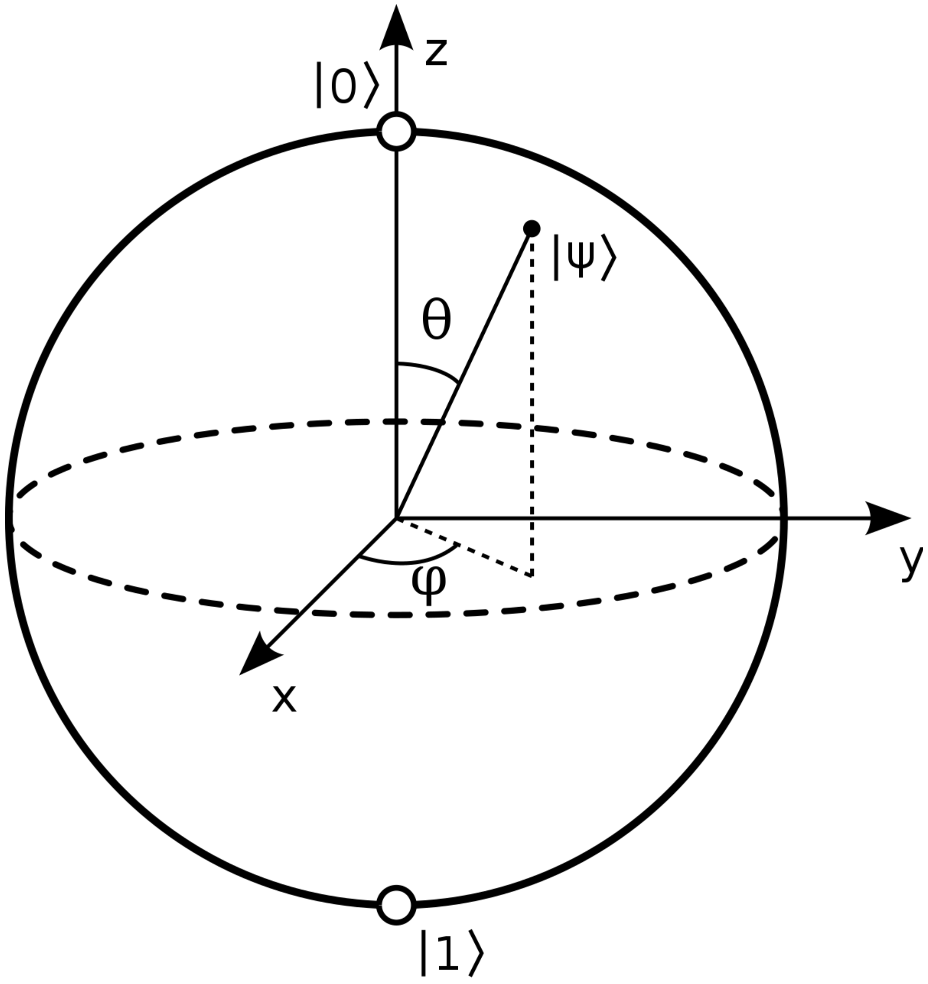
Qubits and Bloch Sphere



- A qubit is defined as a ray of the Bloch sphere.
- The Bloch sphere is a mathematical object defined in C^2, whose elements on the surface can be identified by the coordinates

$$\left(\cos\left(\frac{\theta}{2}\right), e^{i\varphi}\sin\left(\frac{\theta}{2}\right)\right)$$

Qubits and Bloch Sphere

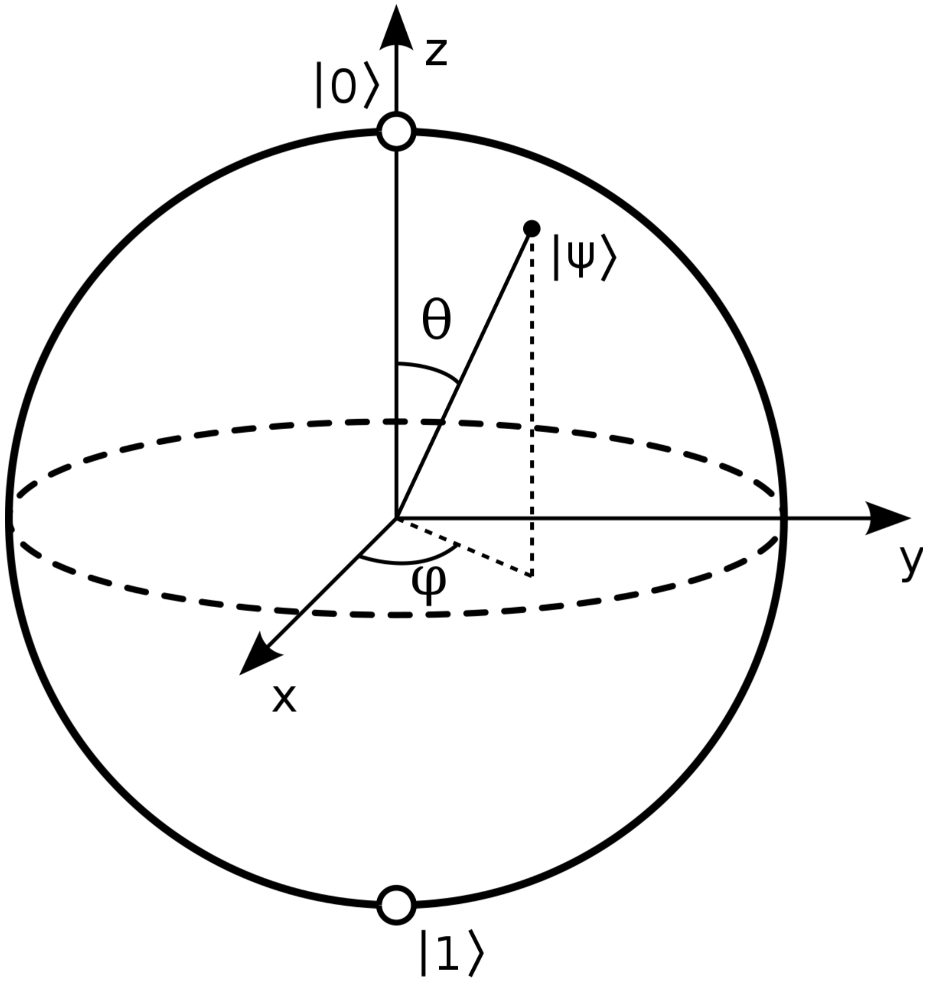


- A qubit is defined as a ray of the Bloch sphere.
- The Bloch sphere is a mathematical object defined in C^2, whose elements on the surface can be identified by the coordinates

$$\left(\cos\left(\frac{\theta}{2}\right), e^{i\varphi} \sin\left(\frac{\theta}{2}\right) \right)$$

- The poles of the Bloch sphere represent the classical states 0 and 1. As we can see, they are highlighted in Dirac notation since they are defined by two vectors

Qubits and Bloch Sphere

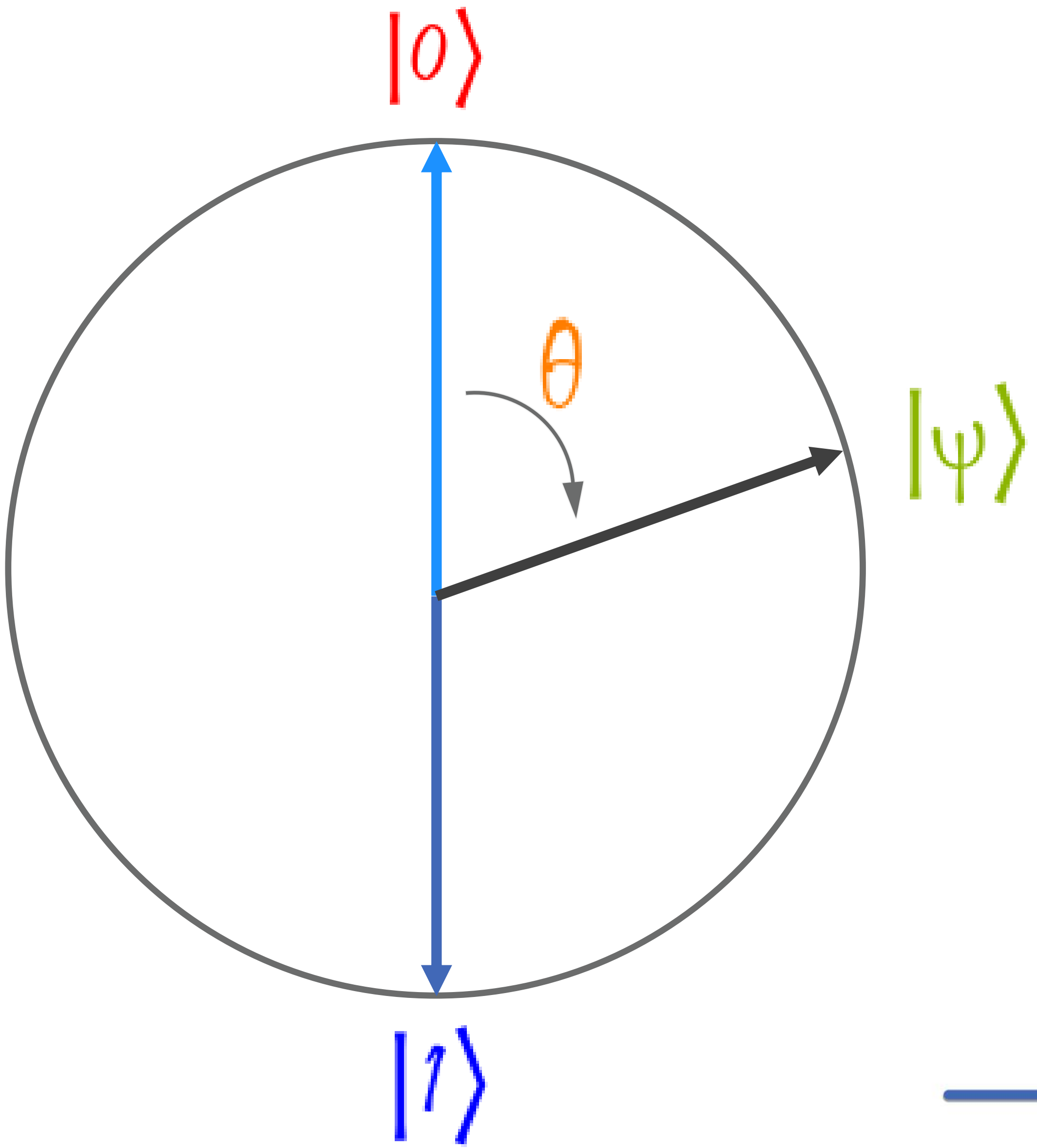


- A qubit is defined as a ray of the Bloch sphere.
- The Bloch sphere is a mathematical object defined in C^2, whose elements on the surface can be identified by the coordinates

$$\left(\cos\left(\frac{\theta}{2}\right), e^{i\varphi}\sin\left(\frac{\theta}{2}\right)\right)$$

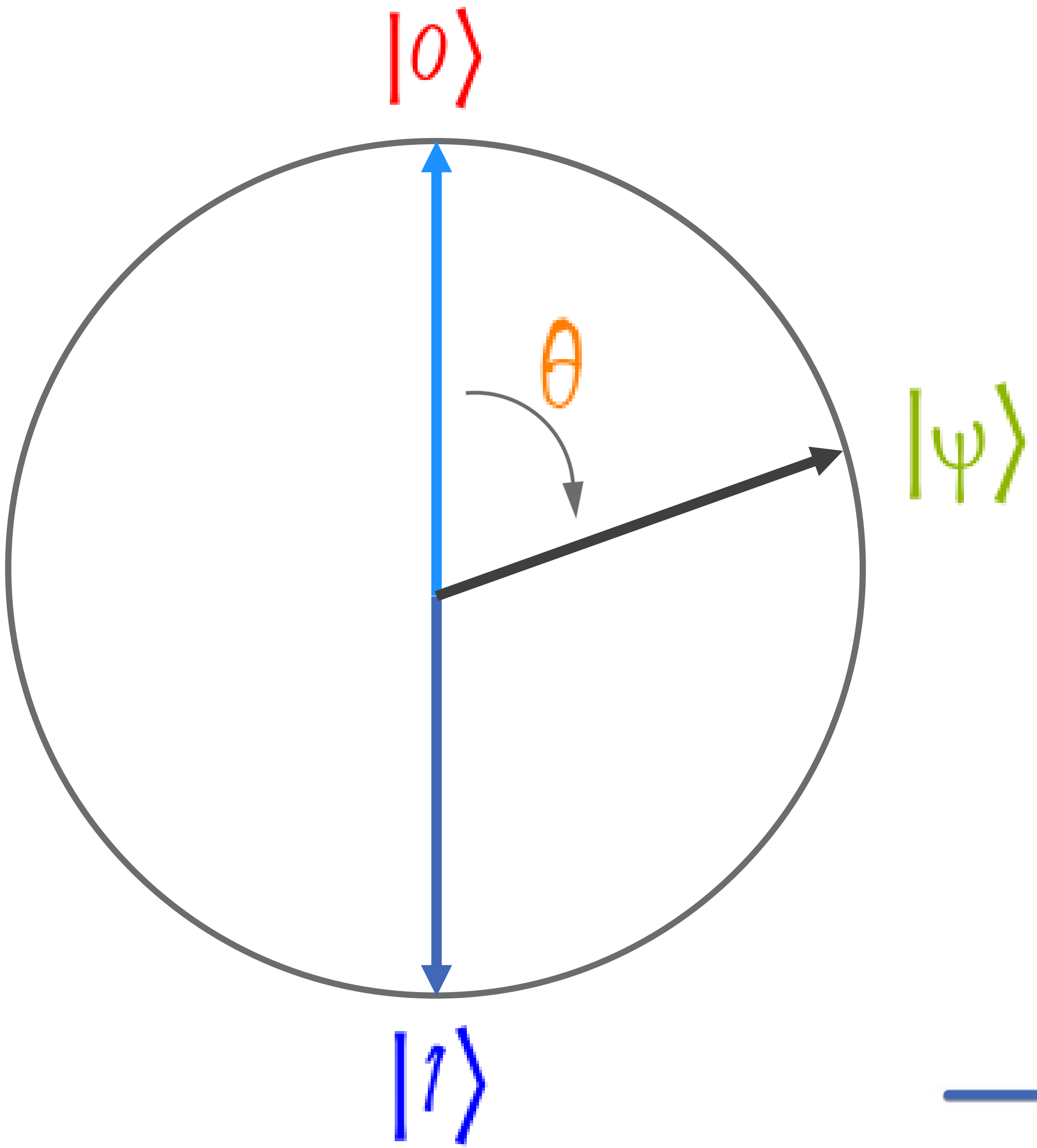
- The poles of the Bloch sphere represent the classical states 0 and 1. As we can see, they are highlighted in Dirac notation since they are defined by two vectors
- Without losing generality, we can analyze the Bloch sphere only in the real field, ignoring the variable phi and bringing us back to a circumference

Qubits and Bloch Sphere



$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

Qubits and Bloch Sphere

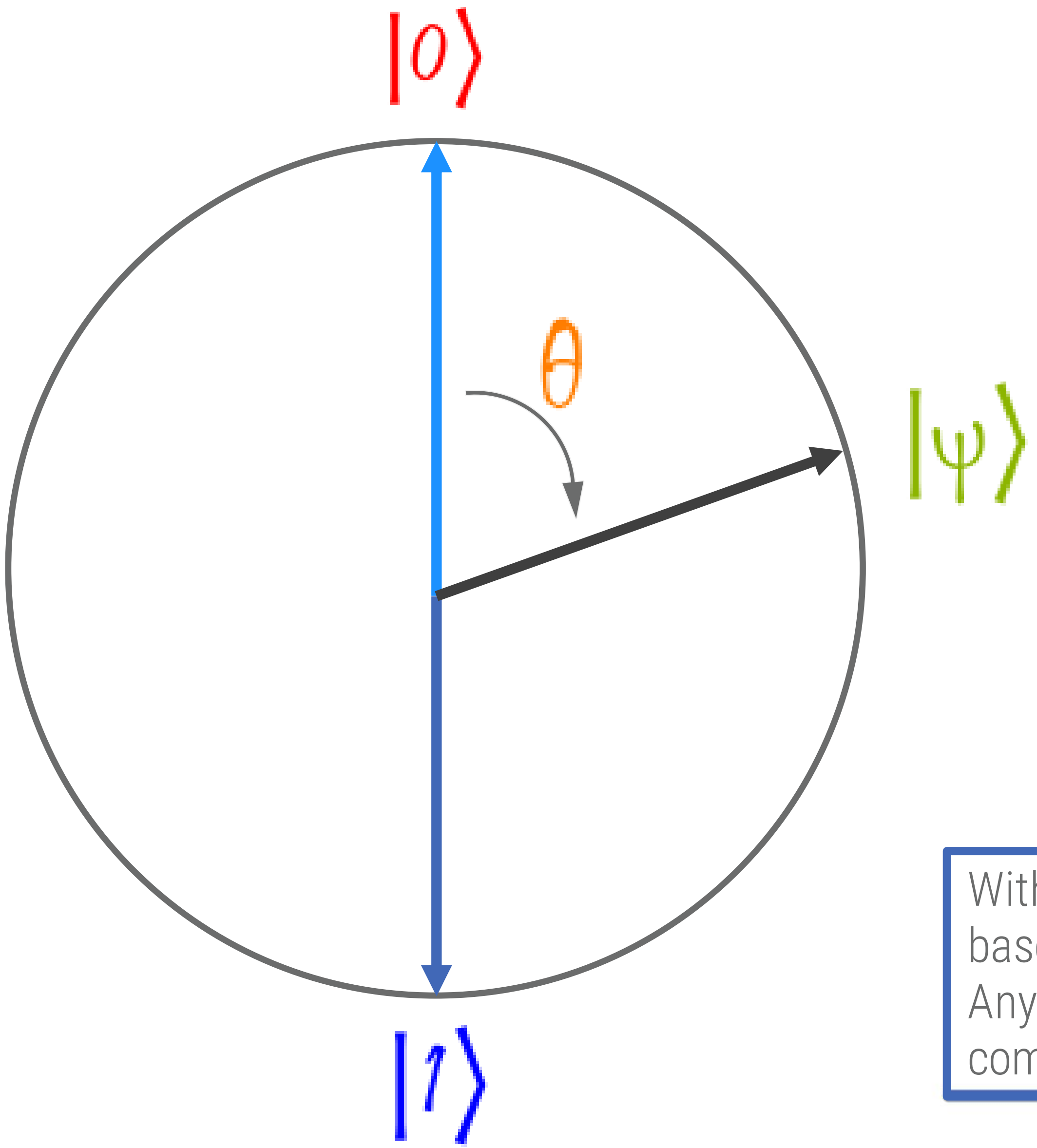


$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$|0\rangle = \left(\cos\left(\frac{0}{2}\right), \sin\left(\frac{0}{2}\right)\right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \left(\cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right)\right) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Qubits and Bloch Sphere



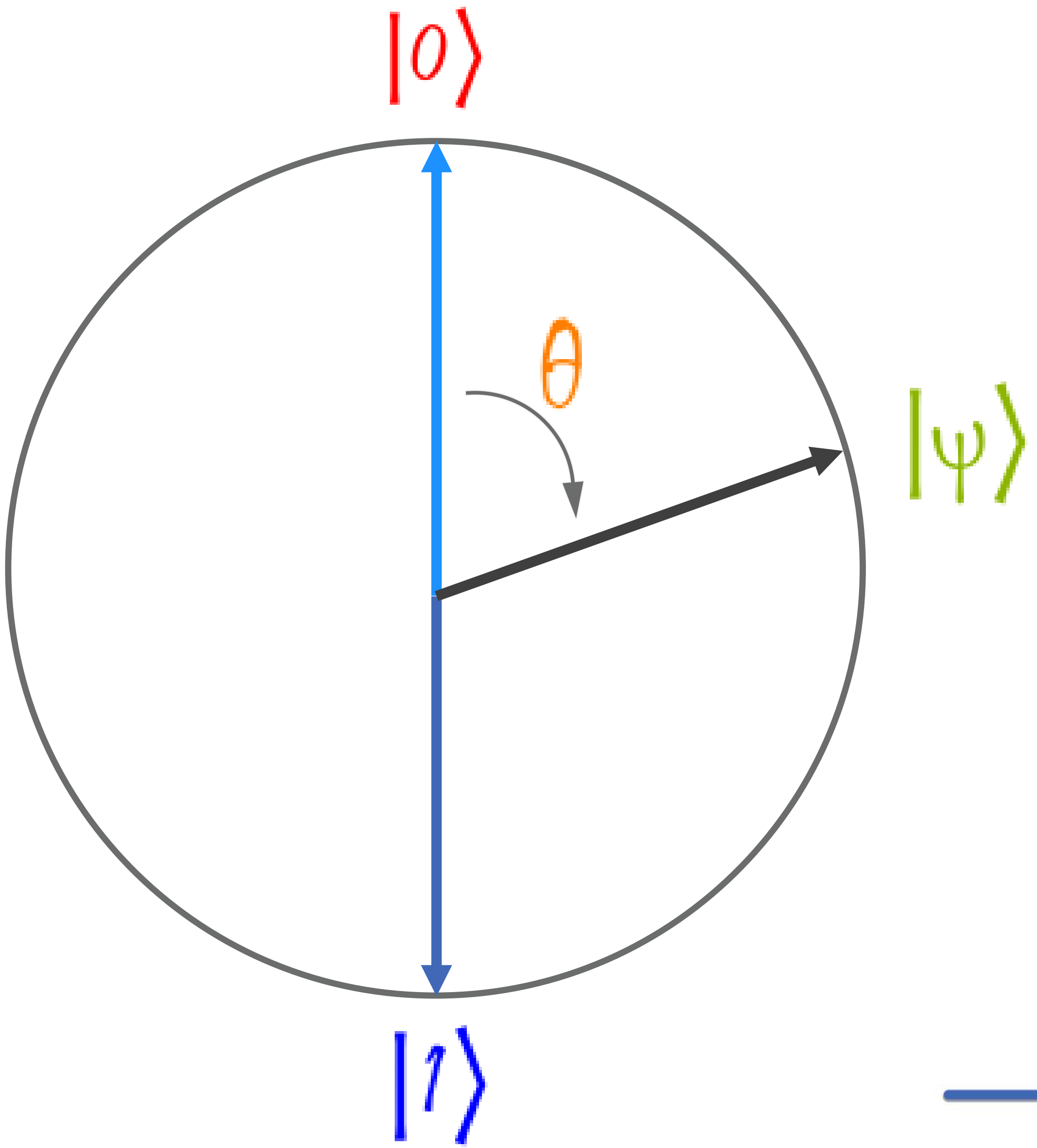
$$|\psi\rangle = \begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$|0\rangle = \left(\cos\left(\frac{0}{2}\right), \sin\left(\frac{0}{2}\right)\right) = \begin{pmatrix}1 \\ 0\end{pmatrix}$$

$$|1\rangle = \left(\cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right)\right) = \begin{pmatrix}0 \\ 1\end{pmatrix}$$

With this choice of coordinates, the classical states |0> and |1> represent a base.
Any other state of the system can therefore be represented as a linear combination of the classical states

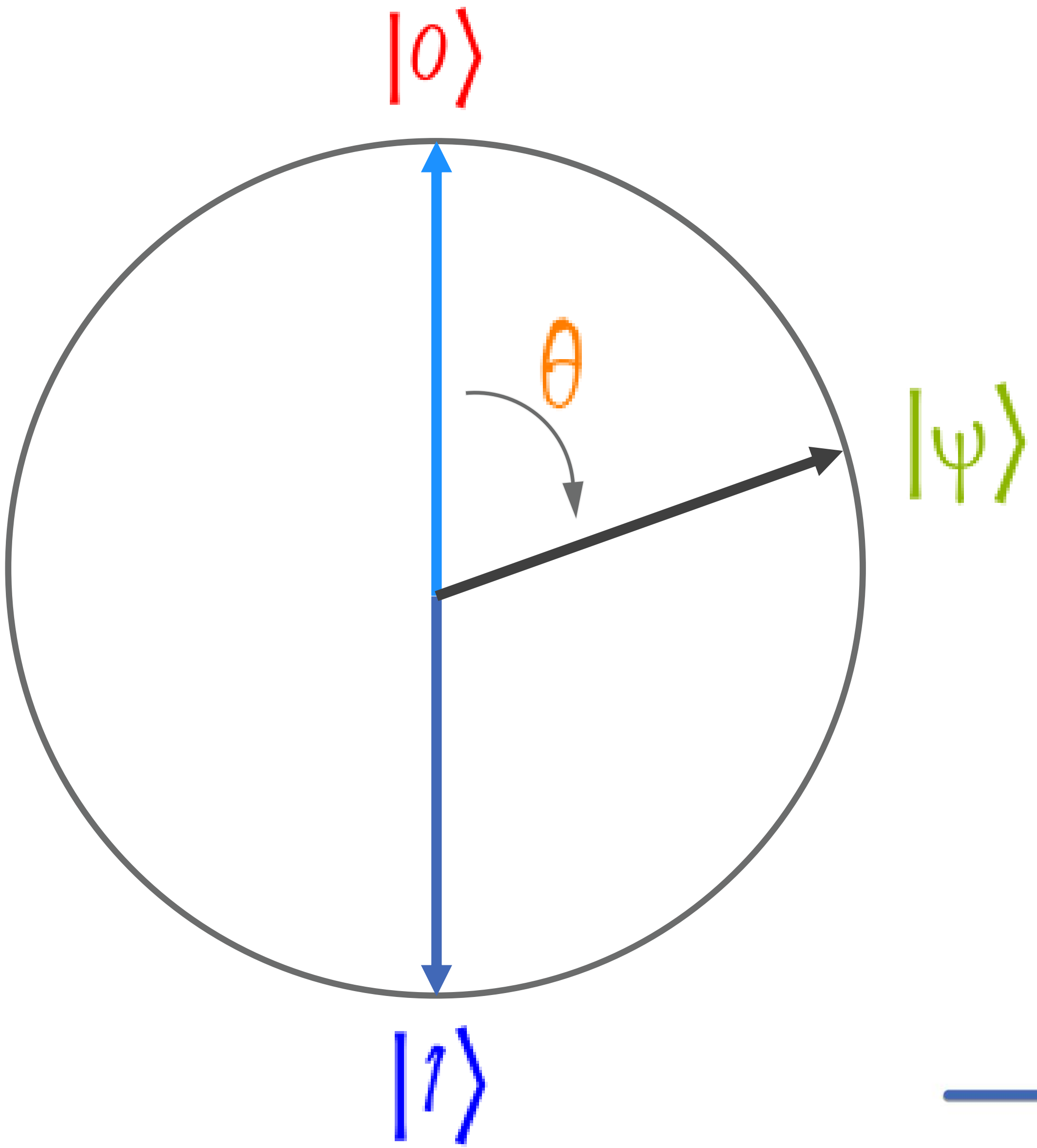
Qubits and Bloch Sphere



$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

CINECA QUANTUM COMPUTING LAB

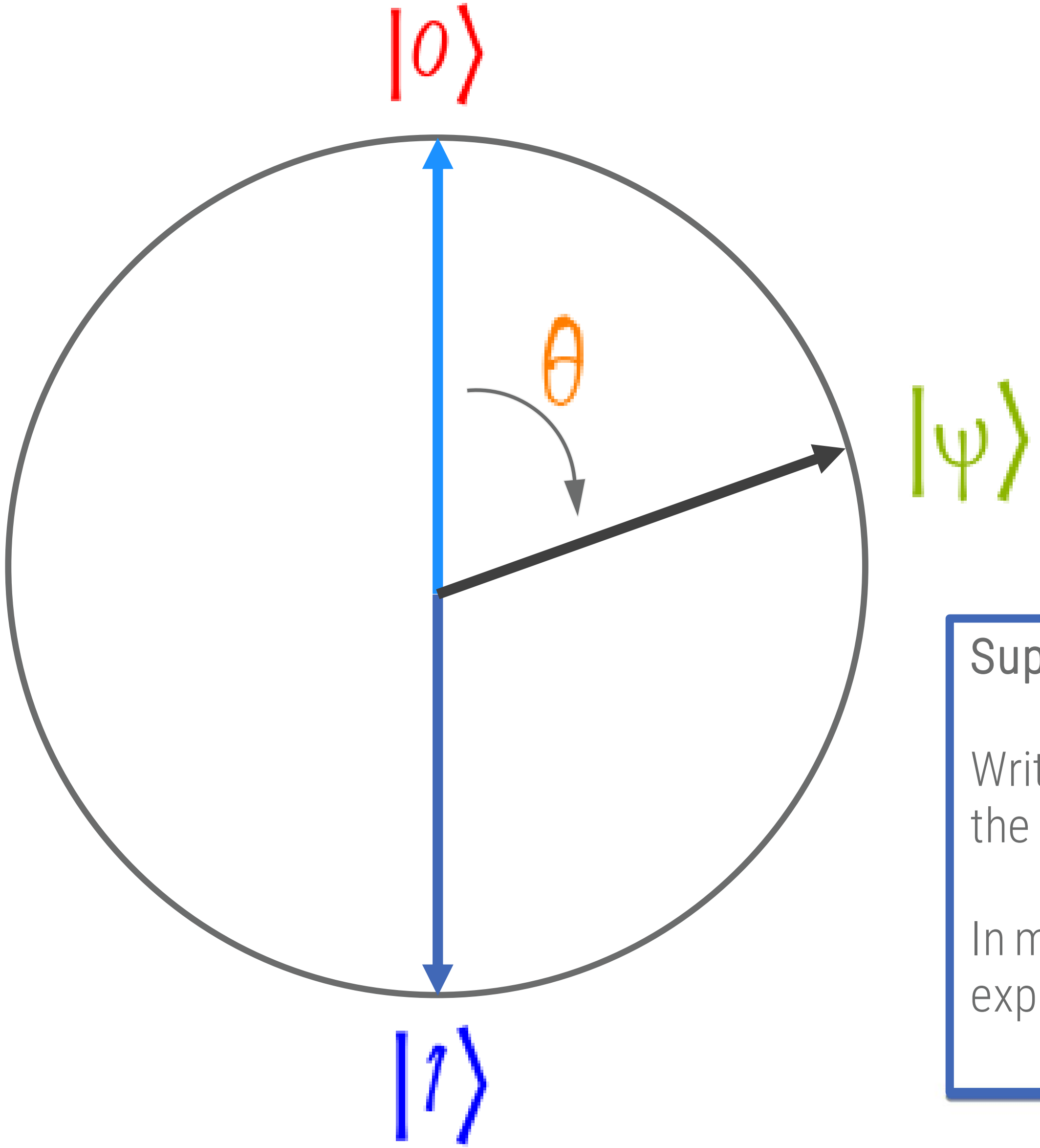
Qubits and Bloch Sphere



$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

$$|\psi\rangle = \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

Qubits and Bloch Sphere



$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

$$|\psi\rangle = \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

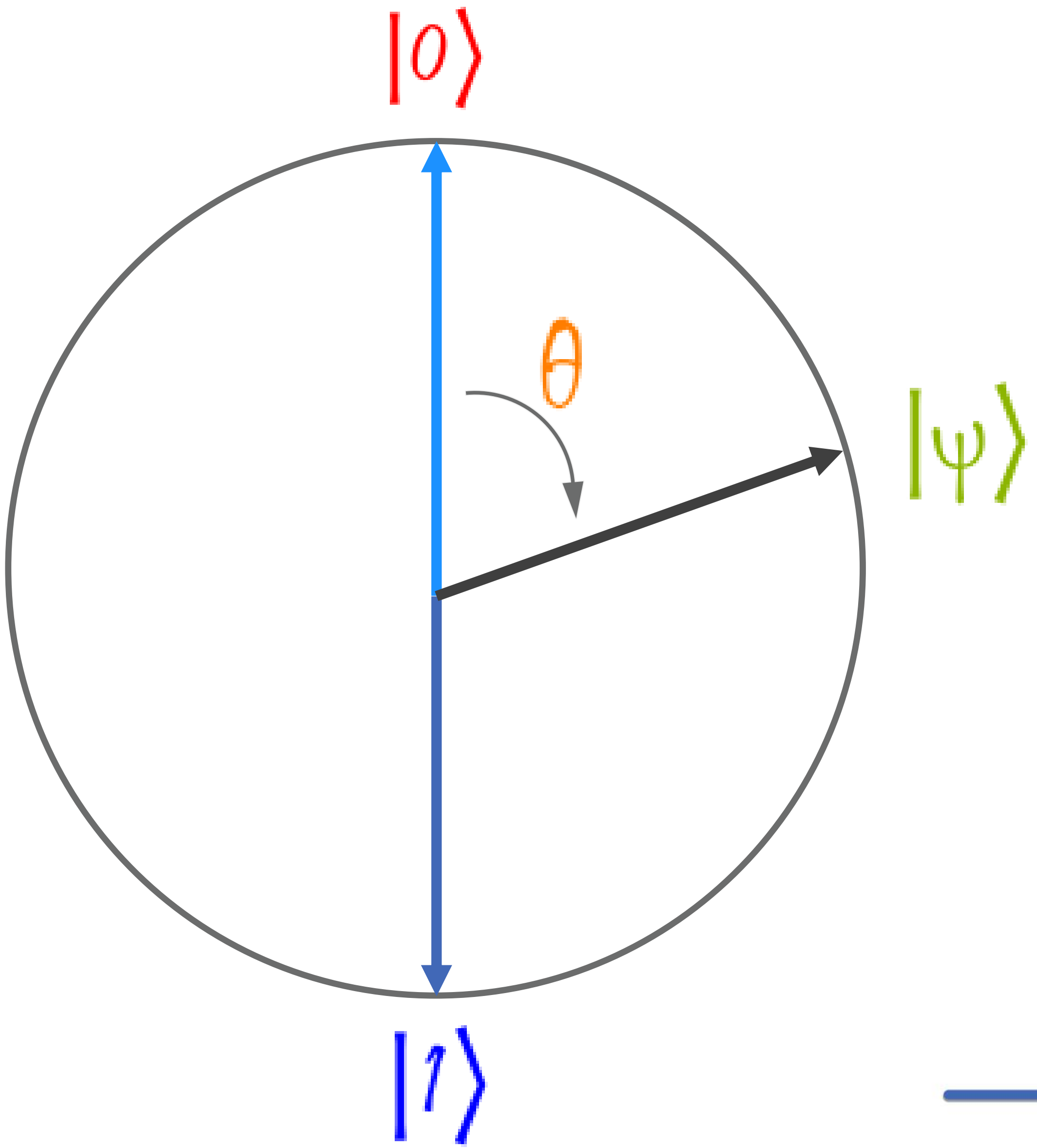
Superposition

Written in this way, the relationship between the coordinates of a qubit and the superposition principle begins to appear clear.

In mathematical terms, the superposition state manifested by a qubit is expressed as a linear combination of the classical states 0 and 1

CINECA QUANTUM COMPUTING LAB

Qubits and Bloch Sphere



$$|\psi\rangle = \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

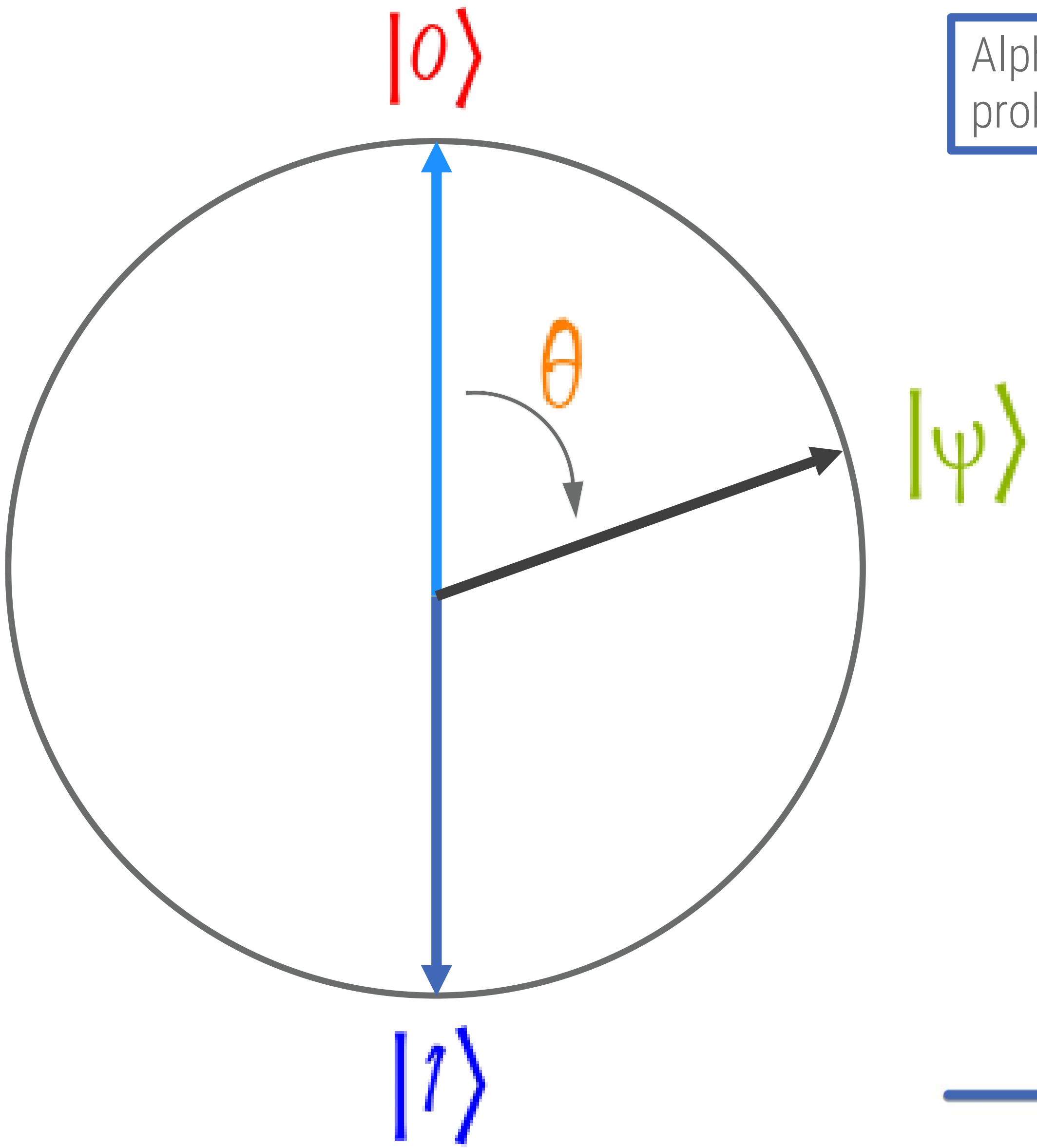
Measuring a qubit: the wave function

Even if it can be represented mathematically, it will never be possible to observe a state of superposition.

Quantum computing, like classical computing, involves the measurement operation, which aims to make known the value of a certain qubit / bit

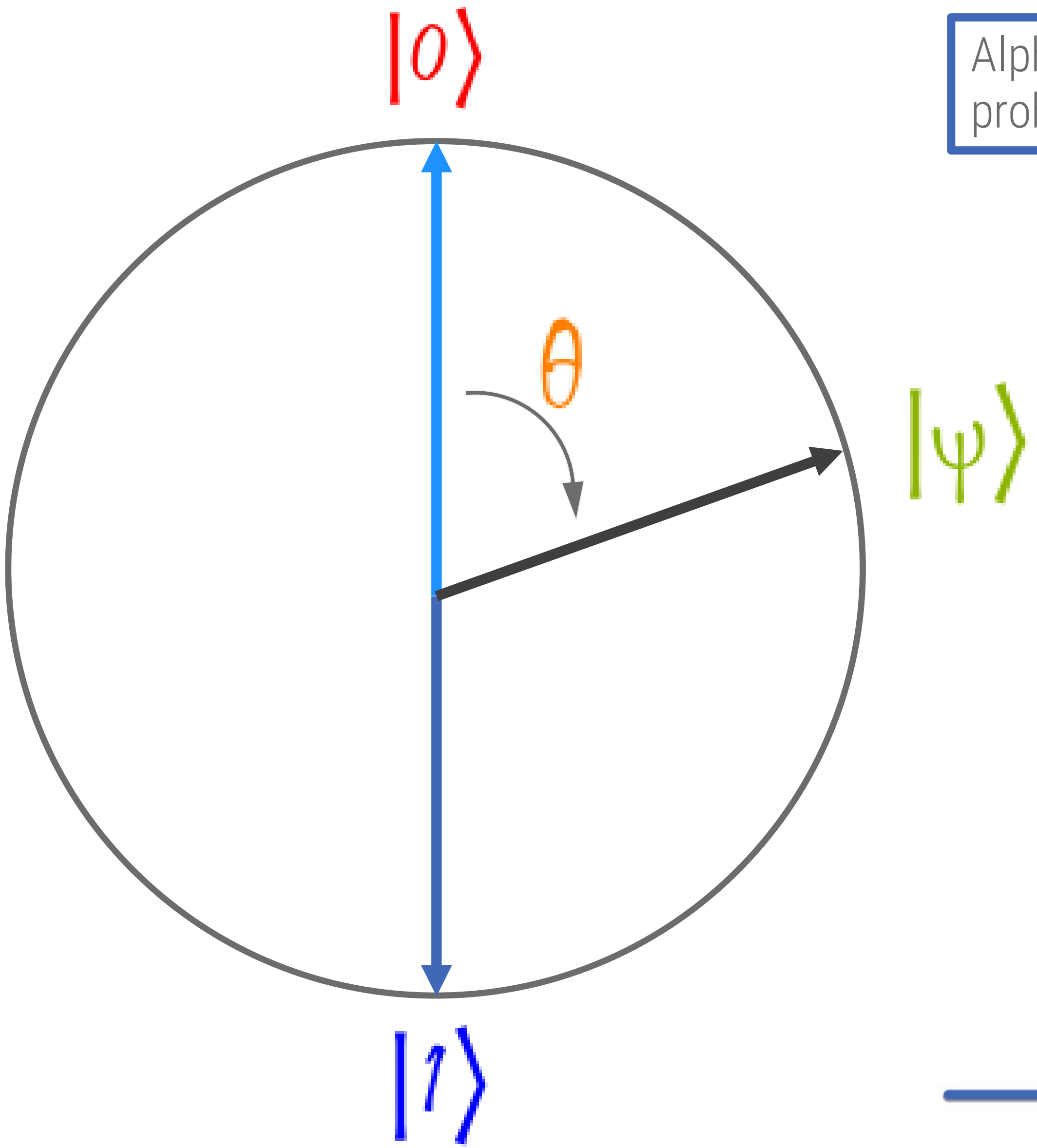
The fundamental difference is that the measurement of a qubit destroys the superposition state, forcing the qubit to assume one of the two classical values (collapse of the wave function)

Qubits and Bloch Sphere



Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

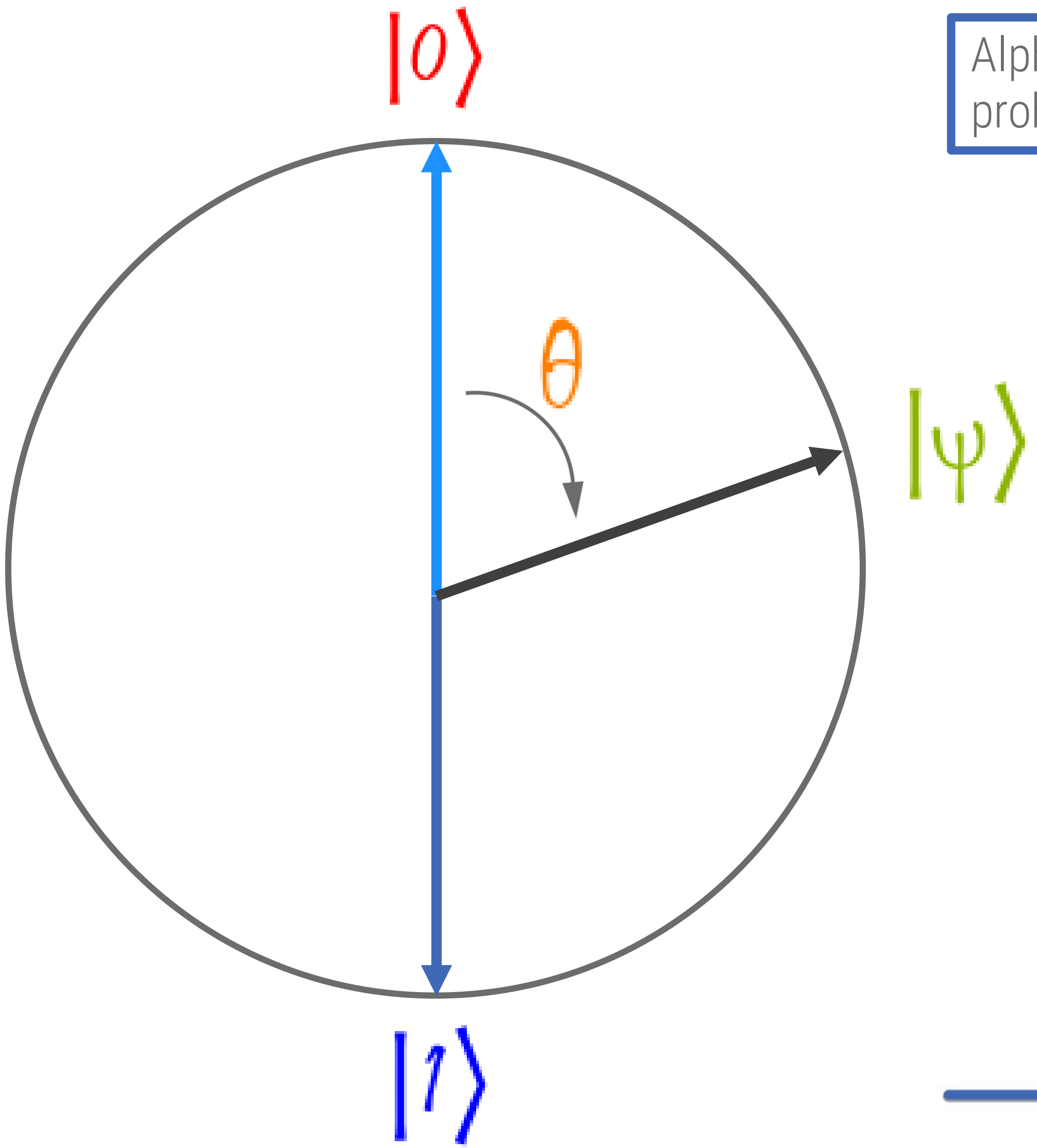
Qubits and Bloch Sphere



Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

Qubits and Bloch Sphere



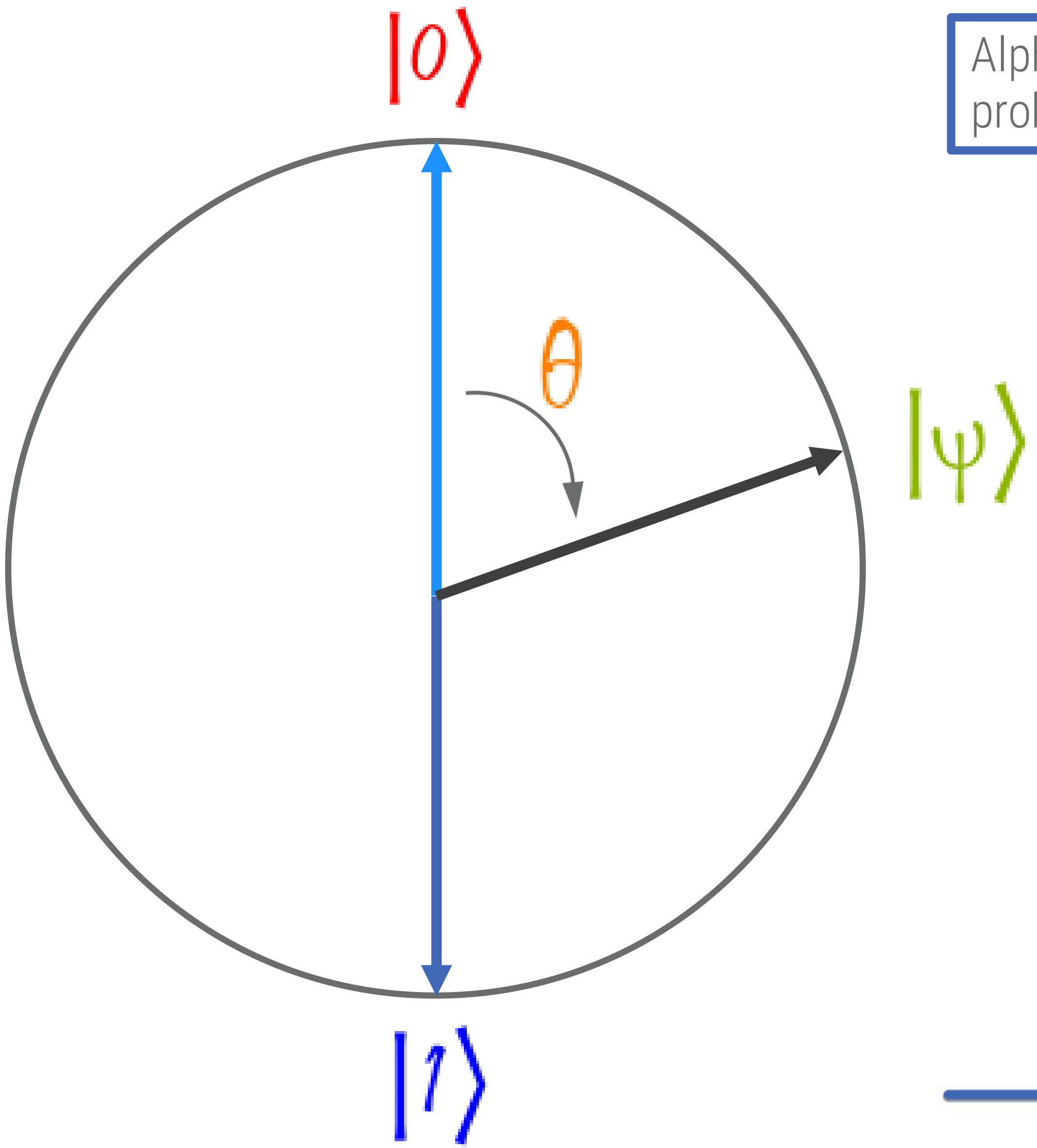
Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

$$\mathbb{P}\left(|\psi\rangle = |0\rangle\right) = \alpha^2$$

$$\mathbb{P}\left(|\psi\rangle = |1\rangle\right) = \beta^2$$

Qubits and Bloch Sphere



Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

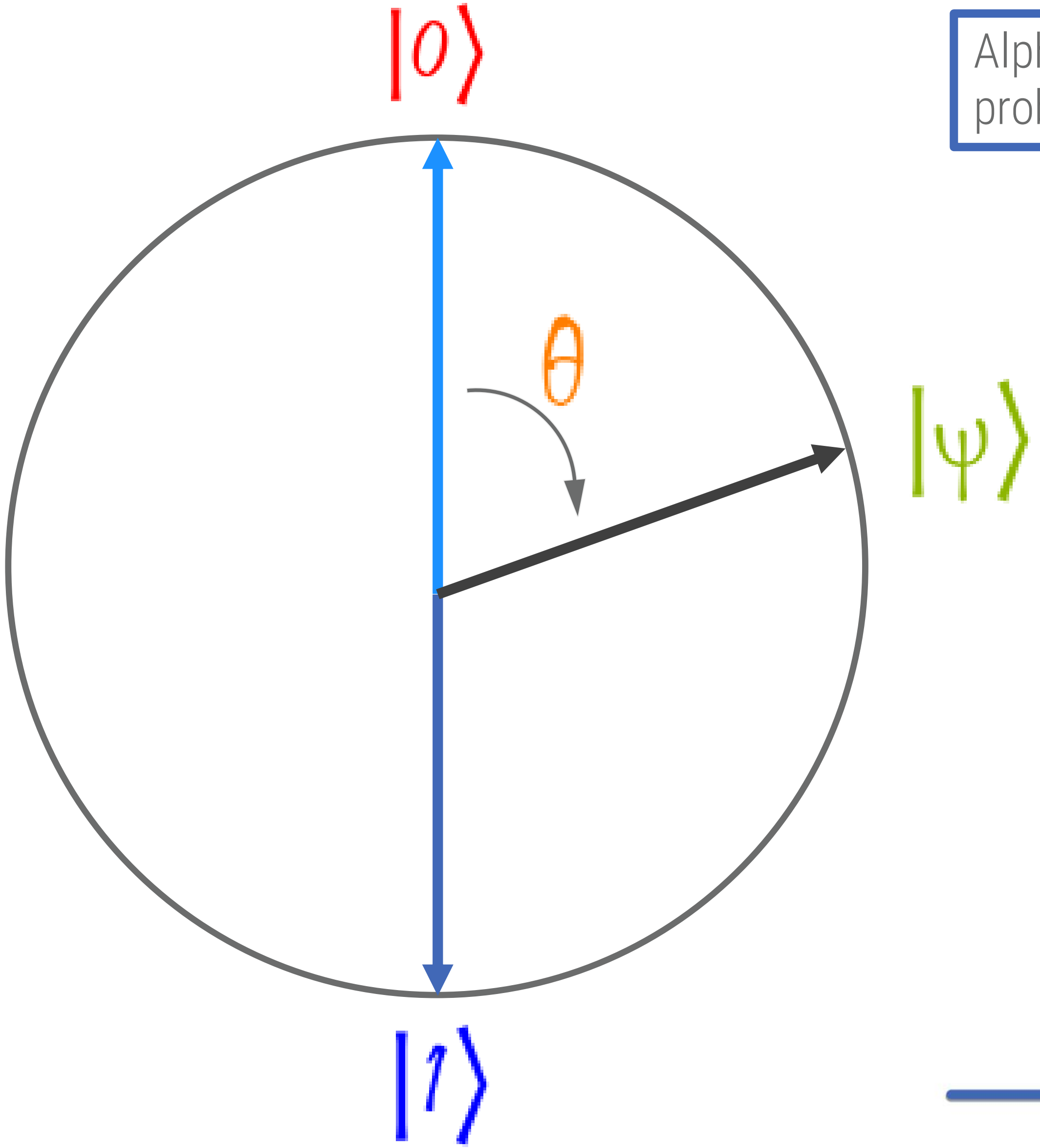
$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

$$\mathbb{P}(|\psi\rangle = |0\rangle) = \alpha^2$$

$$\mathbb{P}(|\psi\rangle = |1\rangle) = \beta^2$$

$$\alpha^2 + \beta^2 = 1$$

Qubits and Bloch Sphere



Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

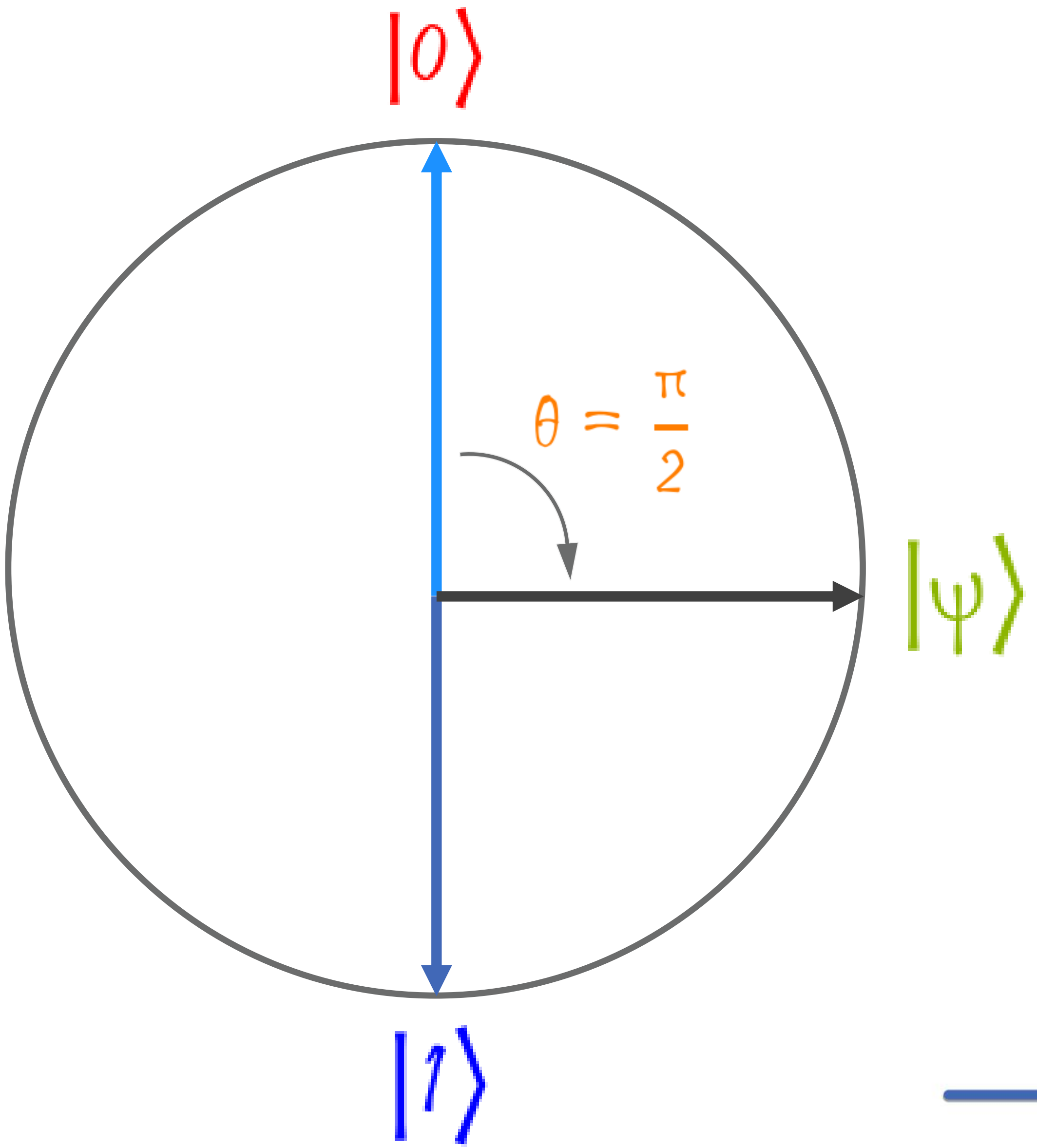
$$\mathbb{P}\left(|\psi\rangle = |0\rangle\right) = \alpha^2$$

$$\mathbb{P}\left(|\psi\rangle = |1\rangle\right) = \beta^2$$

$$\alpha^2 + \beta^2 = 1$$

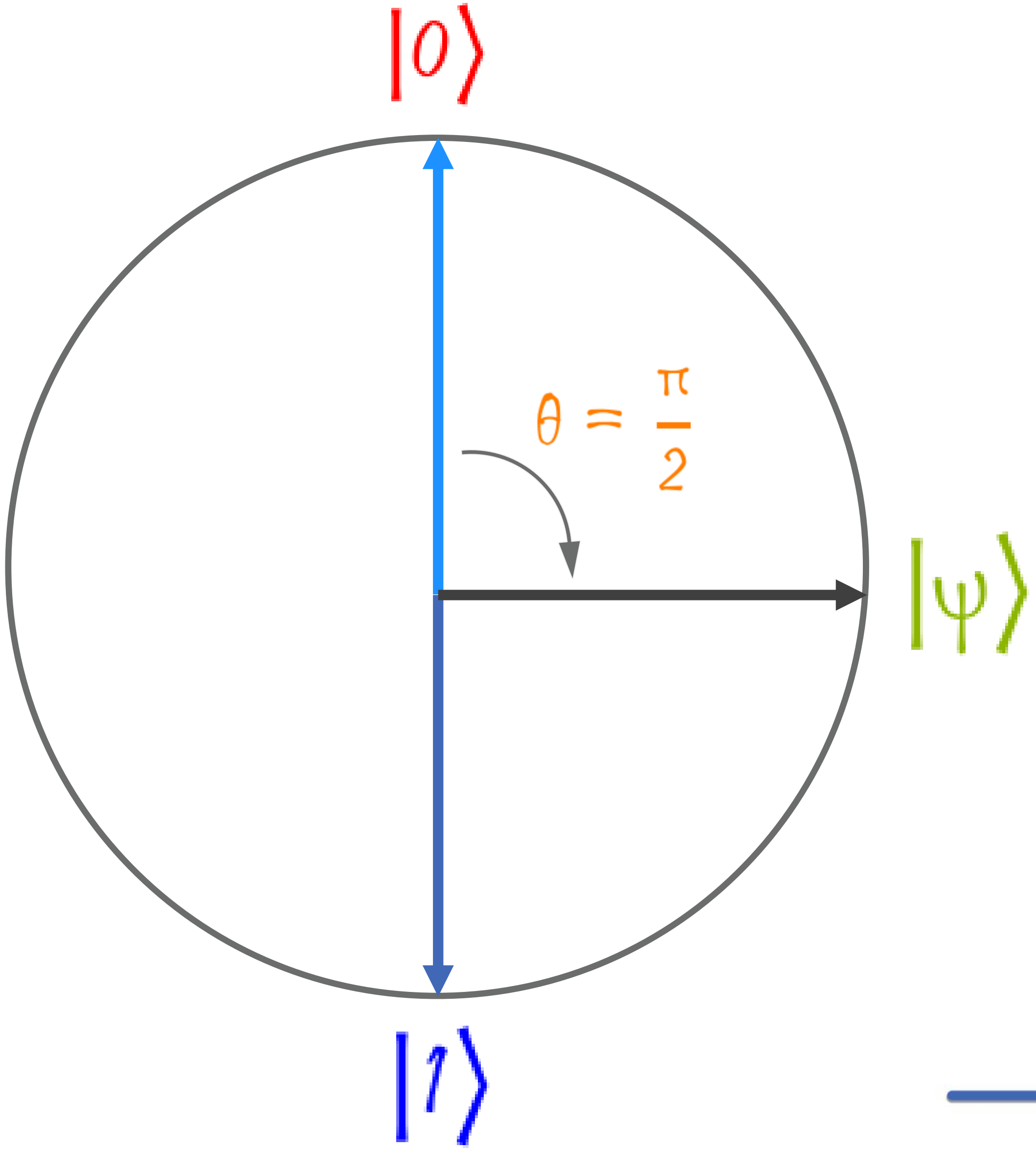
$$\left(\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)\right) = 1 \quad \forall\theta$$

Qubits and Bloch Sphere



$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

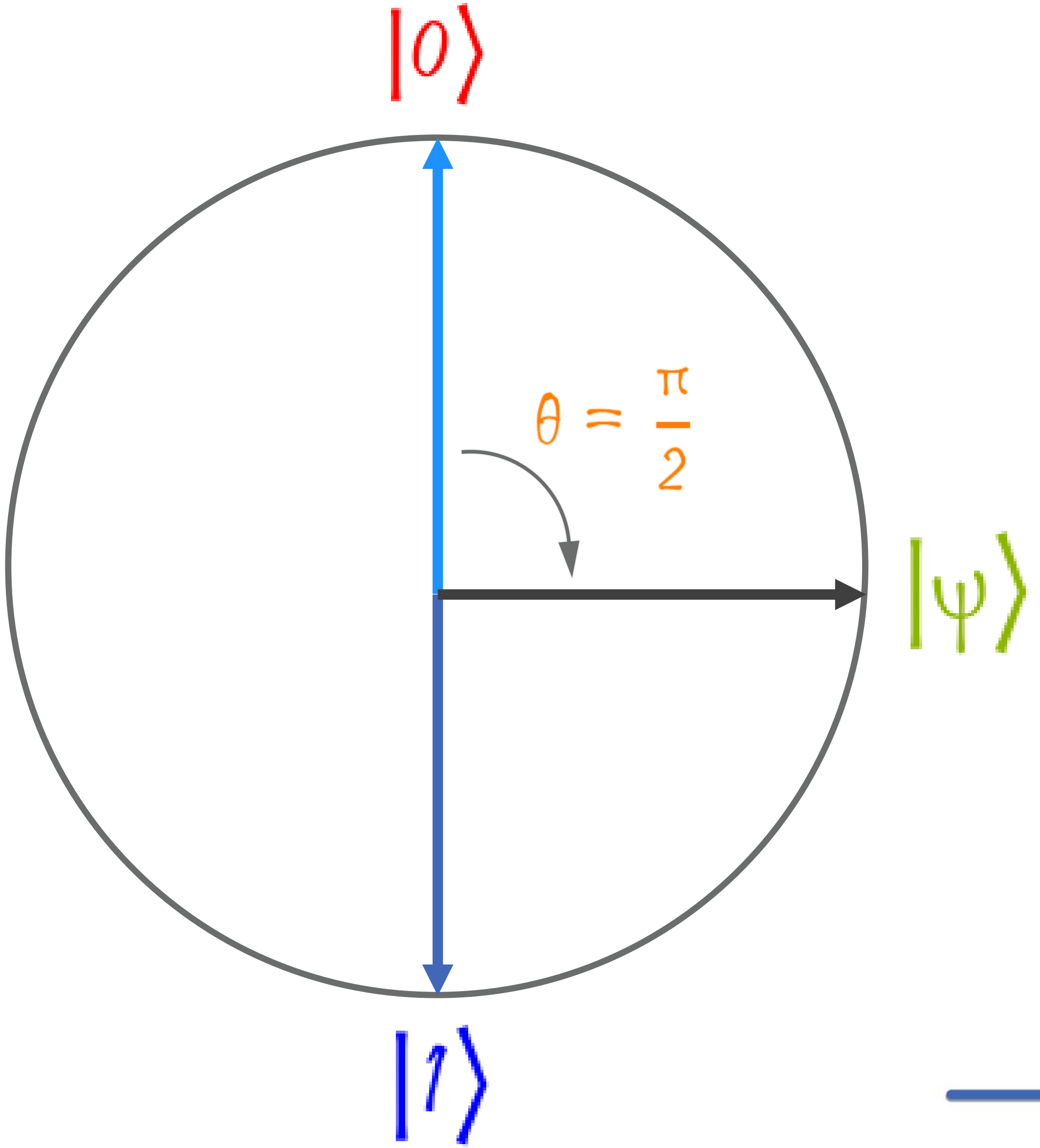
Qubits and Bloch Sphere



$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$= \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right)$$

Qubits and Bloch Sphere

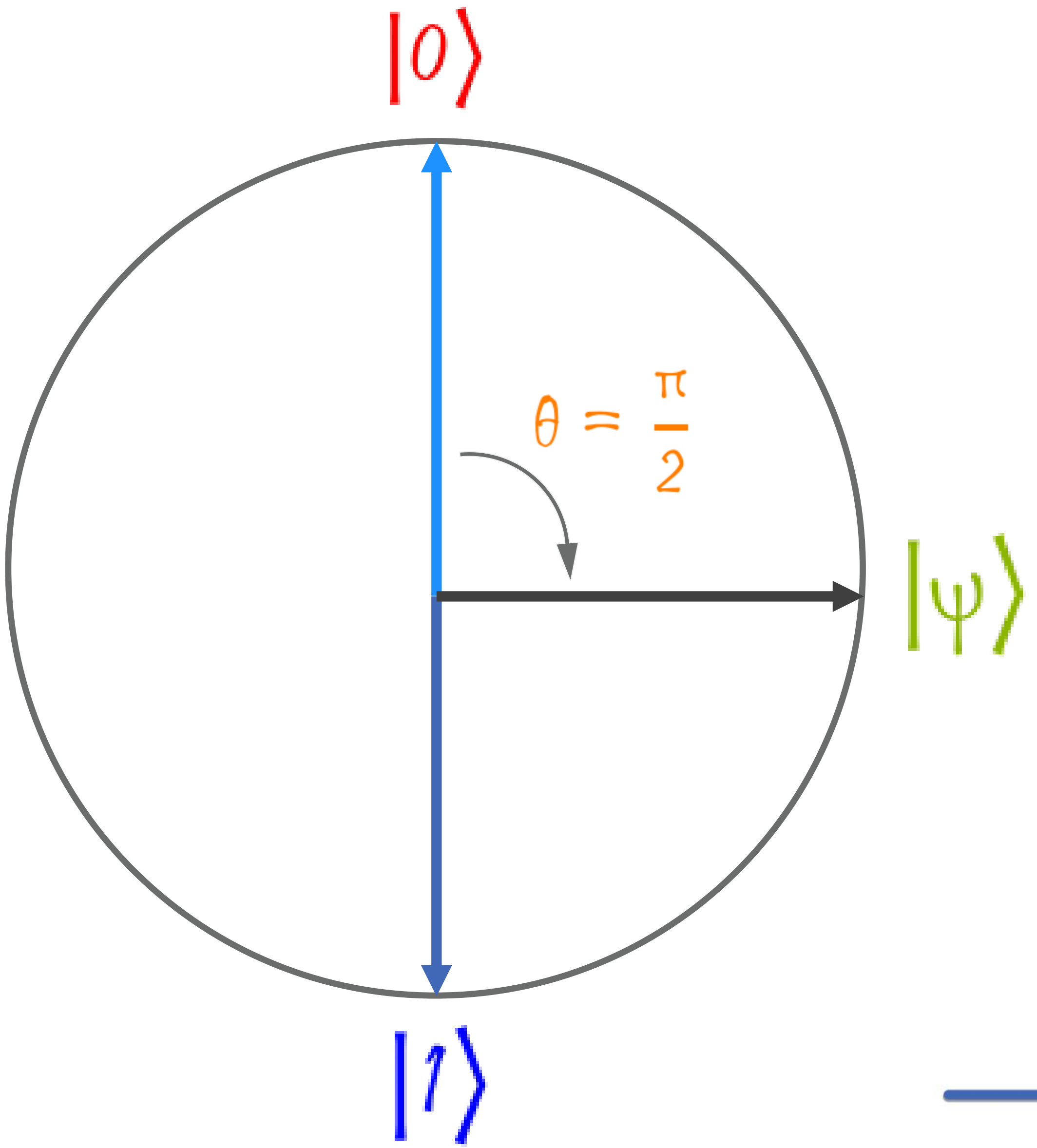


$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$= \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Qubits and Bloch Sphere



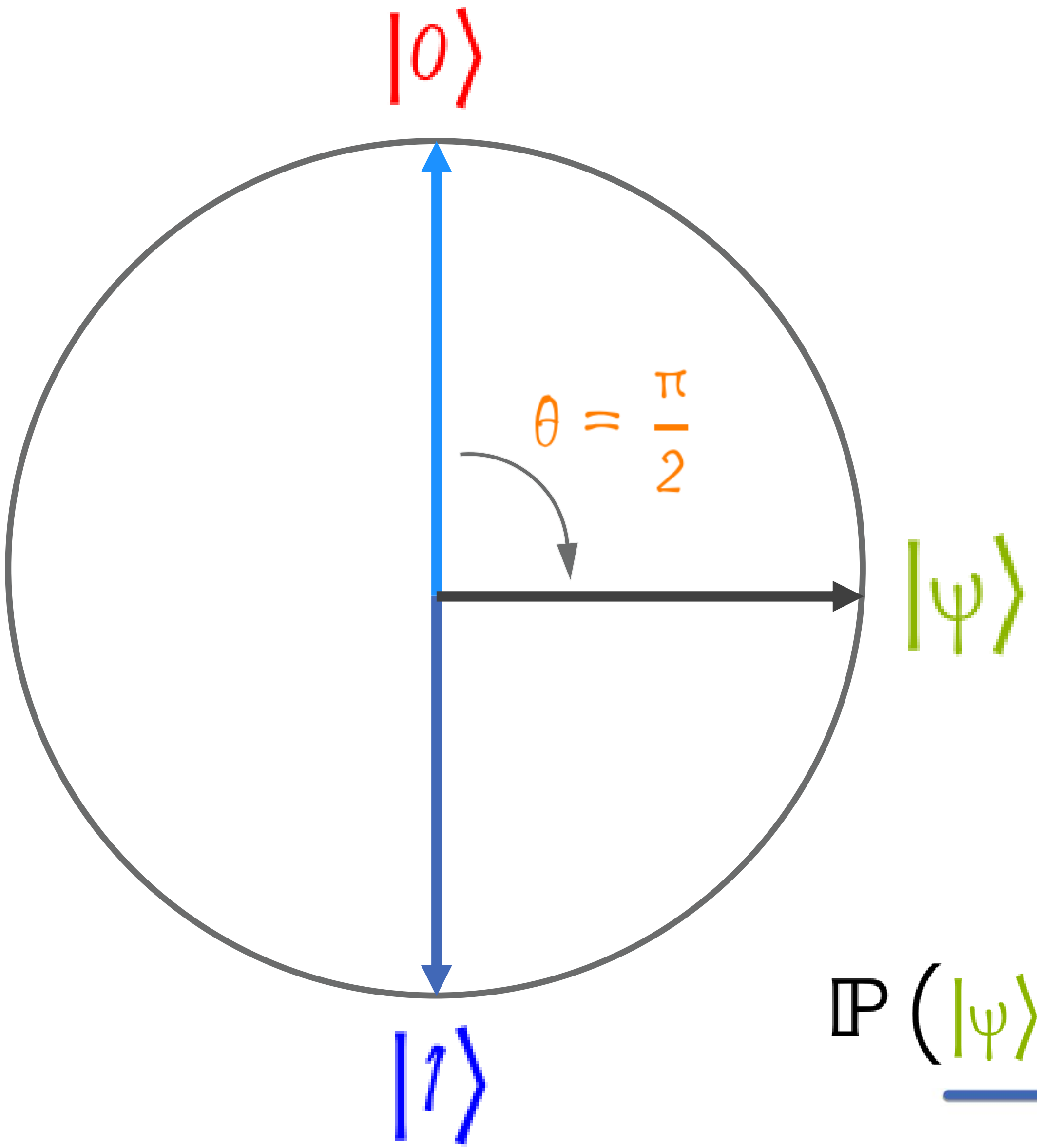
$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$= \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{2} \cdot |0\rangle + \frac{\sqrt{2}}{2} \cdot |1\rangle$$

Qubits and Bloch Sphere



$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

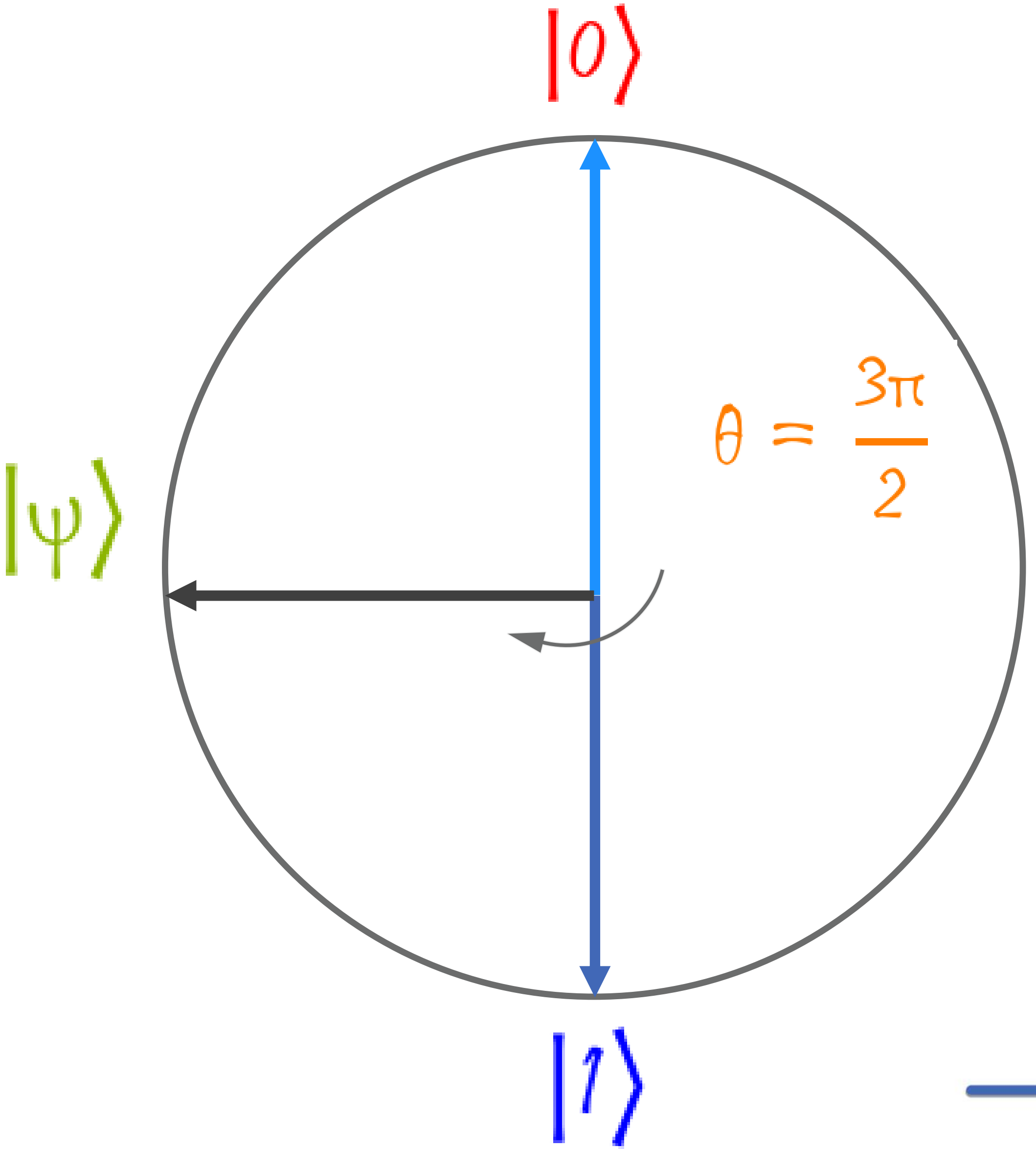
$$= \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{2}\cdot|0\rangle + \frac{\sqrt{2}}{2}\cdot|1\rangle$$

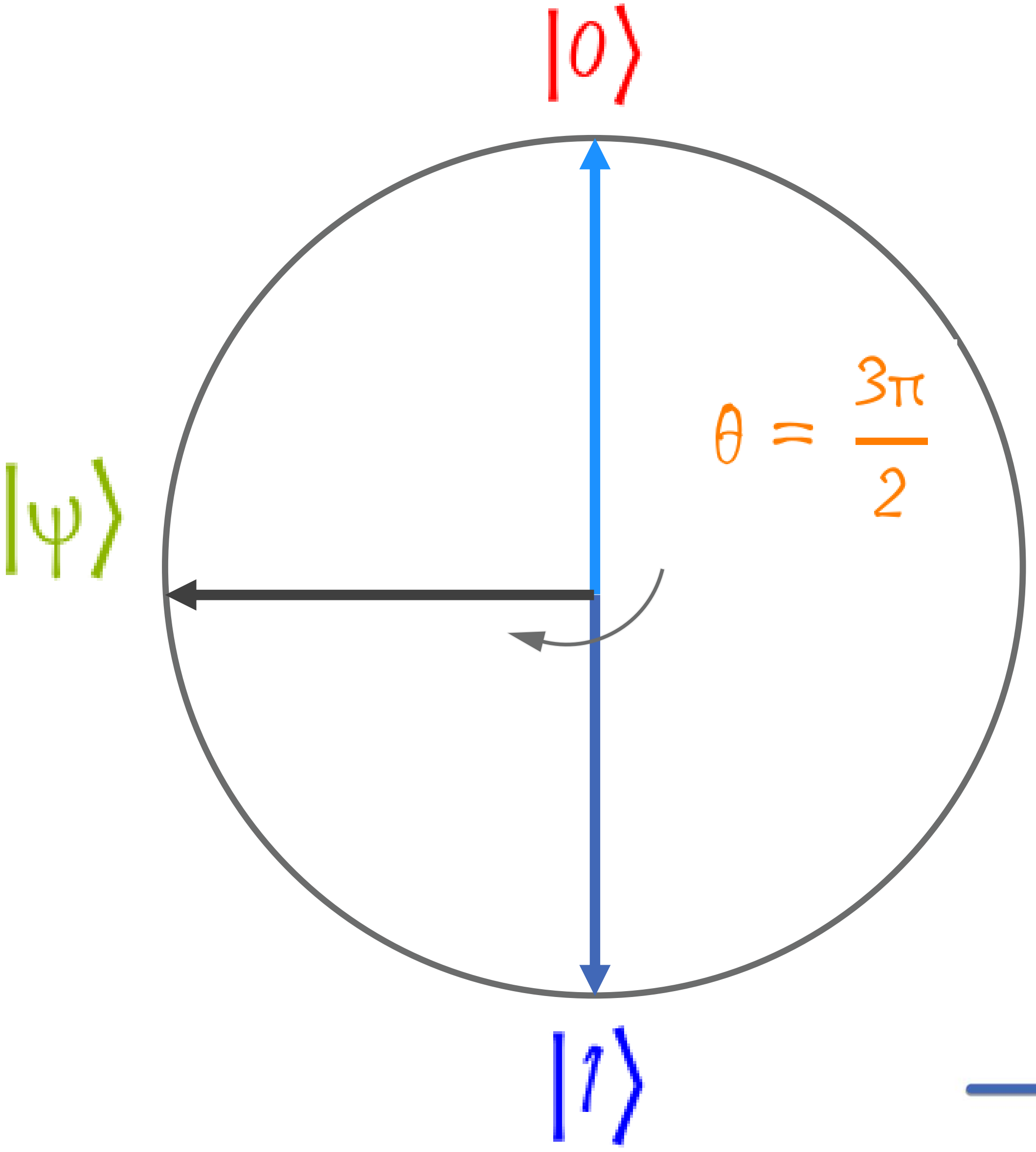
$$\mathbb{P}(|\psi\rangle = |0\rangle) = \mathbb{P}(|\psi\rangle = |1\rangle) = \frac{1}{2} \quad (50\%)$$

Qubits and Bloch Sphere



$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

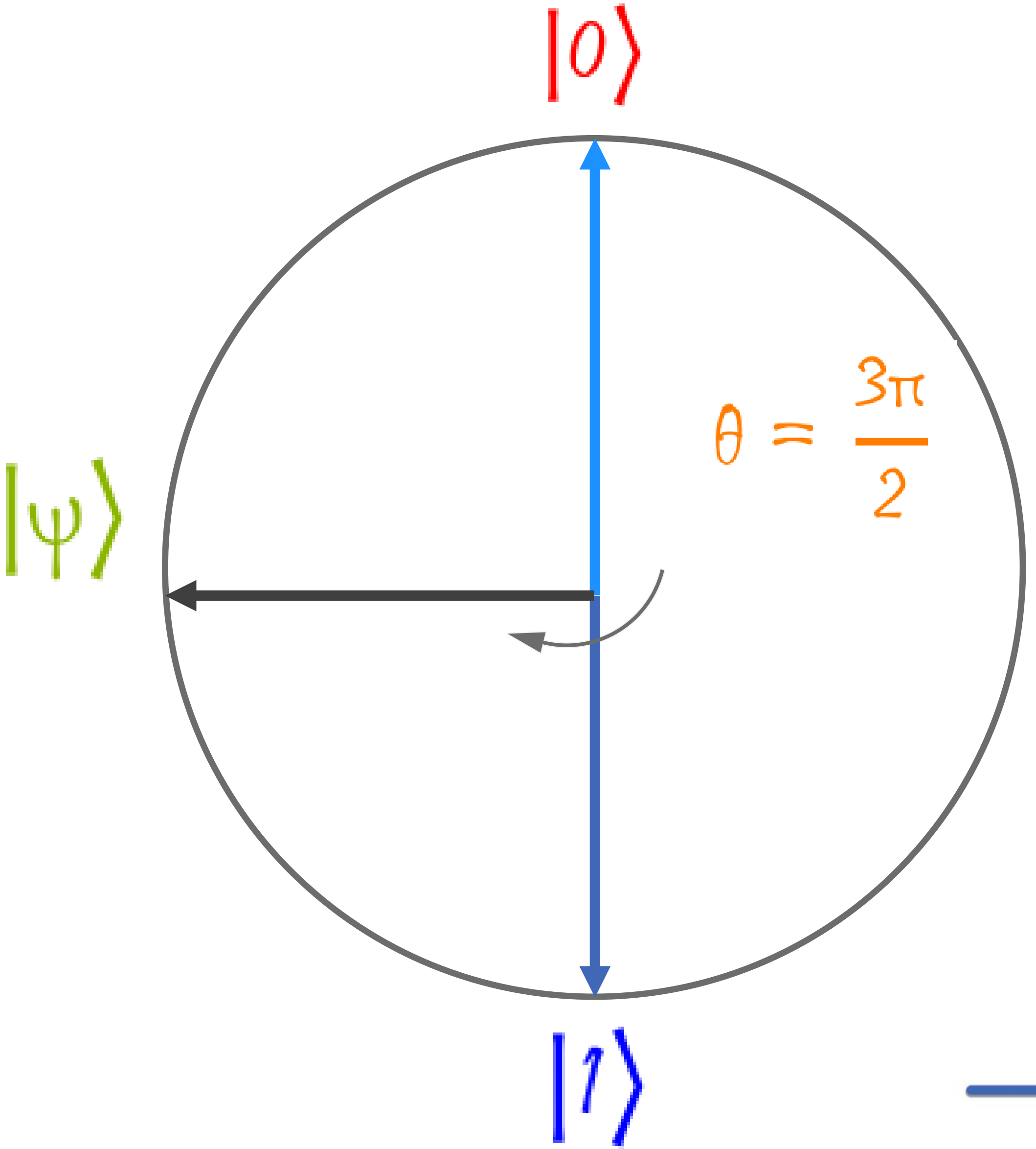
Qubits and Bloch Sphere



$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$= \left(\cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right)\right)$$

Qubits and Bloch Sphere

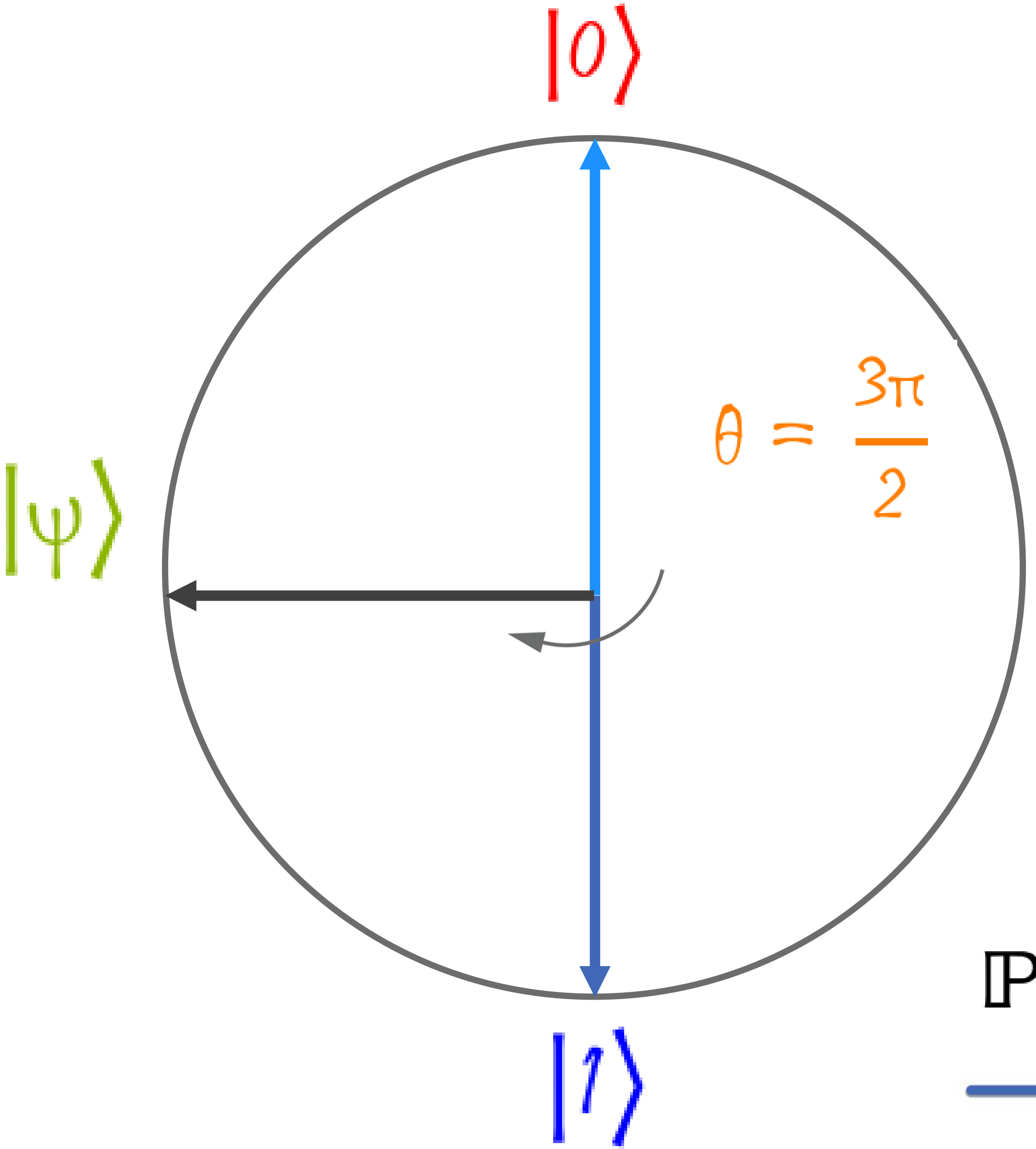


$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

$$= \left(\cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

Qubits and Bloch Sphere



$$|\psi\rangle = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\right)$$

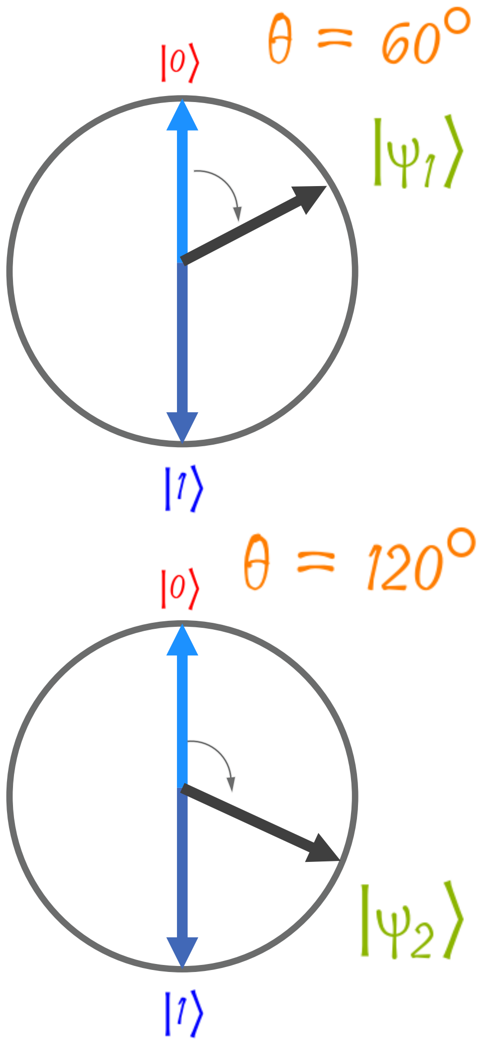
$$= \left(\cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

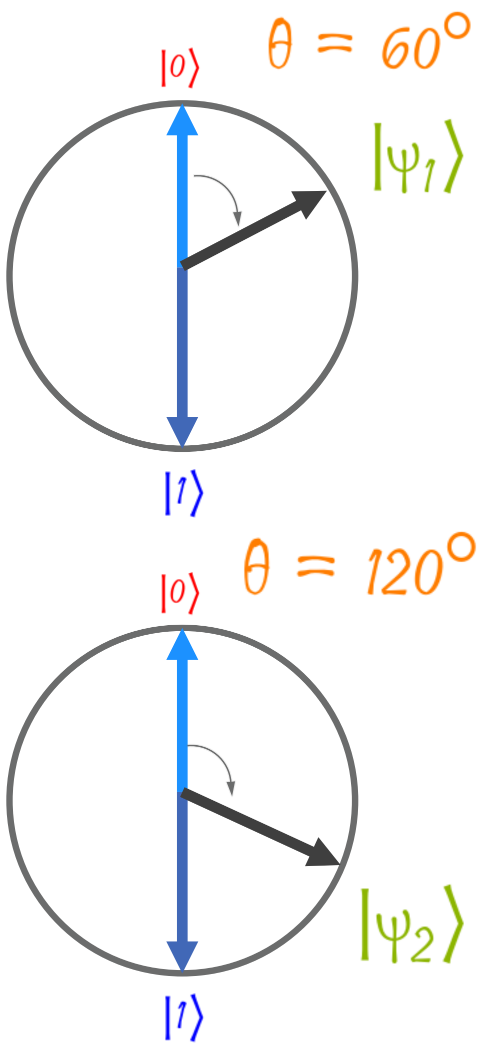
$$= \frac{\sqrt{2}}{2} \cdot |0\rangle - \frac{\sqrt{2}}{2} \cdot |1\rangle$$

$$\mathbb{P}\left(|\psi\rangle = |0\rangle\right) = \mathbb{P}\left(|\psi\rangle = |1\rangle\right) = \frac{1}{2} \quad (50\%)$$

Two Qubits System

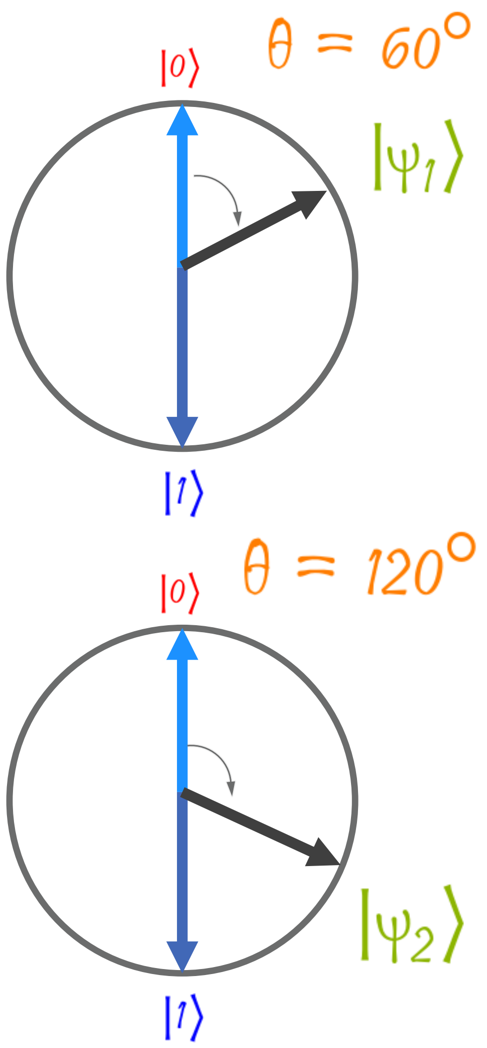


Two Qubits System



- What are the mathematical representations of these two qubits as a function of the classical states?

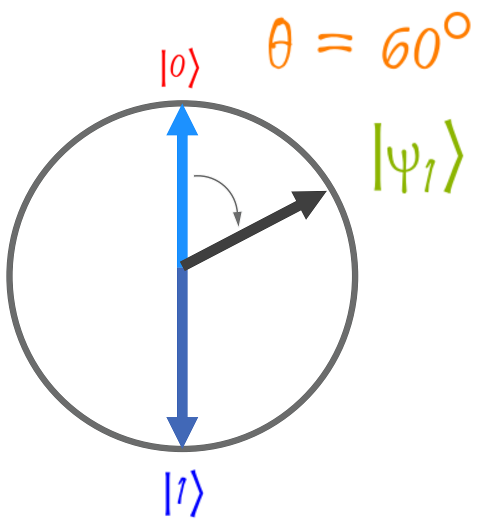
Two Qubits System

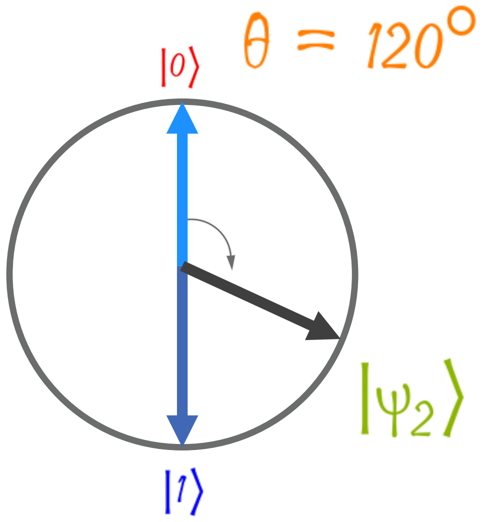


- What are the mathematical representations of these two qubits as a function of the classical states?

$$|\psi_1\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle, \qquad |\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

Two Qubits System



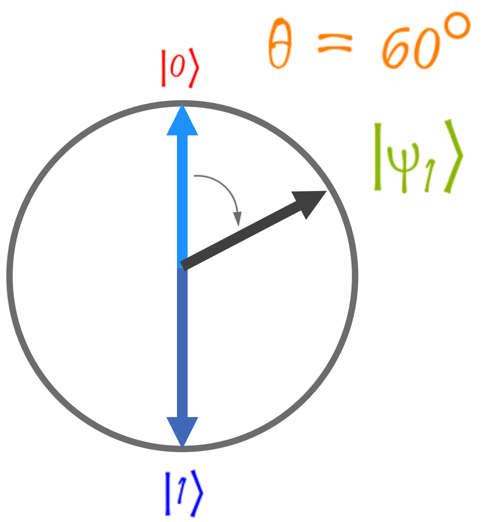


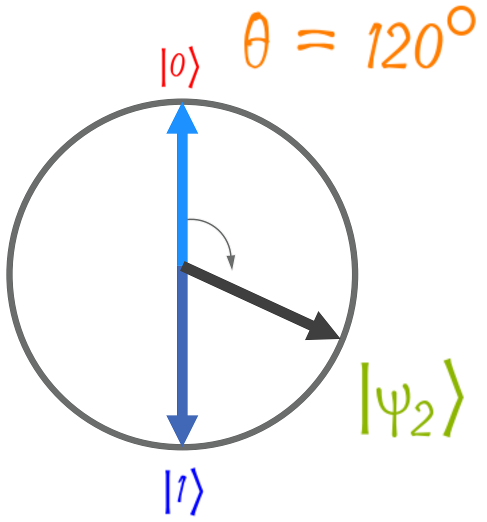
- What are the mathematical representations of these two qubits as a function of the classical states?

$$|\psi_1\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle, \qquad |\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

- What are the probabilities of obtaining the classical states 0 and 1 after taking a measurement?

Two Qubits System





- What are the mathematical representations of these two qubits as a function of the classical states?

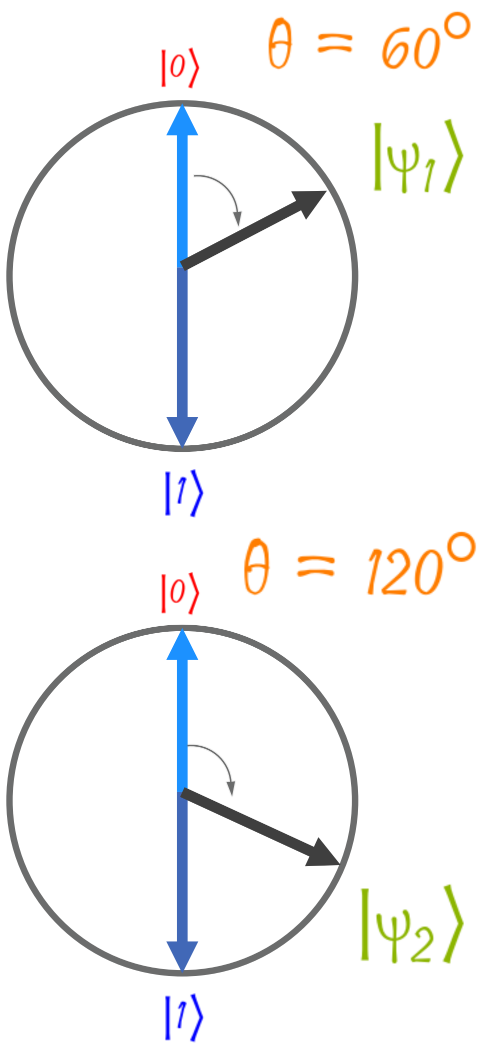
$$|\psi_1\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle, \qquad |\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

- What are the probabilities of obtaining the classical states 0 and 1 after taking a measurement?

$\lvert\psi_1\rangle$	$\mathbb{P}$
$\lvert 0\rangle$	75%
$\lvert 1\rangle$	25%

$\lvert\psi_2\rangle$	$\mathbb{P}$
$\lvert 0\rangle$	25%
$\lvert 1\rangle$	75%

Two Qubits System



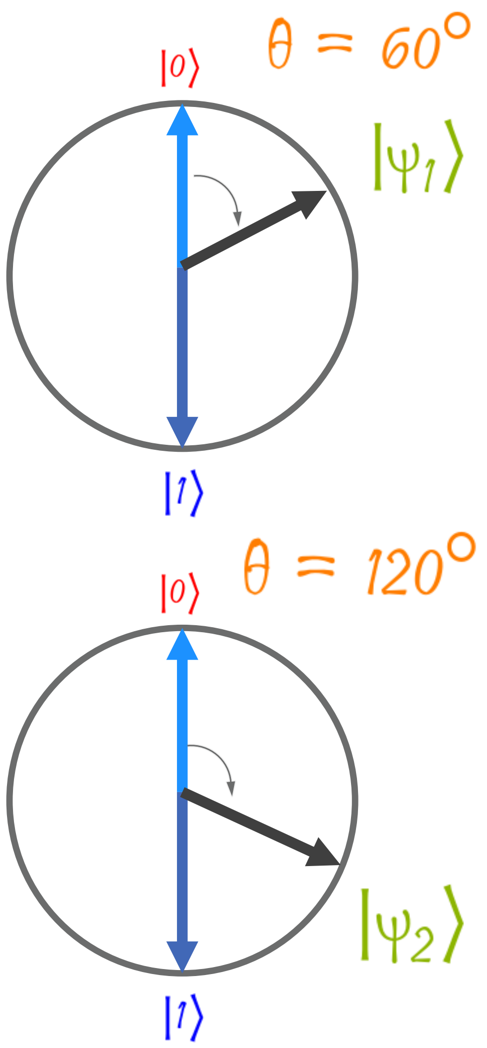
- What are the probabilities of obtaining the classical states 0 and 1 after taking a measurement?

$\|\psi_1\rangle$	$\mathbb{P}$
$\|0\rangle$	75%
$\|1\rangle$	25%

$\|\psi_2\rangle$	$\mathbb{P}$
$\|0\rangle$	25%
$\|1\rangle$	75%

- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

Two Qubits System



- What are the probabilities of obtaining the classical states 0 and 1 after taking a measurement?

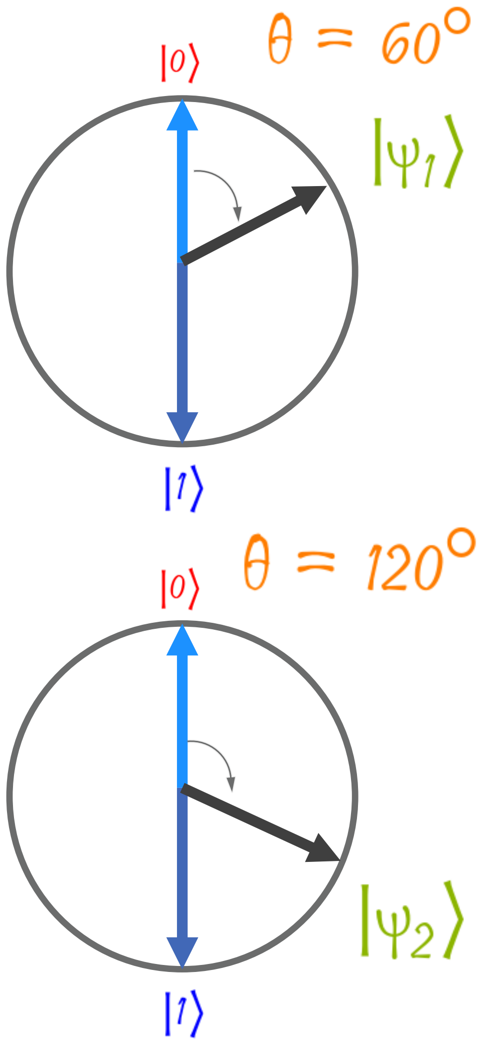
$\|\psi_1\rangle$	$\mathbb{P}$
$\|0\rangle$	75%
$\|1\rangle$	25%

$\|\psi_2\rangle$	$\mathbb{P}$
$\|0\rangle$	25%
$\|1\rangle$	75%

- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

$\|\psi_1\psi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	18.75%	56.25%
$\|1\rangle$	6.25%	18.75%

Two Qubits System

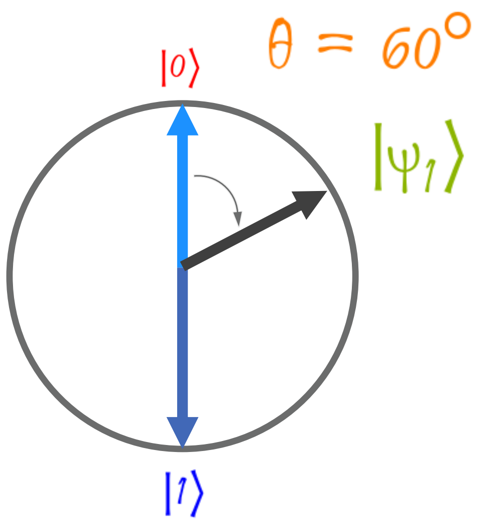


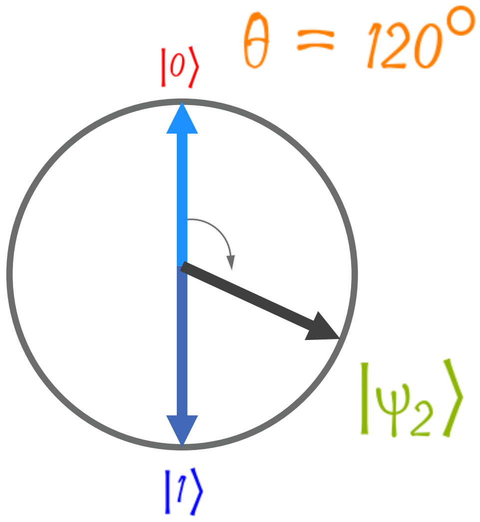
- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

$\|\psi_1\psi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	18.75%	56.25%
$\|1\rangle$	6.25%	18.75%

- The wave function that represents the possible states of a system composed of two qubits is a function with 4 different terms (2^2)

Two Qubits System



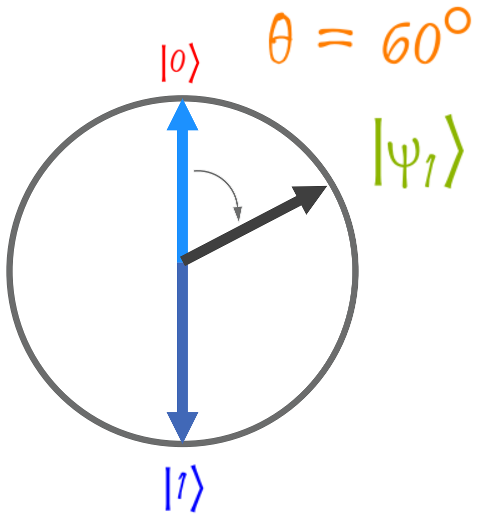


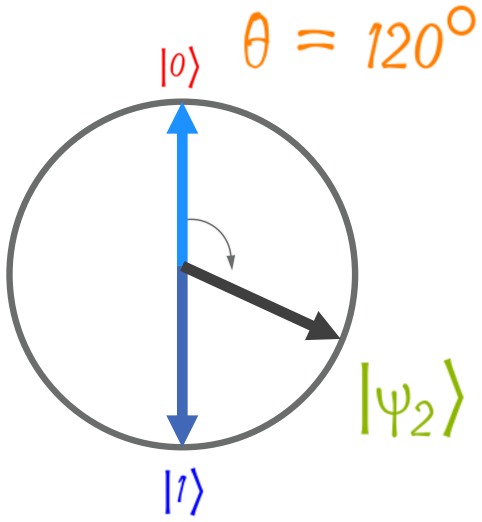
- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

$\|\psi_1\psi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	18.75%	56.25%
$\|1\rangle$	6.25%	18.75%

- By extracting the square root of the values obtained as the probability of obtaining a combination of classical states, I can therefore obtain a description of the wave function relative to the system composed of the two qubits

Two Qubits System





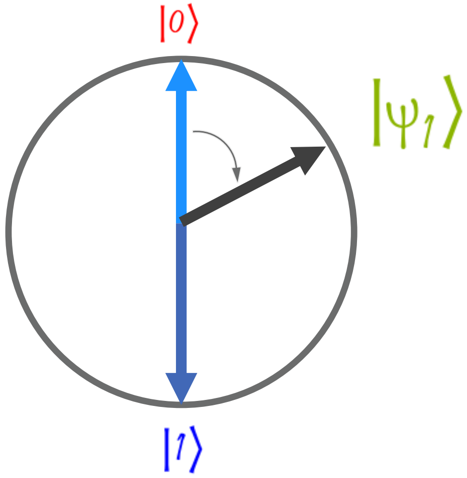
- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

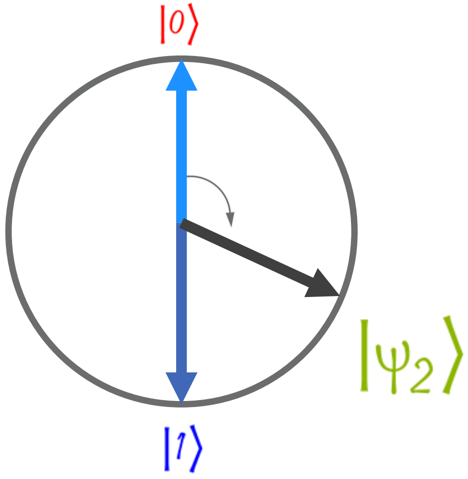
$\|\psi_1\psi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	18.75%	56.25%
$\|1\rangle$	6.25%	18.75%

- By extracting the square root of the values obtained as the probability of obtaining a combination of classical states, I can therefore obtain a description of the wave function relative to the system composed of the two qubits

$$|\psi_1\psi_2\rangle = 0.43|00\rangle + 0.75|01\rangle + 0.25|10\rangle + 0.43|11\rangle$$

Two Qubits System





$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$\|\psi_1\rangle$	$\mathbb{P}$
$\|0\rangle$	α_1^2
$\|1\rangle$	β_1^2

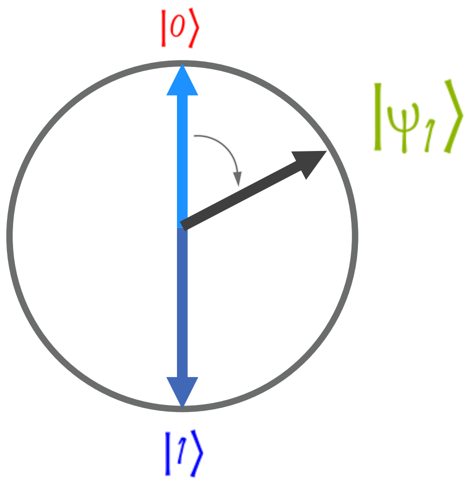
$\|\psi_2\rangle$	$\mathbb{P}$
$\|0\rangle$	α_2^2
$\|1\rangle$	β_2^2

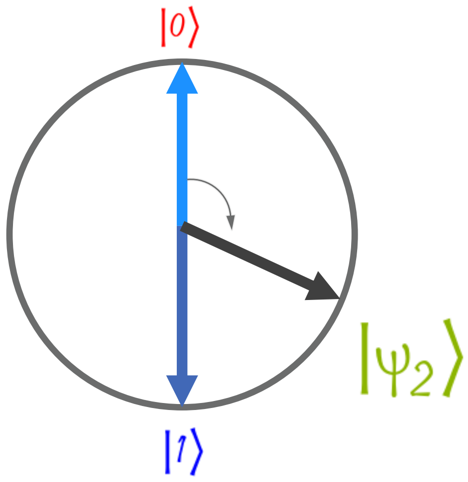
$\|\psi_1\psi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	$\alpha_1^2\alpha_2^2$	$\alpha_1^2\beta_2^2$
$\|1\rangle$	$\beta_1^2\alpha_2^2$	$\beta_1^2\beta_2^2$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

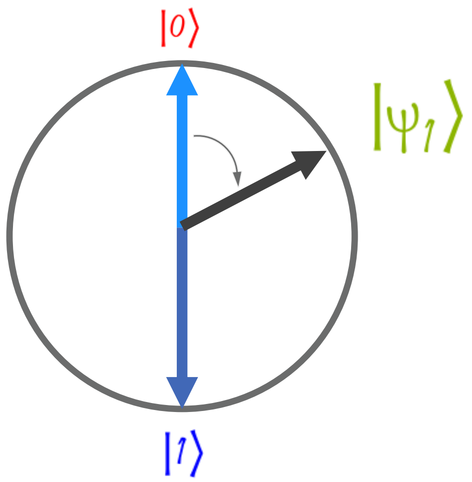
Multi Qubits System

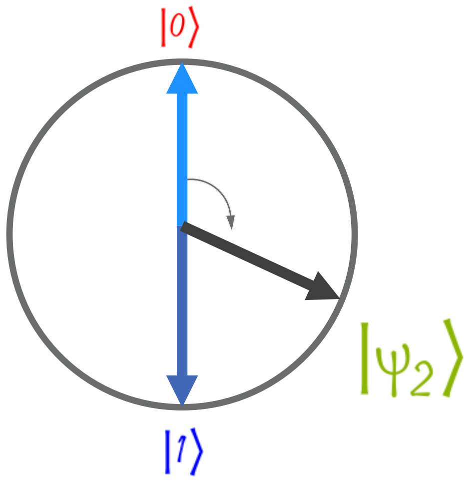
$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$





Multi Qubits System

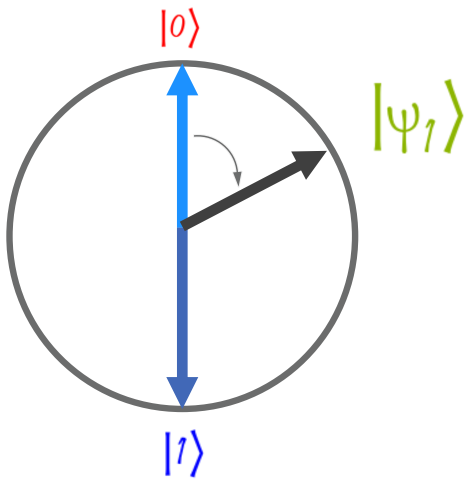


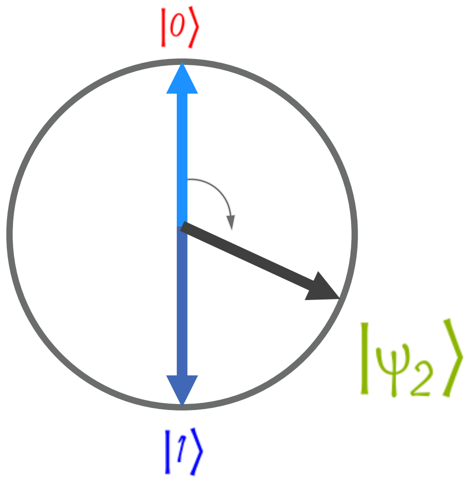


$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\psi_3\rangle = \alpha_1\alpha_2\alpha_3|000\rangle + \alpha_1\alpha_2\beta_3|001\rangle + \dots$$
$$+ \beta_1\beta_2\alpha_3|110\rangle + \beta_1\beta_2\beta_3|111\rangle$$

Multi Qubits System





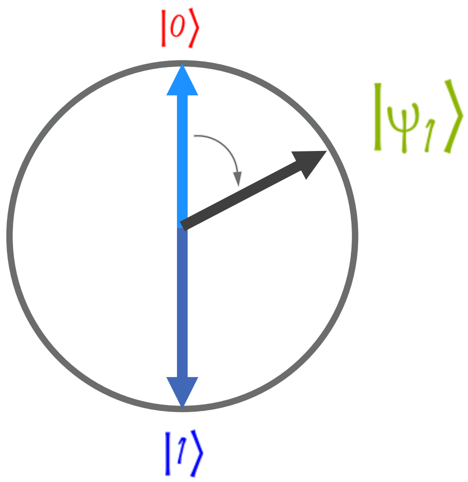
$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\psi_3\rangle = \alpha_1\alpha_2\alpha_3|000\rangle + \alpha_1\alpha_2\beta_3|001\rangle + \dots$$
$$+ \beta_1\beta_2\alpha_3|110\rangle + \beta_1\beta_2\beta_3|111\rangle$$

...

$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots$$
$$+ \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

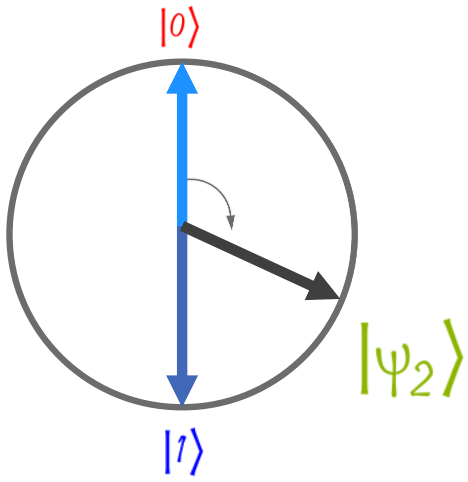
Multi Qubits System



$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\psi_3\rangle = \alpha_1\alpha_2\alpha_3|000\rangle + \alpha_1\alpha_2\beta_3|001\rangle + \dots$$
$$+ \beta_1\beta_2\alpha_3|110\rangle + \beta_1\beta_2\beta_3|111\rangle$$

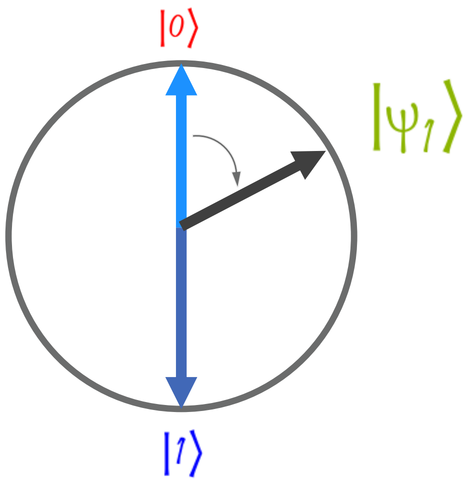
...

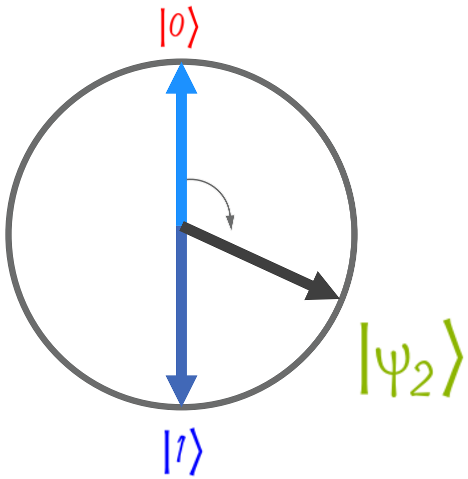


$$|\psi_1\psi_2\dots\psi_N\rangle = \alpha_1\alpha_2\dots\alpha_N|00\dots0\rangle + \alpha_1\alpha_2\dots\alpha_{N-1}\beta_N|00\dots01\rangle + \dots$$
$$+ \beta_1\beta_2\dots\beta_{N-1}\alpha_N|11\dots10\rangle + \beta_1\beta_2\dots\beta_N|11\dots1\rangle$$

In general, a system with **N qubits** is completely described by a vector with **2^N elements**

Multi Qubits System



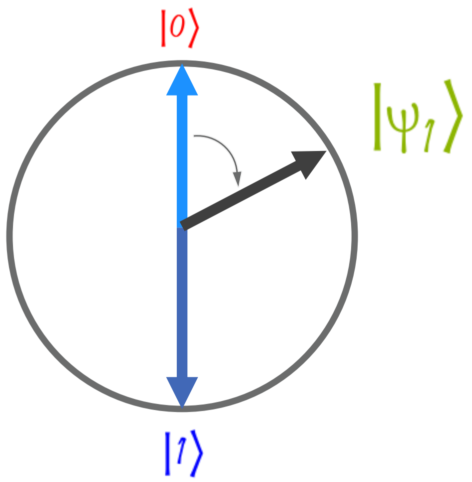


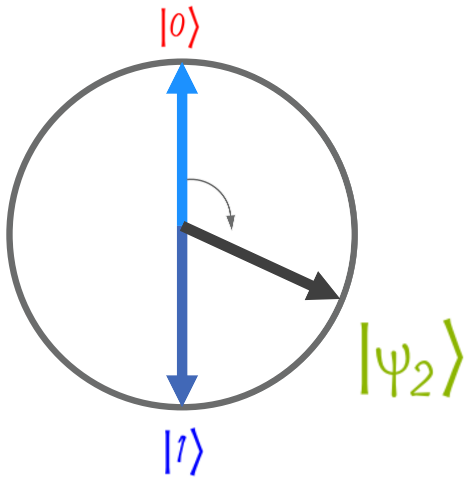
Another way to describe the representation of a system of qubits using the wave functions of the qubits that make up the system is through the concept of outer product (or tensor product). In formulas:

$$|\psi_1\psi_2 \dots \psi_N\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \otimes \dots \otimes |\psi_N\rangle =$$

$$((|\psi_1\rangle\langle\psi_2|)\langle\psi_3|)\dots\langle\psi_N|$$

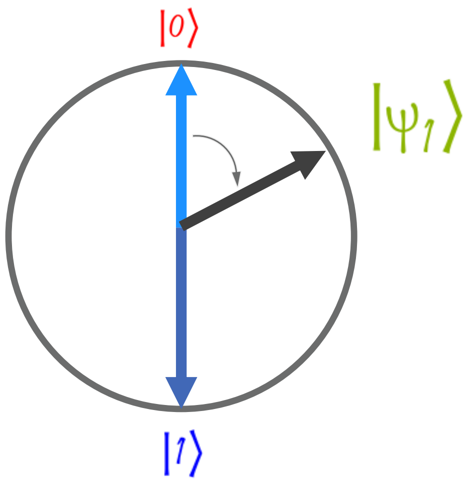
Multi Qubits System

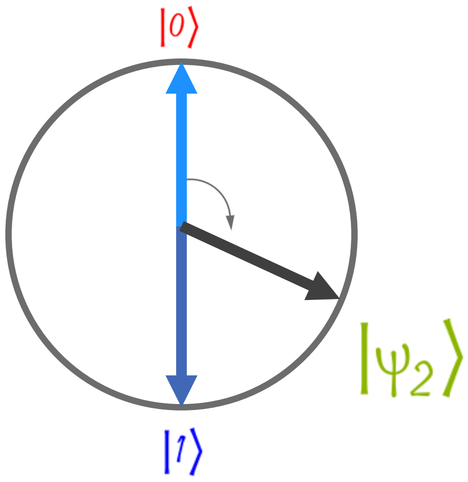




We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

Multi Qubits System

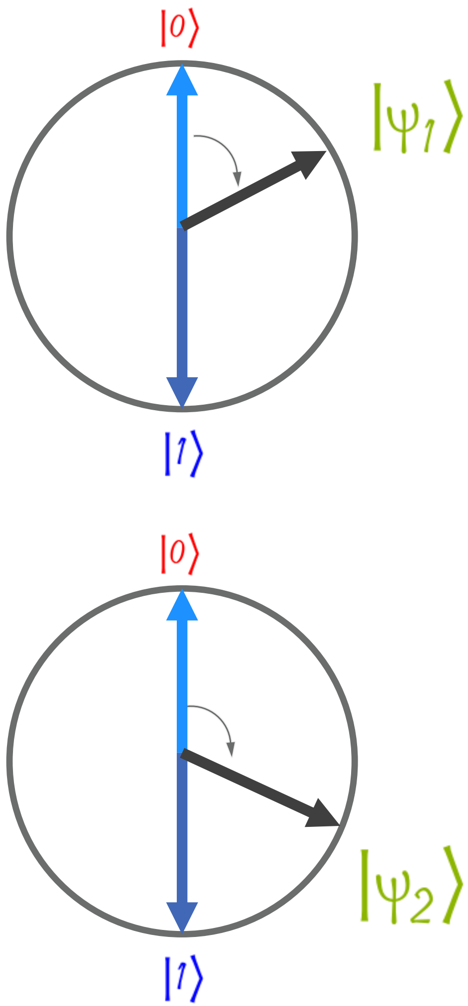




We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots$$
$$+ \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

Multi Qubits System

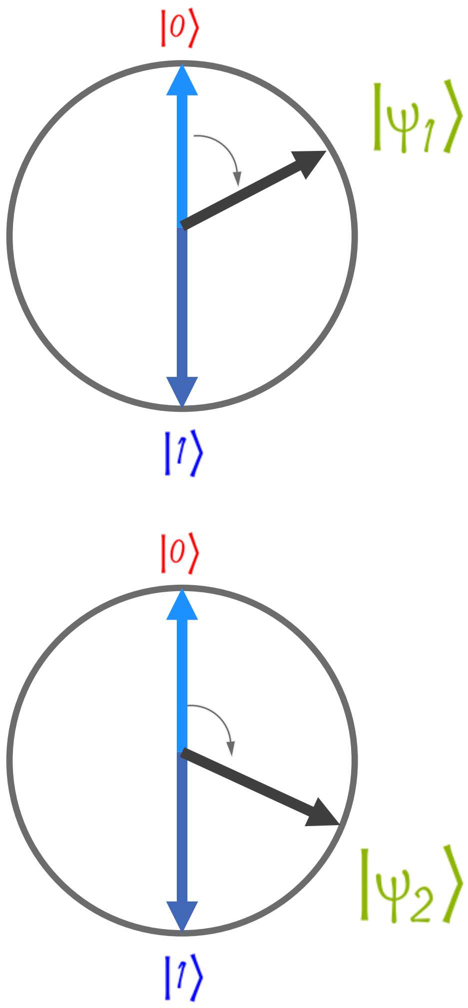


We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots$$
$$+ \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

$$\Downarrow$$

Multi Qubits System



We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

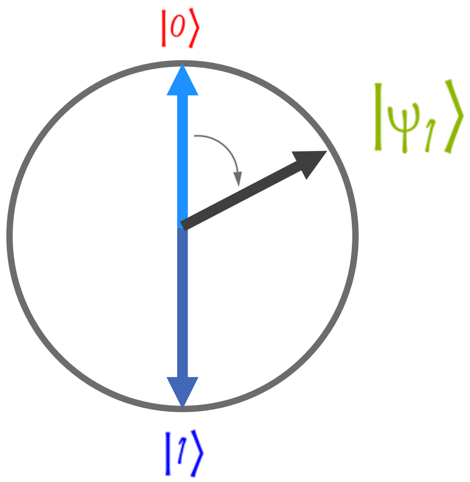
$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots$$

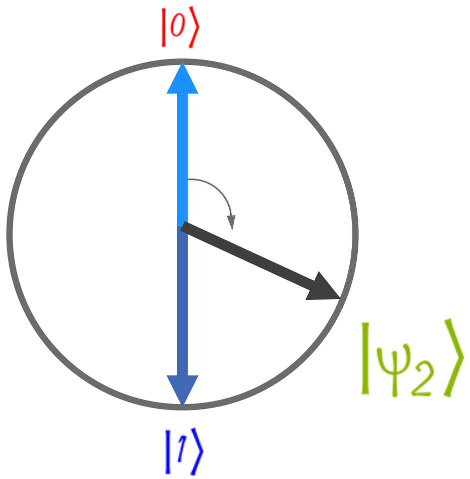
$$+ \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

$$\Downarrow$$

$$|\psi_1\psi_2 \dots \psi_N\rangle = \gamma_1|00 \dots 0\rangle + \gamma_2|00 \dots 01\rangle + \dots + \gamma_{N-1}|11 \dots 10\rangle + \gamma_N|11 \dots 1\rangle$$

Multi Qubits System





We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

However, the reverse statement is not so true. We will see in the course of the lesson an example of a system of qubits that cannot be decomposed into the tensor product of several qubits

$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots$$

$$+ \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

$$\Downarrow \qquad \cancel{\Uparrow}$$

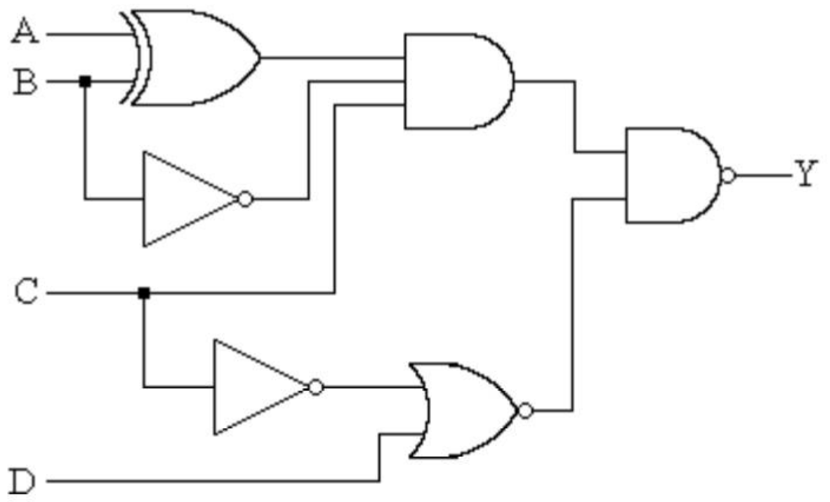
$$|\psi_1\psi_2 \dots \psi_N\rangle = \gamma_1|00 \dots 0\rangle + \gamma_2|00 \dots 01\rangle + \dots + \gamma_{N-1}|11 \dots 10\rangle + \gamma_N|11 \dots 1\rangle$$

Classical bit manipulation

```
#include <stdio.h>
#include <string.h>
#include <math.h>
int main() {
  int input = 0;
  scanf("%d", &input);
  int y = 0;
  int x = 2;
  for (; y = 1; x++) {
    int newInput = input - (x * x);
    newInput = sqrt(newInput);
    if ((input - (x * x)) == ((newInput * newInput) && (newInput != (x * x)))) {
      printf(x, " + ", newInput * newInput);
      y = 1;
    }
  }
  return 0;
}
```

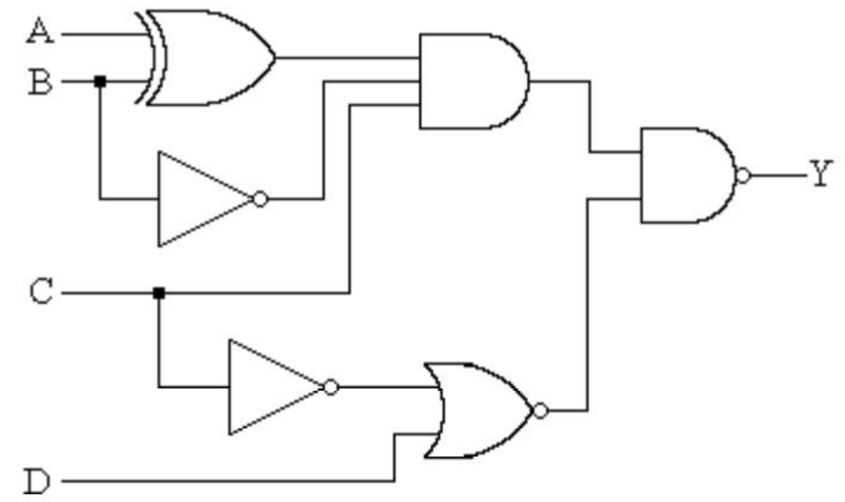
Classical bit manipulation

```c
#include <stdio.h>
#include <string.h>
#include <math.h>
int main() {
  int input = 0;
  scanf("%d", &input);
  int y = 0;
  int x = 2;
  for (; y = 1; x++) {
    int newInput = input - (x * x);
    newInput = sqrt(newInput);
    if ((input - (x * x)) == ((newInput * newInput) && (newInput != (x * x)))) {
      printf(x, " + ", newInput * newInput);
      y = 1;
    }
  }
  return 0;
}
```

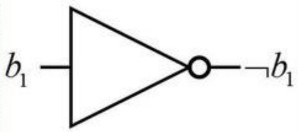


Classical bit manipulation

```
#include <stdio.h>
#include <string.h>
#include <math.h>
int main() {
  int input = 0;
  scanf("%d", &input);
  int y = 0;
  int x = 2;
  for (; y = 1; x++) {
    int newInput = input - (x * x);
    newInput = sqrt(newInput);
    if ((input - (x * x)) == ((newInput * newInput) && (newInput != (x * x)))) {
      printf(x, " + ", newInput * newInput);
      y = 1;
    }
  }
  return 0;
}
```

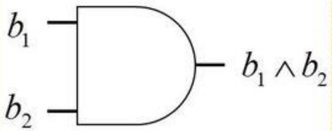


NOT gate



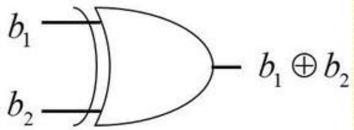
b_1	$\neg b_1$
0	1
1	0

AND gate



b_1	b_2	$b_1 \wedge b_2$
0	0	0
0	1	0
1	0	0
1	1	1

XOR gate



b_1	b_2	$b_1 \oplus b_2$
0	0	0
0	1	1
1	0	1
1	1	0

Quantum Gates

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1M} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2M} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MM} \end{pmatrix}}_{M=2^N} \left.\vphantom{\begin{pmatrix} a \\ a \\ a \\ \vdots \\ a \end{pmatrix}}\right\} M=2^N$$

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to 2^N

CINECA QUANTUM COMPUTING LAB

Quantum Gates

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1M} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2M} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MM} \end{pmatrix}}_{M=2^N} \left.\vphantom{\begin{pmatrix} a \\ a \\ a \\ \vdots \\ a \end{pmatrix}}\right\} M=2^N$$

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to 2^N
- In order to act as quantum gates, matrices must meet a couple of important prerequisites

Quantum Gates

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

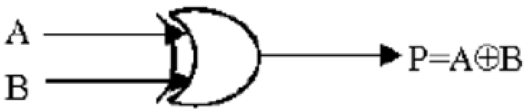
$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1M} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2M} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & a_{M3} & \cdots & a_{MM} \end{pmatrix}}_{M=2^N} \left.\vphantom{\begin{pmatrix} a \\ a \\ a \\ \vdots \\ a \end{pmatrix}}\right\} M=2^N$$

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to 2^N
- In order to act as quantum gates, matrices must meet a couple of important prerequisites

  1: They must be unitary matrices: one of the reasons for this choice is that quantum gates are defined as operators that take qubits in input and return qubits in output. Consequently, it is important that the application of quantum gates does not modify the length of the input vector

Quantum Gates

A	B	P
0	0	0
0	1	1
1	0	1
1	1	0



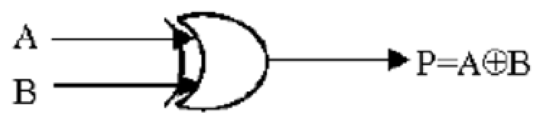
Irreversible EX-OR Gate

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to 2^N
- In order to act as quantum gates, matrices must meet a couple of important prerequisites

  2: The number of input qubits must equal the number of output qubits. This implies that it is not possible to construct quantum gates for non-reversible operations, ie where it is not possible to trace the input states by observing only the output states (such as the XOR gate). However, it is possible to make these operations reversible

Quantum Gates

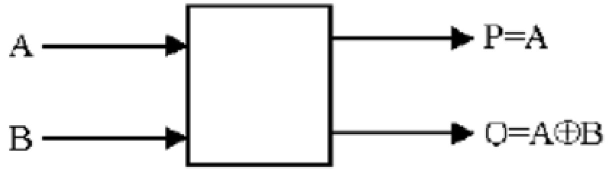
A	B	P
0	0	0
0	1	1
1	0	1
1	1	0



Irreversible EX-OR Gate

Garbage Output

A	B	P	Q
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



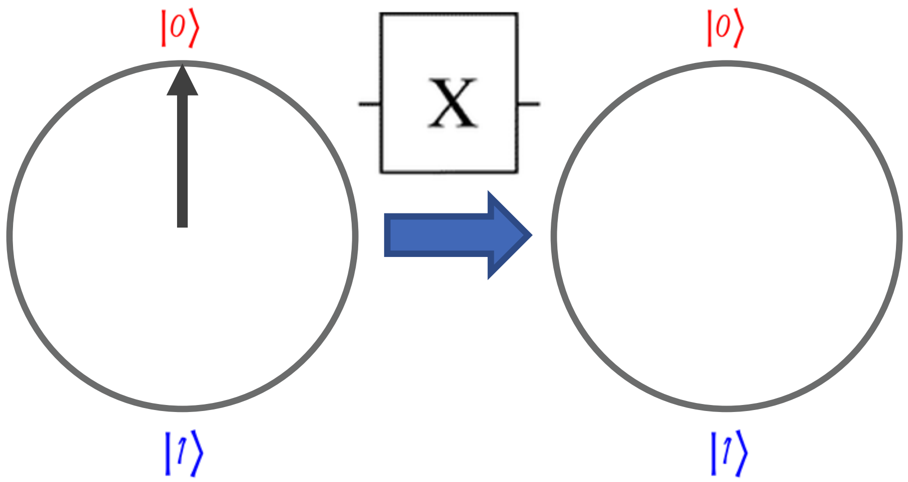
Reversible EX-OR Gate

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to 2^N

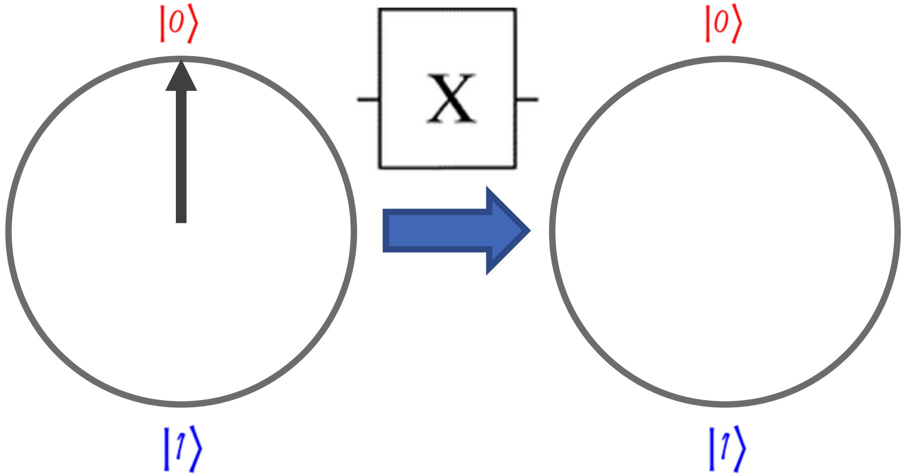
  In order to act as quantum gates, matrices must meet a couple of important prerequisites

  2: The number of input qubits must equal the number of output qubits. This implies that it is not possible to construct quantum gates for non-reversible operations, ie where it is not possible to trace the input states by observing only the output states (such as the XOR gate). However, it is possible to make these operations reversible

X (NOT) Gate

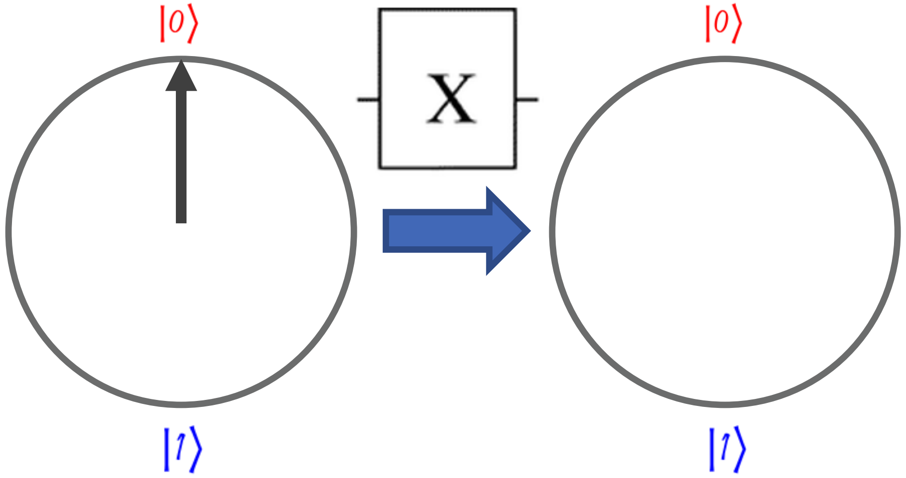


X (NOT) Gate



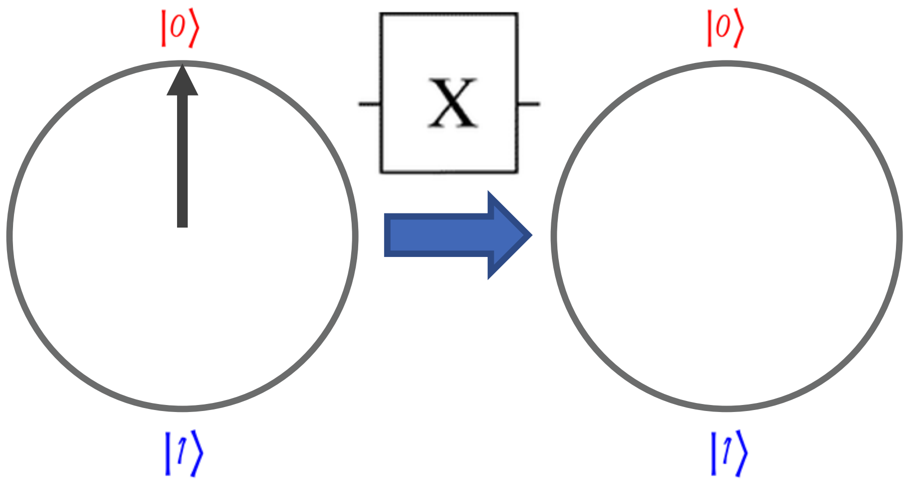
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

X (NOT) Gate



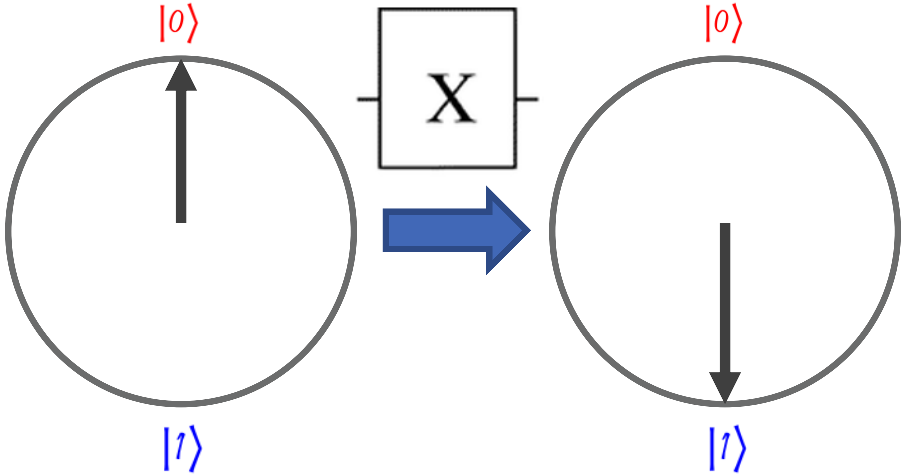
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

X (NOT) Gate



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

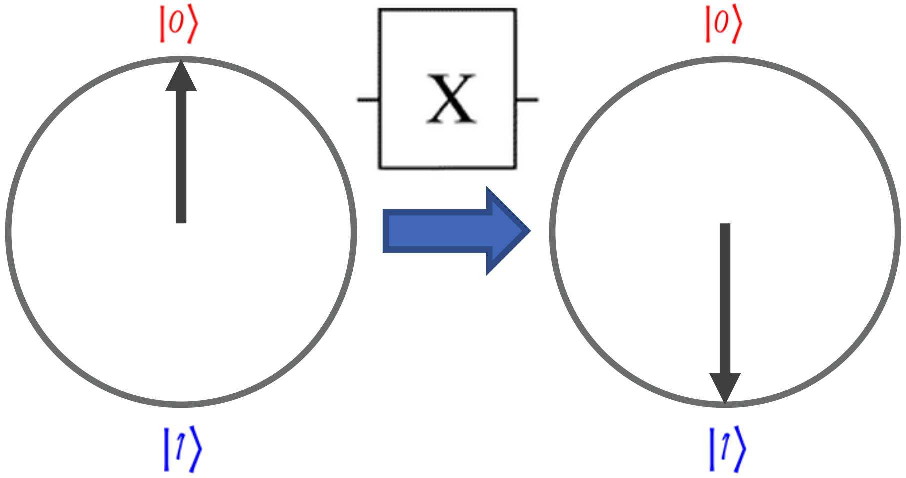
X (NOT) Gate

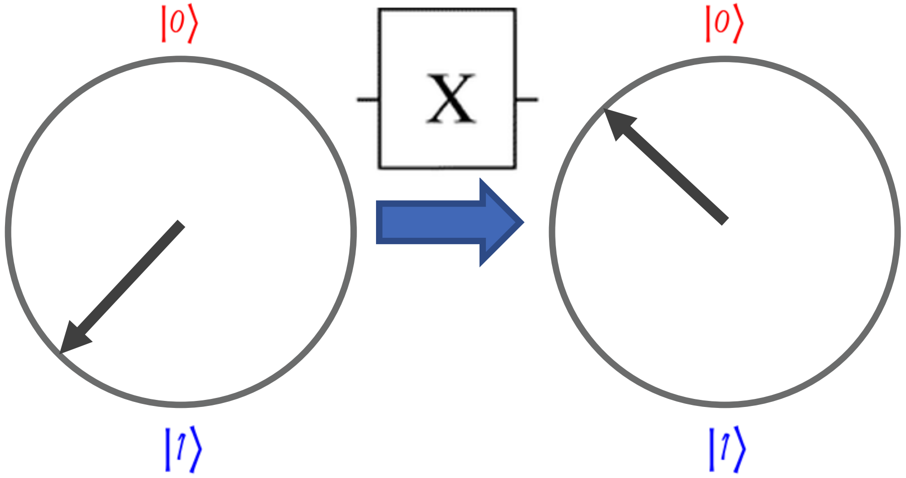


$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |1\rangle$$

X (NOT) Gate





$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |1\rangle$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

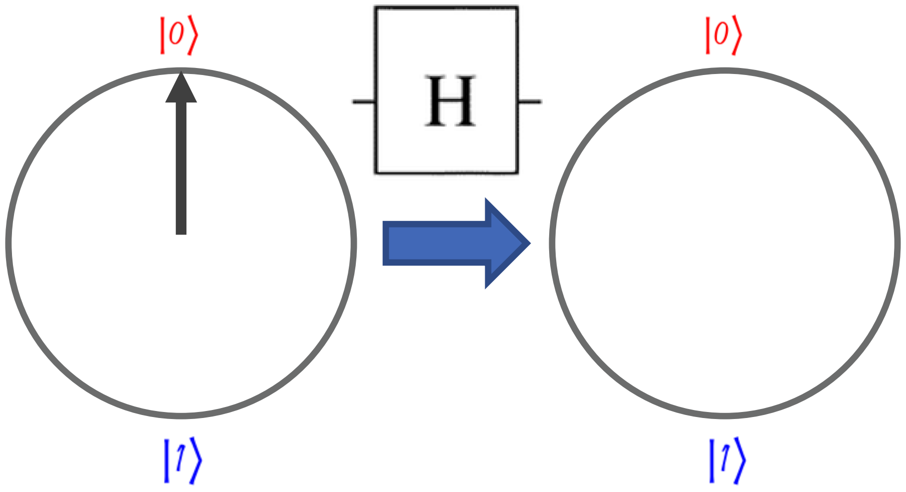
$$\alpha \cdot |0\rangle + \beta \cdot |1\rangle \rightarrow \beta \cdot |0\rangle + \alpha \cdot |1\rangle$$

Single Qubit Quantum Gates

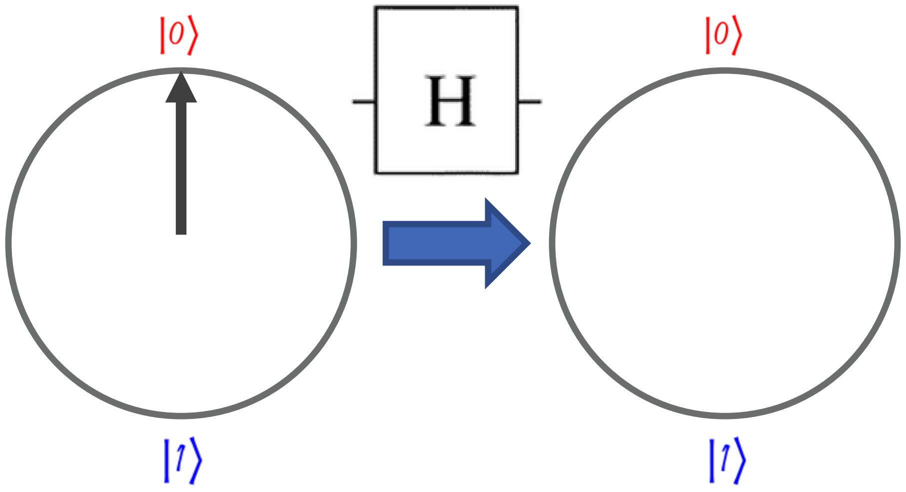
X Gate
Bit–flip, Not

$$\text{—}\boxed{X}\text{—} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta|0\rangle + \alpha|1\rangle$$

Hadamard Gate

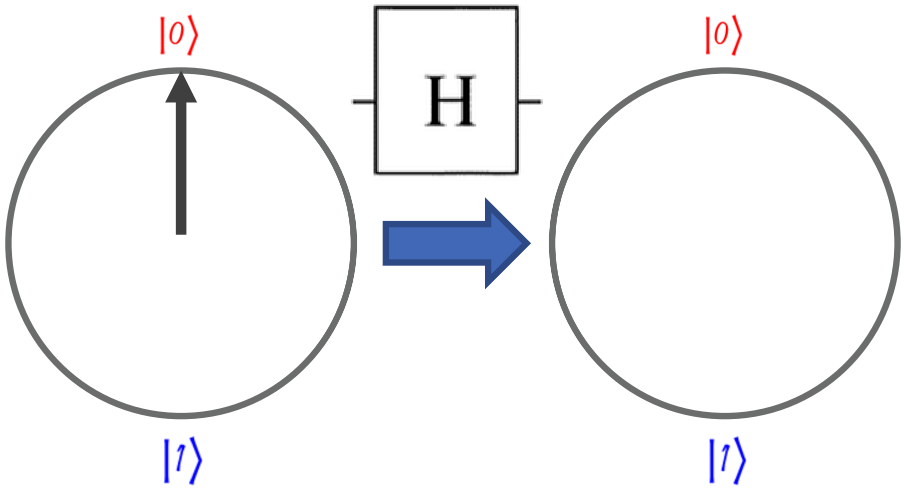


Hadamard Gate



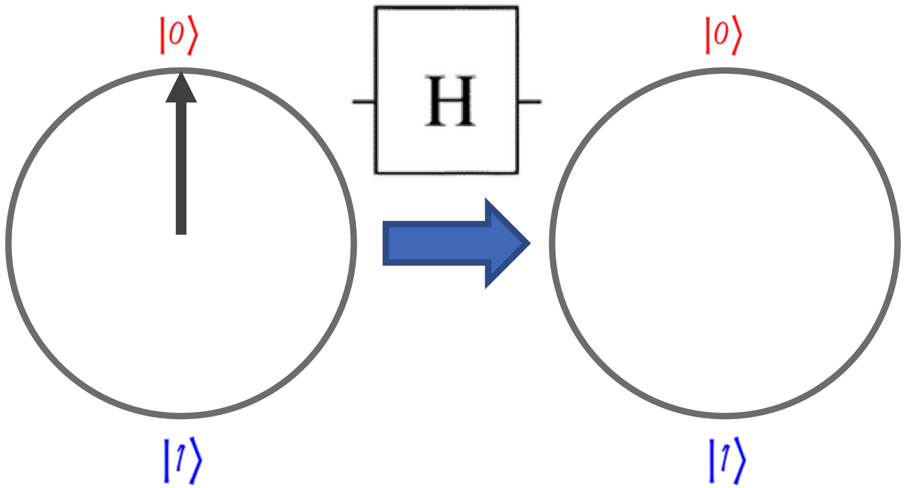
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Hadamard Gate



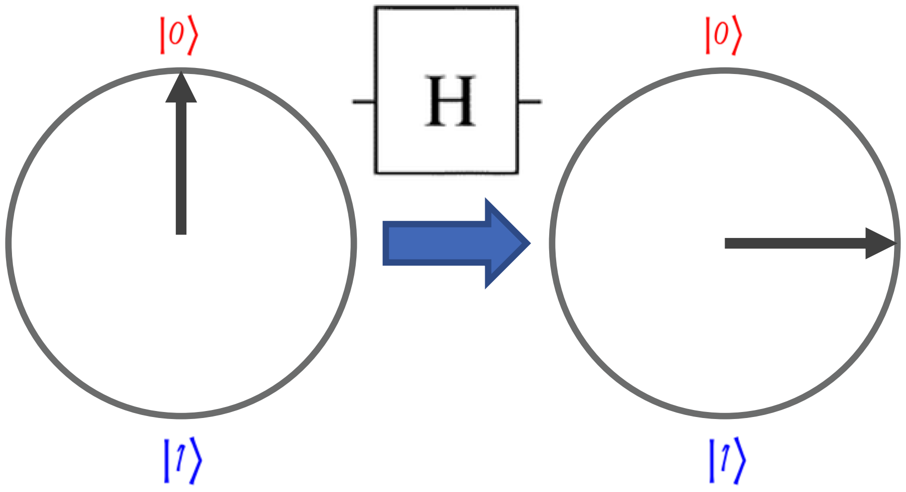
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Hadamard Gate



$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\ 0\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$$

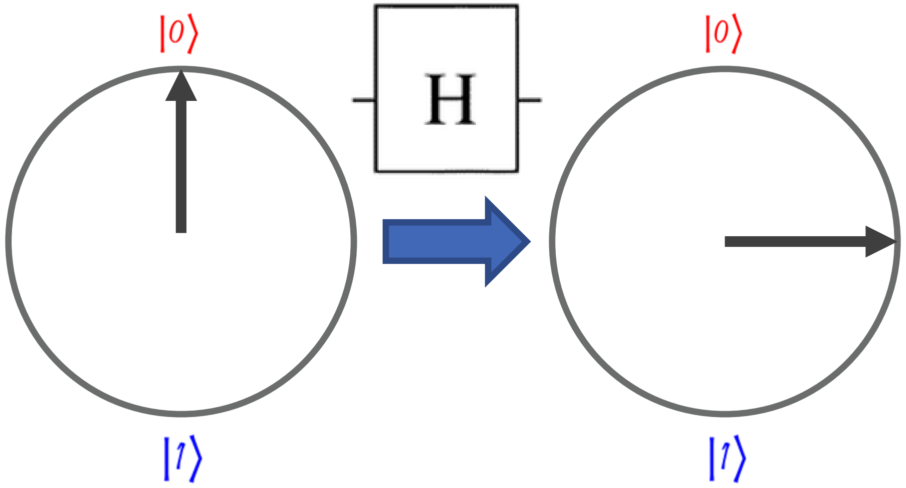
Hadamard Gate

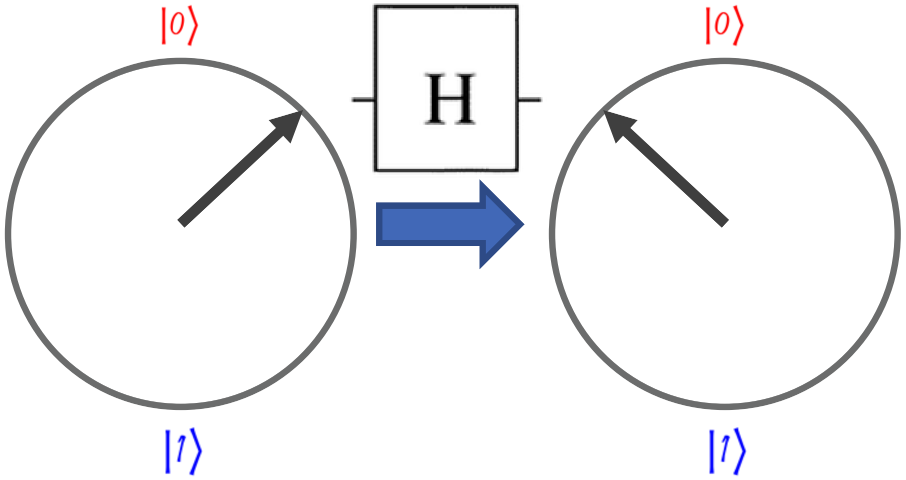


$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle$$

Hadamard Gate





$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \alpha + \beta \\ \alpha - \beta \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle \qquad \alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha + \beta}{\sqrt{2}} \cdot |0\rangle + \frac{\alpha - \beta}{\sqrt{2}} \cdot |1\rangle$$

Single Qubit Quantum Gates

X Gate
Bit–flip, Not

$$-\boxed{X}- \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta|0\rangle + \alpha|1\rangle$$

Z Gate
Phase–flip

$$-\boxed{Z}- \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle$$

H Gate
Hadamard

$$-\boxed{H}- \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{\alpha+\beta|0\rangle + \alpha-\beta|1\rangle}{\sqrt{2}}$$

T Gate

$$-\boxed{T}- \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle + e^{i\pi/4}\beta|1\rangle$$

Two qubits Gates: Swap gate

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

Two qubits Gates: Swap gate

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\psi_1\rangle = \alpha_2|0\rangle + \beta_2|1\rangle, \qquad |\psi_2\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

Two qubits Gates: Swap gate

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\psi_1\rangle = \alpha_2|0\rangle + \beta_2|1\rangle, \qquad |\psi_2\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$\Downarrow$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

Two qubits Gates: Swap gate

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\psi_1\rangle = \alpha_2|0\rangle + \beta_2|1\rangle, \qquad |\psi_2\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$\Downarrow$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_2\beta_1|01\rangle + \beta_2\alpha_1|10\rangle + \beta_1\beta_2|11\rangle$$

Two qubits Gates: Swap gate

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\psi_1\rangle = \alpha_2|0\rangle + \beta_2|1\rangle, \qquad |\psi_2\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$\Downarrow$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_2\beta_1|01\rangle + \beta_2\alpha_1|10\rangle + \beta_1\beta_2|11\rangle$$

Two qubits Gates: Swap gate

$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \quad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$

$|\psi_1\rangle = \alpha_2|0\rangle + \beta_2|1\rangle, \quad |\psi_2\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$

$\Downarrow$

$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$

$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_2\beta_1|01\rangle + \beta_2\alpha_1|10\rangle + \beta_1\beta_2|11\rangle$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ c \\ b \\ d \end{pmatrix}$$

Two qubits Gates: Swap gate

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \qquad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\psi_1\rangle = \alpha_2|0\rangle + \beta_2|1\rangle, \qquad |\psi_2\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$\Downarrow$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_2\beta_1|01\rangle + \beta_2\alpha_1|10\rangle + \beta_1\beta_2|11\rangle$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ c \\ b \\ d \end{pmatrix}$$

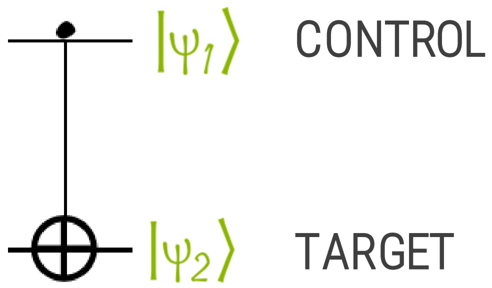
$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle$$

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



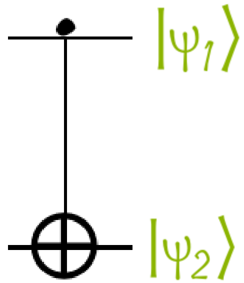
The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



CONTROL

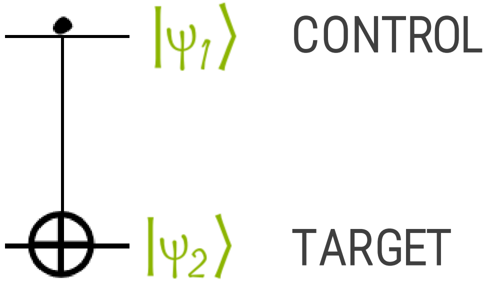
TARGET

The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\mathbb{P}\left(\psi_2 = |1\rangle \,|\, \psi_1 = |1\rangle\right)$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \,|\, \psi_1 = |1\rangle\right)$$

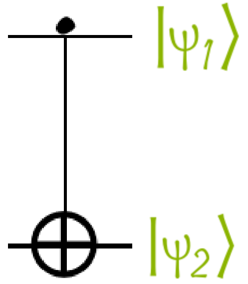
$$\mathbb{P}\left(\psi_2 = |1\rangle \,|\, \psi_1 = |0\rangle\right)$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \,|\, \psi_1 = |0\rangle\right)$$

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\mathbb{P}\left(\psi_2 = |1\rangle \middle| \psi_1 = |1\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |1\rangle, \psi_2 = |1\rangle\right)}{\mathbb{P}\left(\psi_1 = |1\rangle\right)}$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \middle| \psi_1 = |1\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |1\rangle, \psi_2 = |0\rangle\right)}{\mathbb{P}\left(\psi_1 = |1\rangle\right)}$$

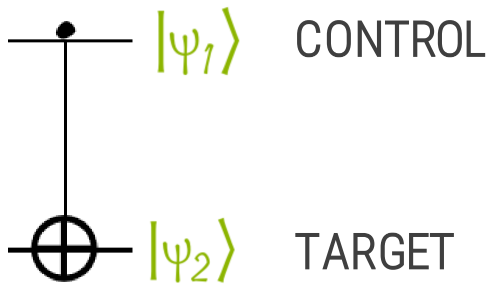
$$\mathbb{P}\left(\psi_2 = |1\rangle \middle| \psi_1 = |0\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |0\rangle, \psi_2 = |1\rangle\right)}{\mathbb{P}\left(\psi_1 = |0\rangle\right)}$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \middle| \psi_1 = |0\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |0\rangle, \psi_2 = |0\rangle\right)}{\mathbb{P}\left(\psi_1 = |0\rangle\right)}$$

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\mathbb{P}\left(\psi_2 = |1\rangle \middle| \psi_1 = |1\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |1\rangle, \psi_2 = |1\rangle\right)}{\mathbb{P}\left(\psi_1 = |1\rangle\right)} = \frac{d^2}{c^2 + d^2}$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \middle| \psi_1 = |1\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |1\rangle, \psi_2 = |0\rangle\right)}{\mathbb{P}\left(\psi_1 = |1\rangle\right)} = \frac{c^2}{c^2 + d^2}$$

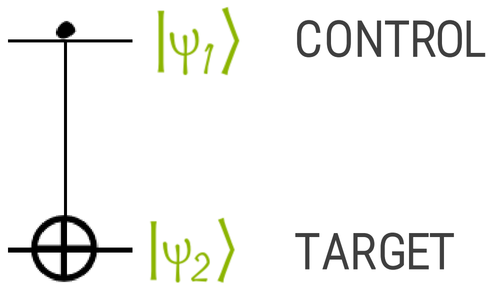
$$\mathbb{P}\left(\psi_2 = |1\rangle \middle| \psi_1 = |0\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |0\rangle, \psi_2 = |1\rangle\right)}{\mathbb{P}\left(\psi_1 = |0\rangle\right)} = \frac{b^2}{a^2 + b^2}$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \middle| \psi_1 = |0\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |0\rangle, \psi_2 = |0\rangle\right)}{\mathbb{P}\left(\psi_1 = |0\rangle\right)} = \frac{a^2}{a^2 + b^2}$$

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\mathbb{P}\left(\psi_2 = |1\rangle \middle| \psi_1 = |1\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |1\rangle, \psi_2 = |1\rangle\right)}{\mathbb{P}\left(\psi_1 = |1\rangle\right)} = \frac{d^2}{c^2 + d^2}$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \middle| \psi_1 = |1\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |1\rangle, \psi_2 = |0\rangle\right)}{\mathbb{P}\left(\psi_1 = |1\rangle\right)} = \frac{c^2}{c^2 + d^2}$$

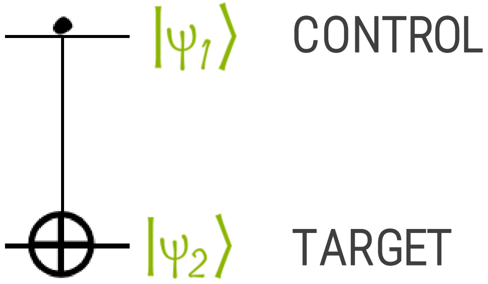
$$\mathbb{P}\left(\psi_2 = |1\rangle \middle| \psi_1 = |0\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |0\rangle, \psi_2 = |1\rangle\right)}{\mathbb{P}\left(\psi_1 = |0\rangle\right)} = \frac{b^2}{a^2 + b^2}$$

$$\mathbb{P}\left(\psi_2 = |0\rangle \middle| \psi_1 = |0\rangle\right) = \frac{\mathbb{P}\left(\psi_1 = |0\rangle, \psi_2 = |0\rangle\right)}{\mathbb{P}\left(\psi_1 = |0\rangle\right)} = \frac{a^2}{a^2 + b^2}$$

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\mathbb{P}(\psi_2 = |1\rangle \,|\, \psi_1 = |1\rangle) = \frac{\mathbb{P}(\psi_1 = |1\rangle, \psi_2 = |1\rangle)}{\mathbb{P}(\psi_1 = |1\rangle)} = \frac{d^2}{c^2 + d^2}$$

$$\mathbb{P}(\psi_2 = |0\rangle \,|\, \psi_1 = |1\rangle) = \frac{\mathbb{P}(\psi_1 = |1\rangle, \psi_2 = |0\rangle)}{\mathbb{P}(\psi_1 = |1\rangle)} = \frac{c^2}{c^2 + d^2}$$

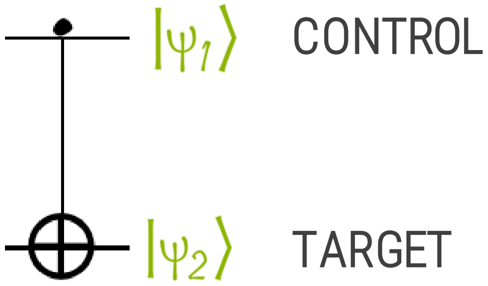
$$\mathbb{P}(\psi_2 = |1\rangle \,|\, \psi_1 = |0\rangle) = \frac{\mathbb{P}(\psi_1 = |0\rangle, \psi_2 = |1\rangle)}{\mathbb{P}(\psi_1 = |0\rangle)} = \frac{b^2}{a^2 + b^2}$$

$$\mathbb{P}(\psi_2 = |0\rangle \,|\, \psi_1 = |0\rangle) = \frac{\mathbb{P}(\psi_1 = |0\rangle, \psi_2 = |0\rangle)}{\mathbb{P}(\psi_1 = |0\rangle)} = \frac{a^2}{a^2 + b^2}$$

Two qubits Gates: CNOT (Controlled-NOT) Gate

The Controlled-NOT logic gate (abbreviated to CNOT) is a quantum logic gate that acts on two qubits.

The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ b \\ d \\ c \end{pmatrix}$$

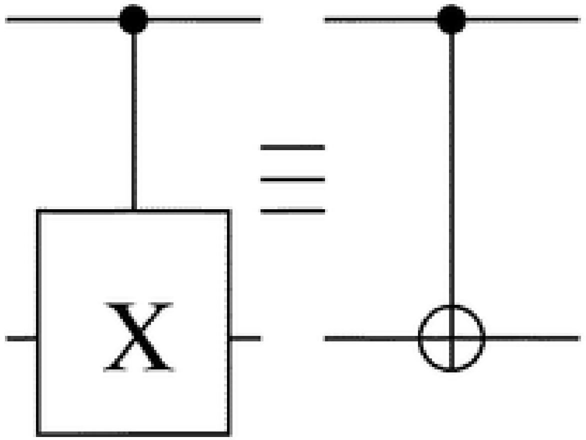
$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$$

The purpose of a CNOT gate is:
invert the amplitudes of the target qubit if and only if the control qubit is in state | 1>

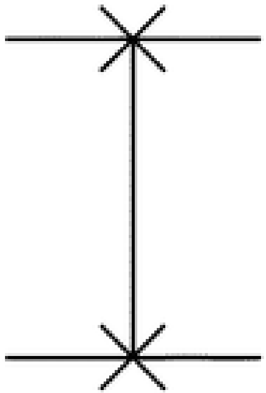
Two qubits Gates

Controlled Not
Controlled X
CNot



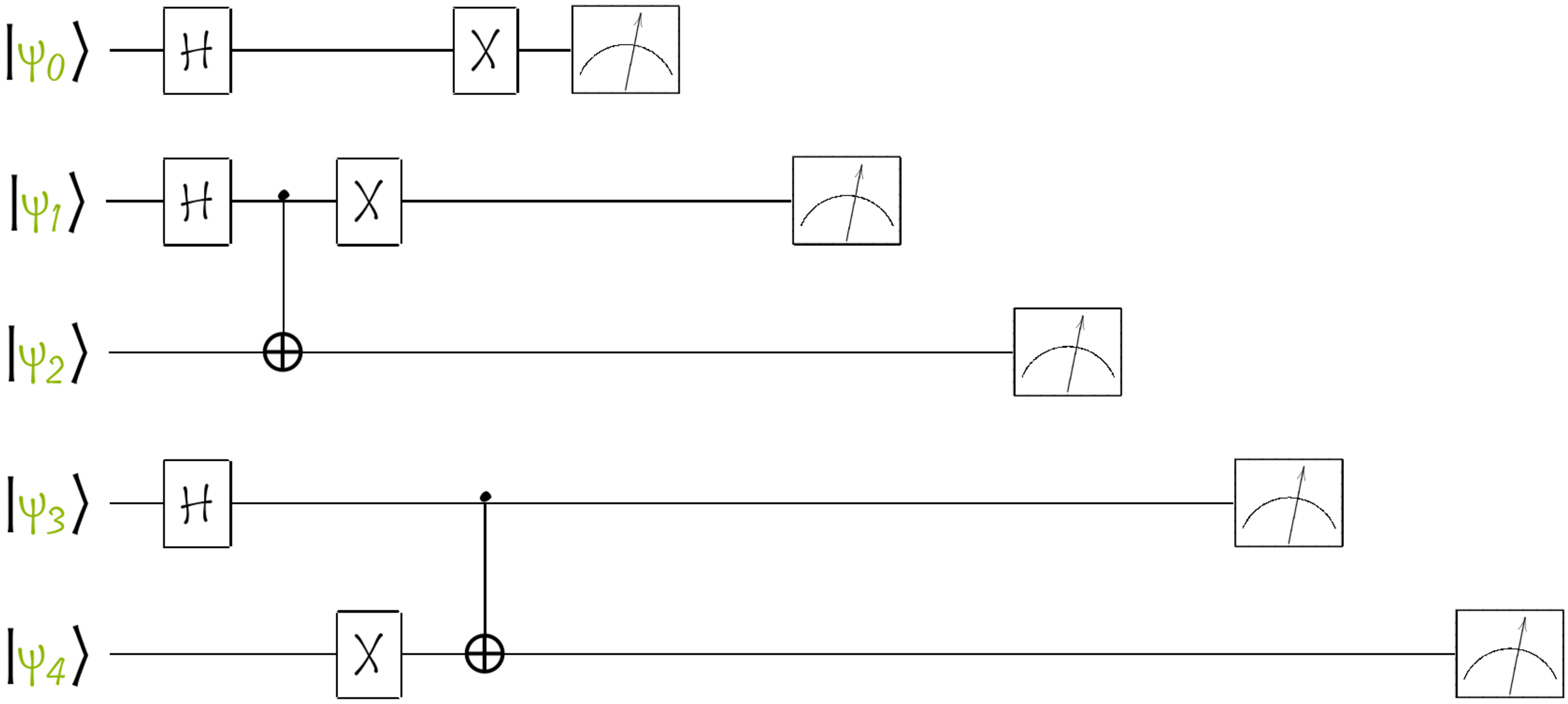
$$\equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$$

Swap

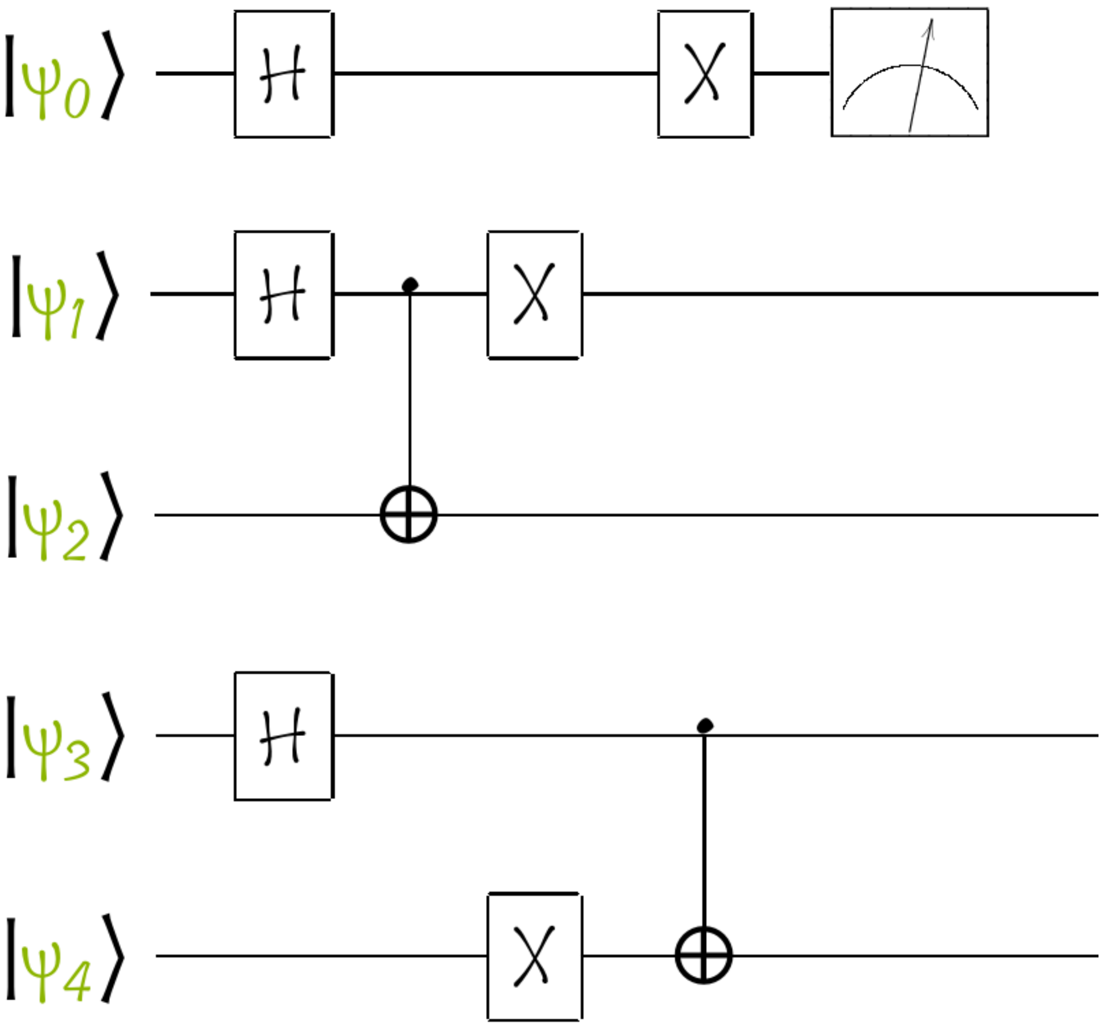


$$\equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle$$

Quantum Circuits

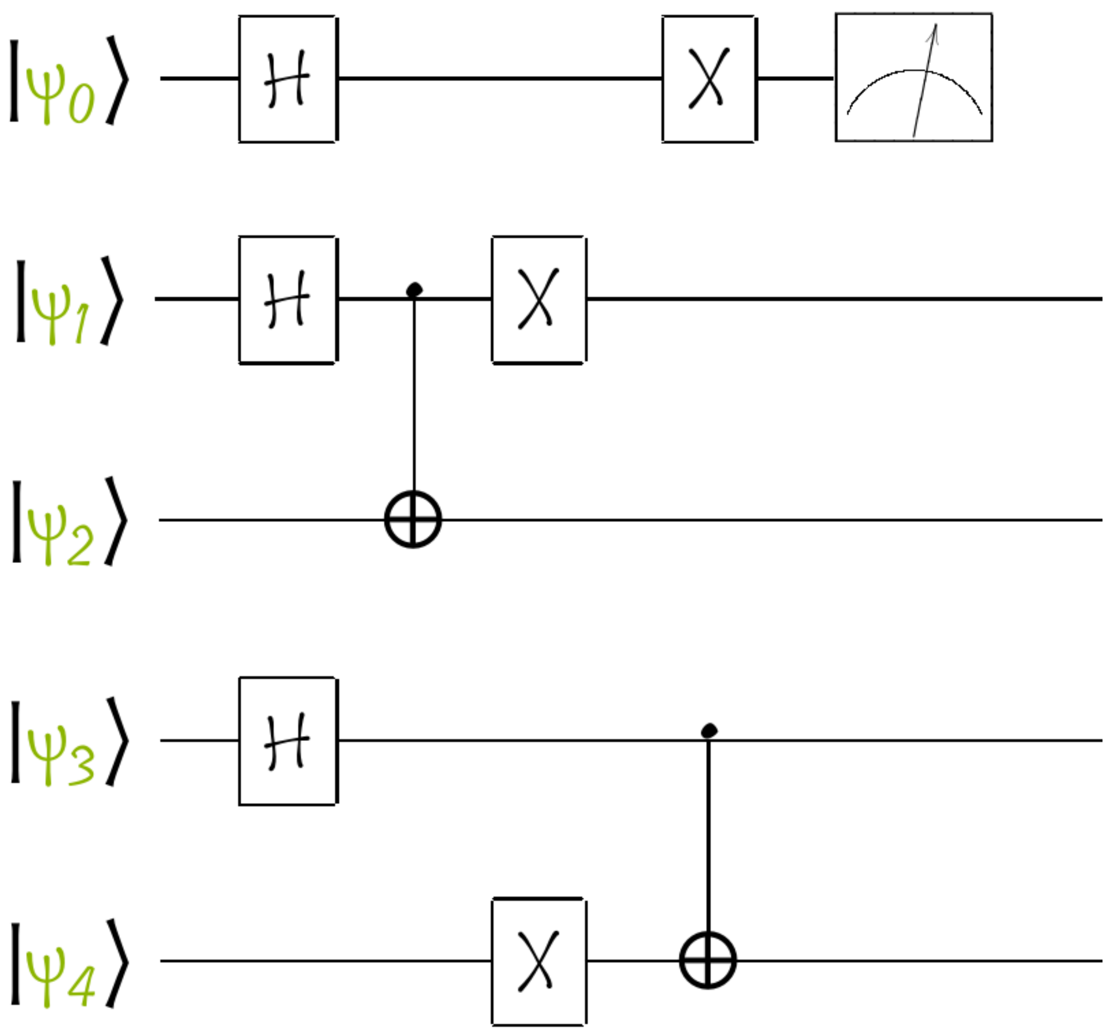


Quantum Circuits



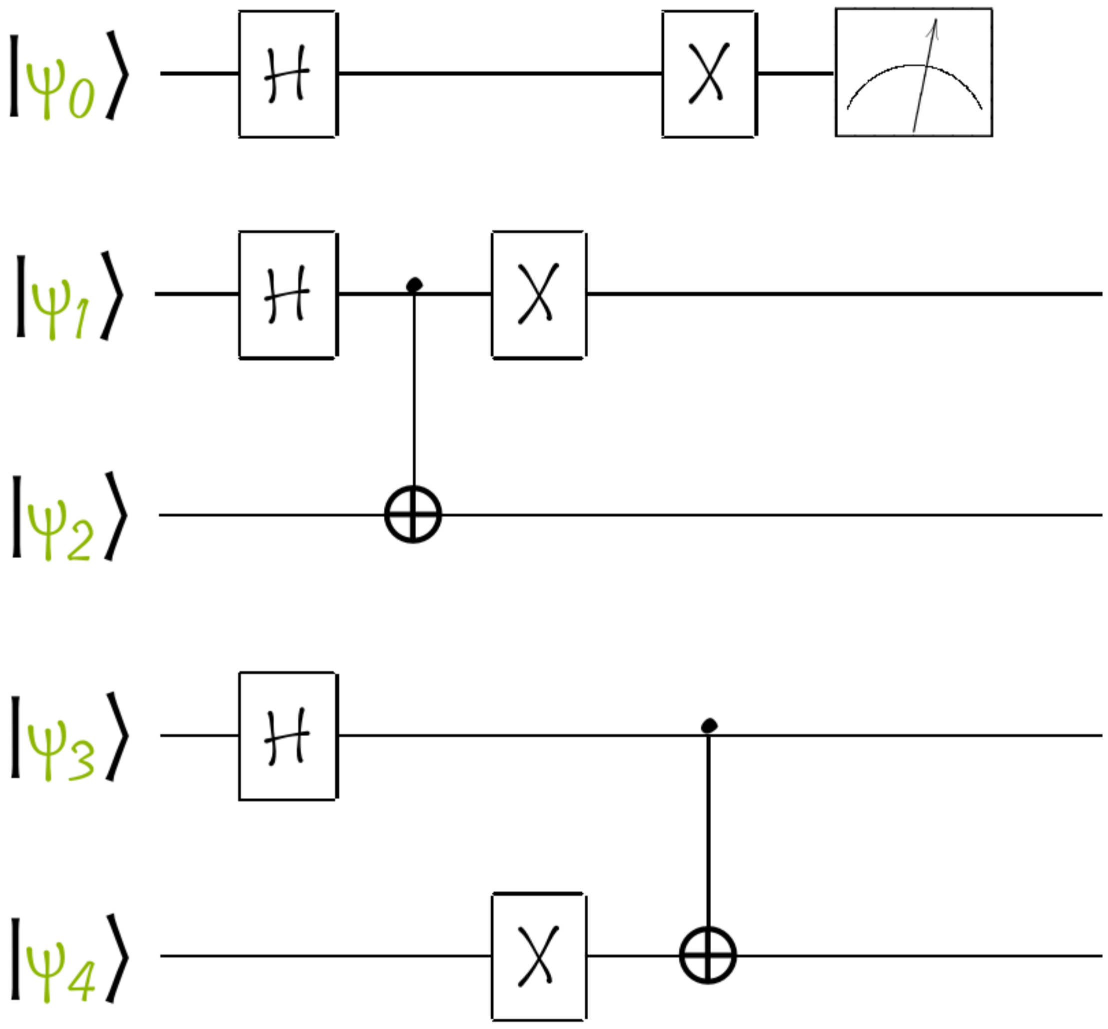
- A quantum circuit is a set of quantum gates acting on a system of qubits

Quantum Circuits



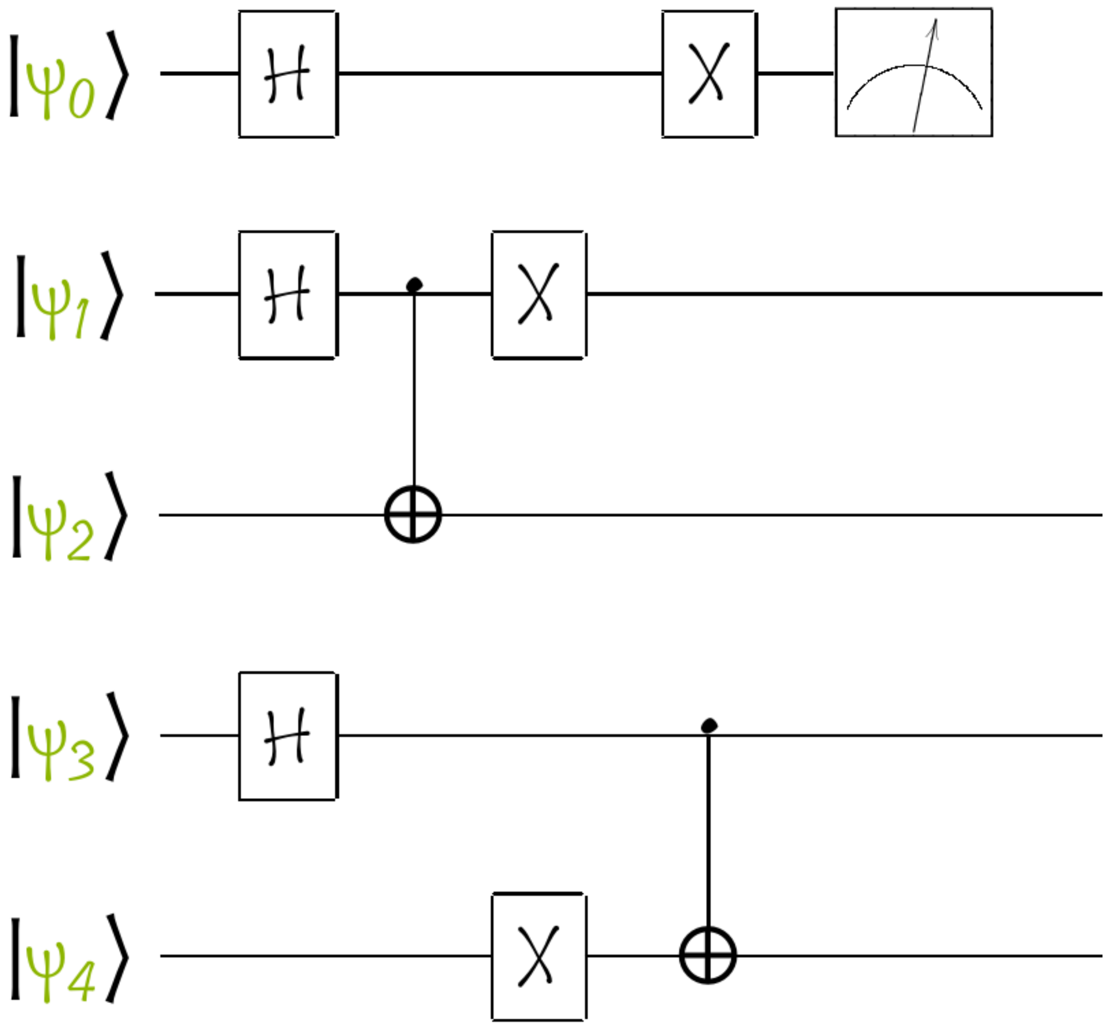
- A quantum circuit is a set of quantum gates acting on a system of qubits
- Each quantum circuit is characterized by a depth, i.e. the longest path from the input (or from a preparation) to the output (or a measurement gate), moving forward in time along qubit wires.

Quantum Circuits



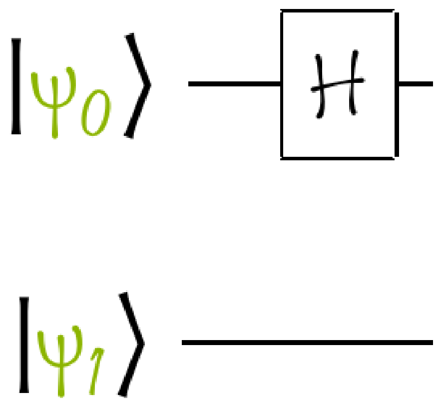
- A quantum circuit is a set of quantum gates acting on a system of qubits
- Each quantum circuit is characterized by a depth, i.e. the longest path from the input (or from a preparation) to the output (or a measurement gate), moving forward in time along qubit wires.
- Gates on the same level act simultaneously on the entire system of qubits. The order is from left to right

Quantum Circuits



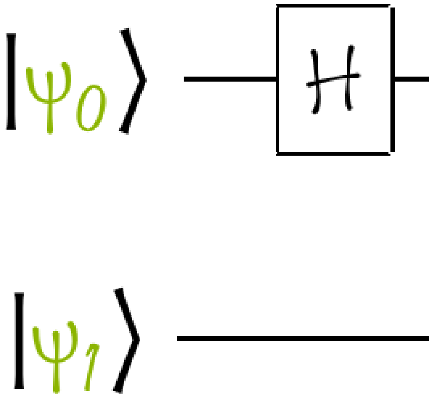
- A quantum circuit is a set of quantum gates acting on a system of qubits
- Each quantum circuit is characterized by a depth, i.e. the longest path from the input (or from a preparation) to the output (or a measurement gate), moving forward in time along qubit wires.
- Gates on the same level act simultaneously on the entire system of qubits. The order is from left to right
- By convention, the system always starts from a state of all at rest (i.e. all qubits in the classic state 0)

Combine Quantum Gates



- Let's suppose we have a circuit like the one in the figure

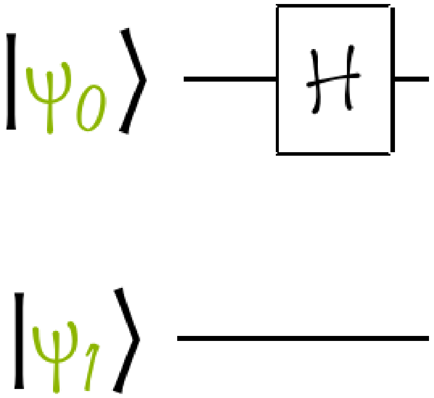
Combine Quantum Gates



- Let's suppose we have a circuit like the one in the figure
- Furthermore, suppose we are in the middle of a larger quantum circuit, which however only involves two qubits. In this case, the wave function of the system can be represented by the product of the wave functions of the individual qubits

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

Combine Quantum Gates

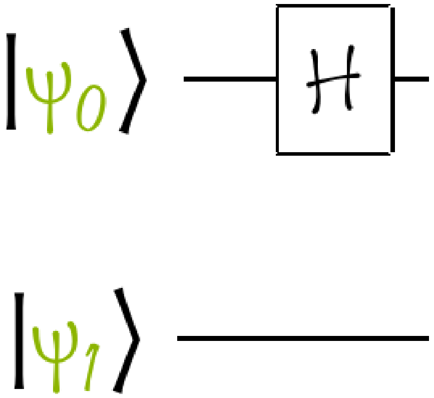


$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- Let's suppose we have a circuit like the one in the figure
- Furthermore, suppose we are in the middle of a larger quantum circuit, which however only involves two qubits. In this case, the wave function of the system can be represented by the product of the wave functions of the individual qubits
- Or, more generally, it can be represented by non-decomposable coefficients (we will see an example shortly)

Combine Quantum Gates

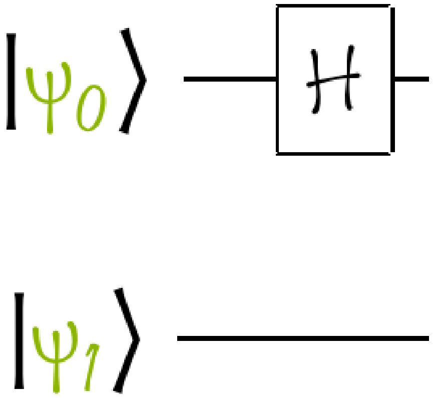


$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- Let's suppose we have a circuit like the one in the figure
- Furthermore, suppose we are in the middle of a larger quantum circuit, which however only involves two qubits. In this case, the wave function of the system can be represented by the product of the wave functions of the individual qubits
- Or, more generally, it can be represented by non-decomposable coefficients (we will see an example shortly)
- The question therefore is: how should I act on the total wave function of the system to reflect the application of the Hadamard gate only on the first qubit?

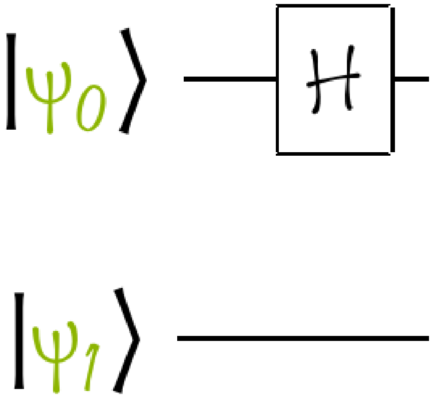
Combine Quantum Gates



- The answer is: Tensor Product

$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

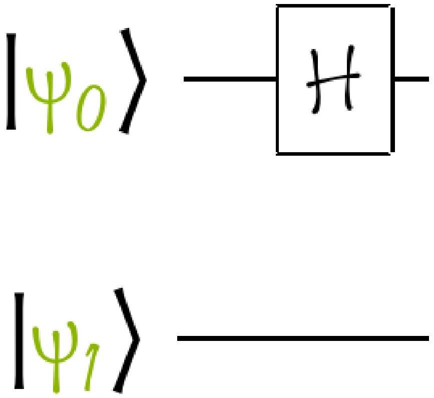
Combine Quantum Gates



$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- The answer is: Tensor Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)

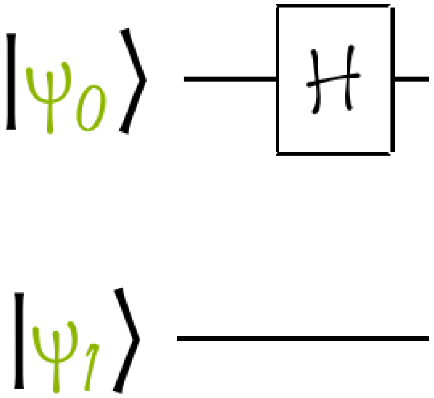
Combine Quantum Gates



$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- The answer is: Tensor Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)
- The desired matrix will therefore be the tensor product of all gates at the same level, starting from the top and proceeding downwards.

Combine Quantum Gates

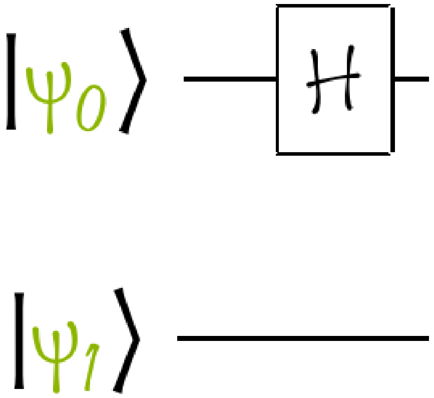


$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The answer is: Tensor Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)
- The desired matrix will therefore be the tensor product of all gates at the same level, starting from the top and proceeding downwards.

Combine Quantum Gates

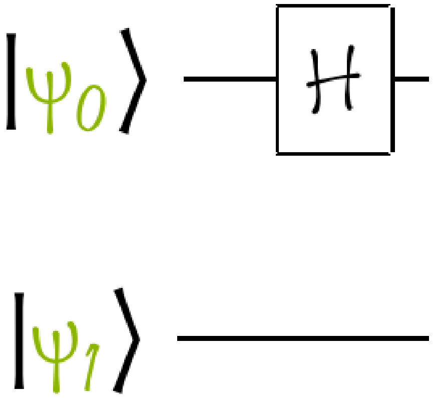


$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & -1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}$$

- The answer is: Tensor Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)
- The desired matrix will therefore be the tensor product of all gates at the same level, starting from the top and proceeding downwards.

Combine Quantum Gates

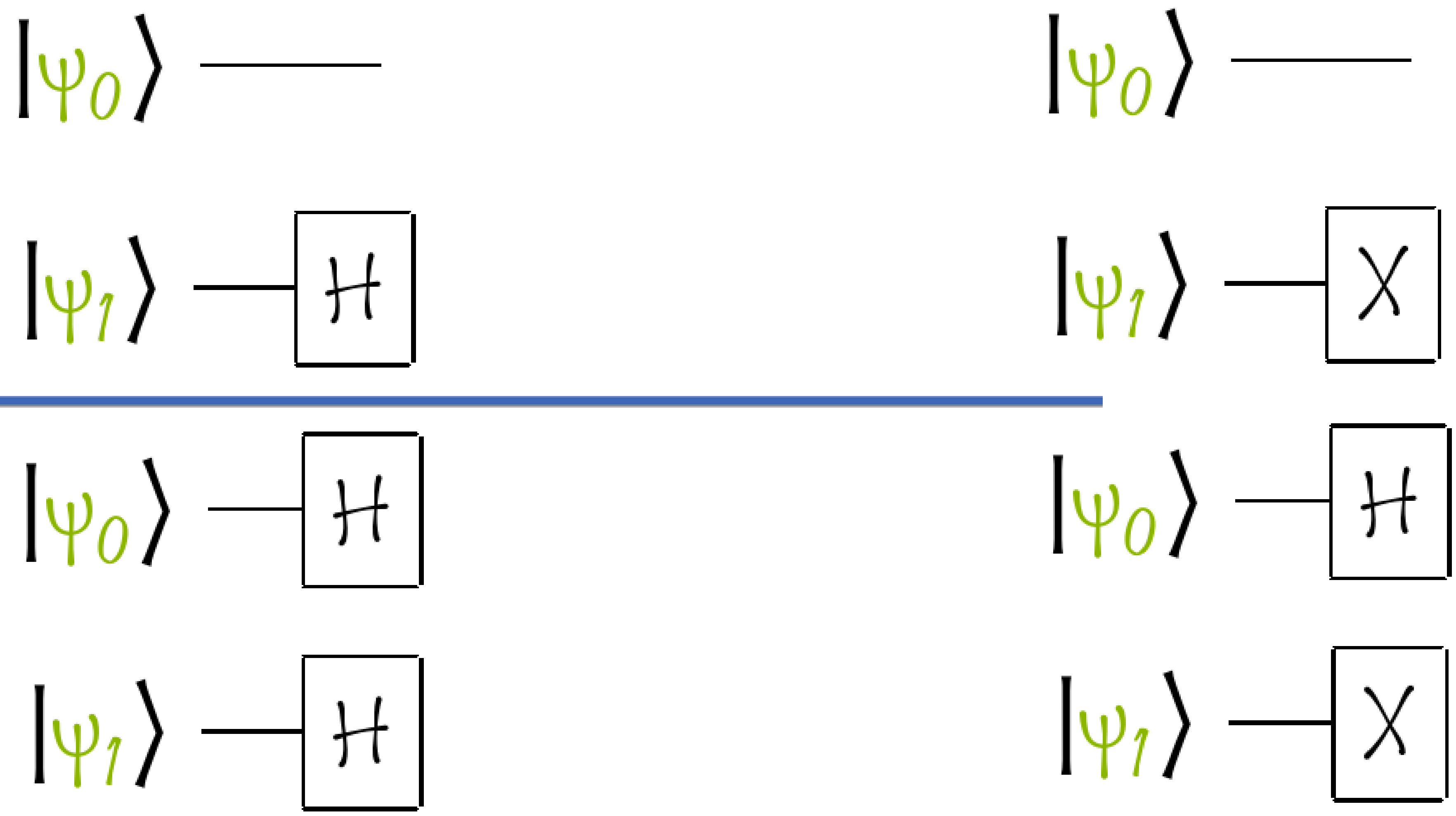


$$|\psi_1\psi_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- The answer is: Tensor Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)
- The desired matrix will therefore be the tensor product of all gates at the same level, starting from the top and proceeding downwards.

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & -1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}.$$

Combine Quantum Gates



Single Qubit Quantum Gates

X Gate
Bit–flip, Not

$$\boxed{X} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta|0\rangle + \alpha|1\rangle$$

Z Gate
Phase–flip

$$\boxed{Z} \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle$$

H Gate
Hadamard

$$\boxed{H} \equiv \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{\alpha+\beta|0\rangle + \alpha-\beta|1\rangle}{\sqrt{2}}$$

T Gate

$$\boxed{T} \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle + e^{i\pi/4}\beta|1\rangle$$

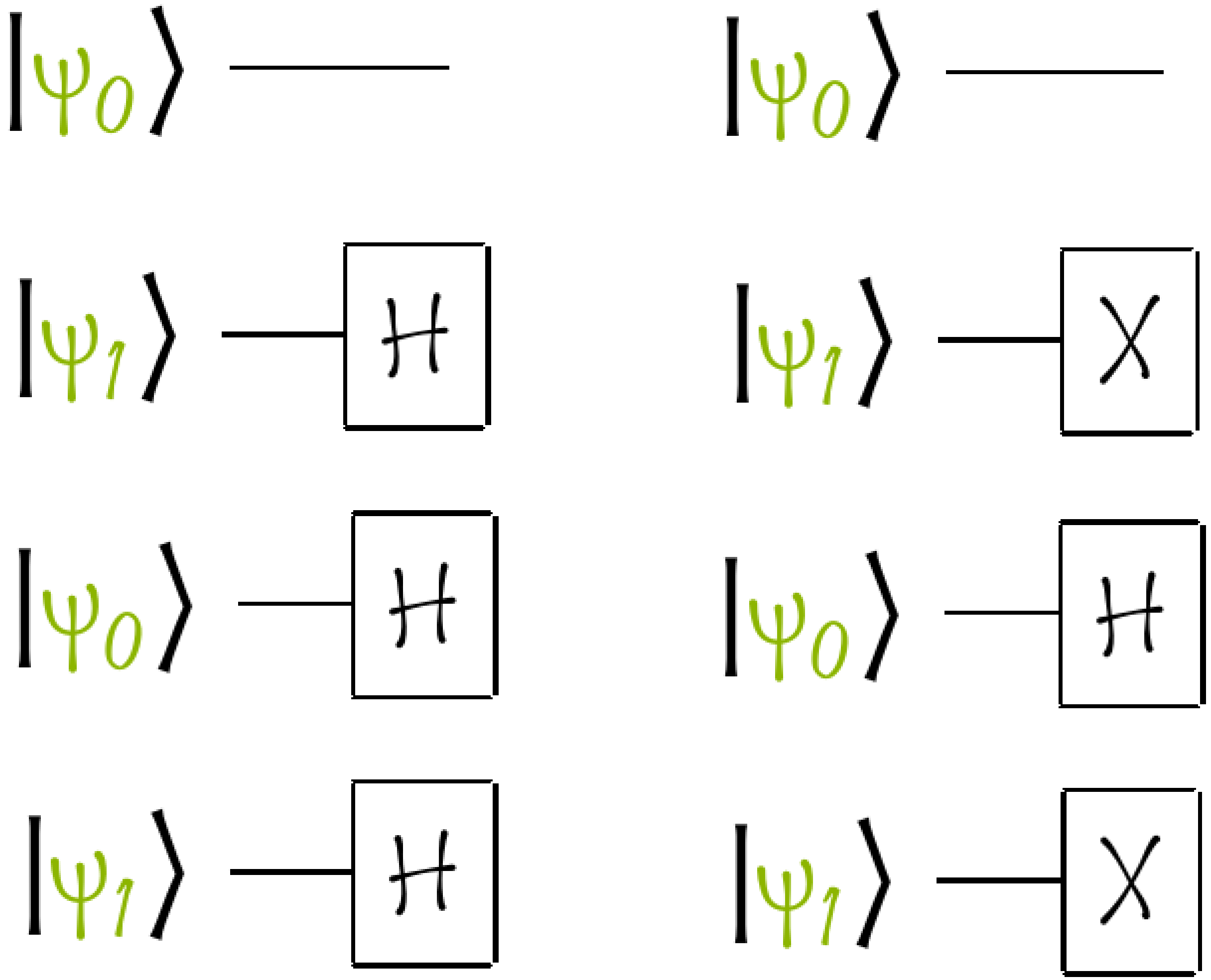
Single Qubit Quantum Gates

$$-\boxed{X}- \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

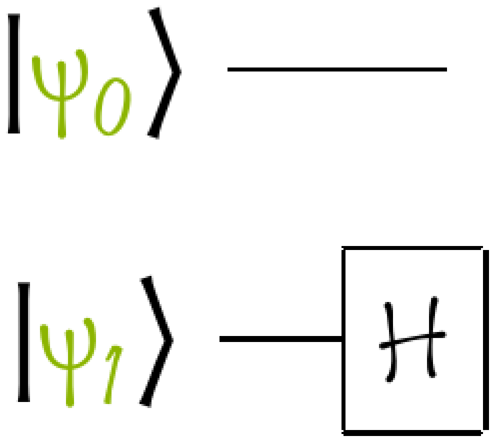
$$-\boxed{Z}- \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$-\boxed{H}- \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

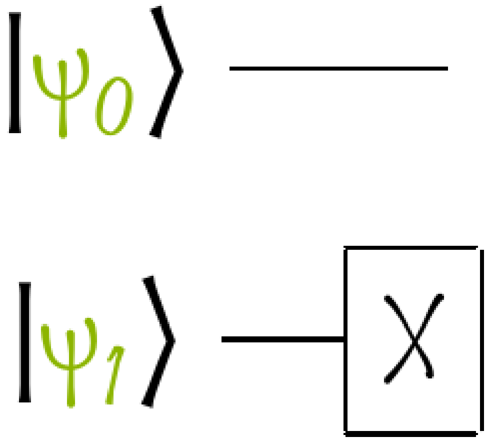
$$-\boxed{T}- \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$



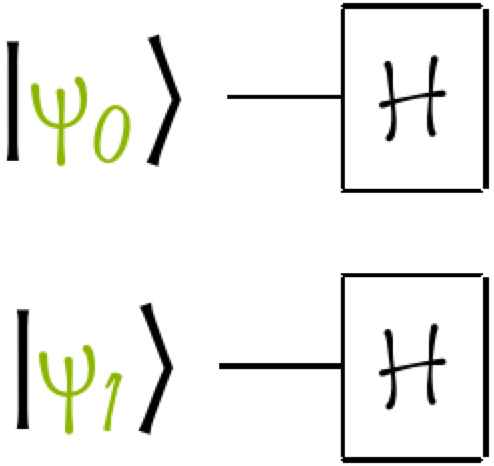
Combine Quantum Gates



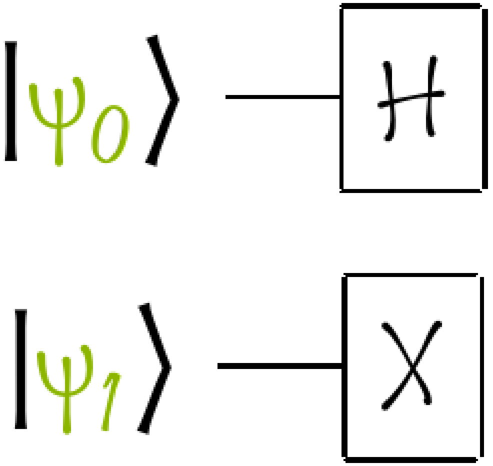
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



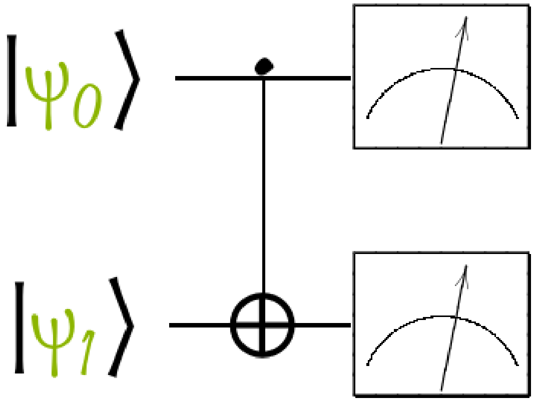
$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

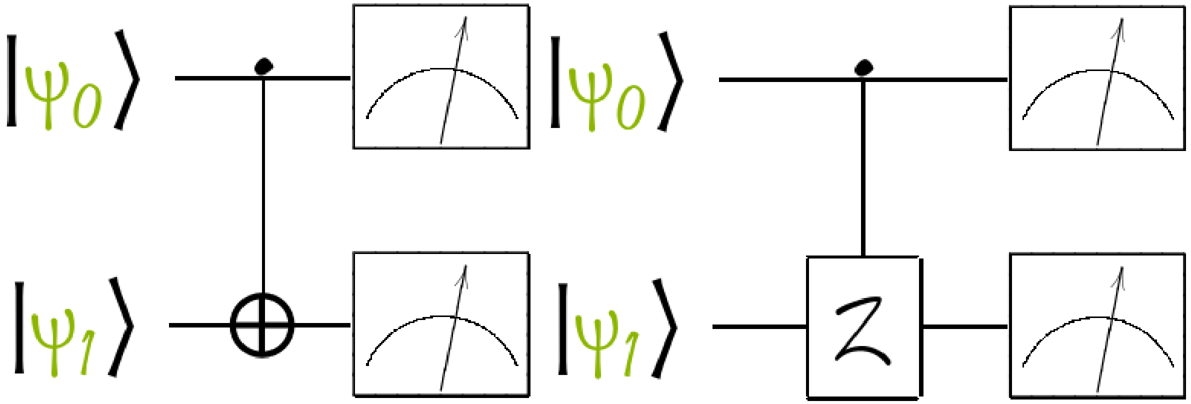
Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate



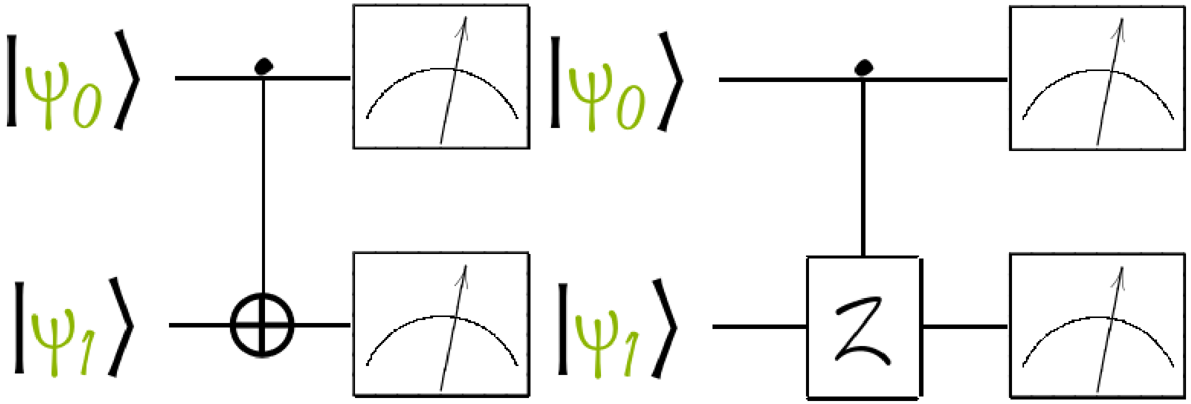
Alternative Controlled Gates

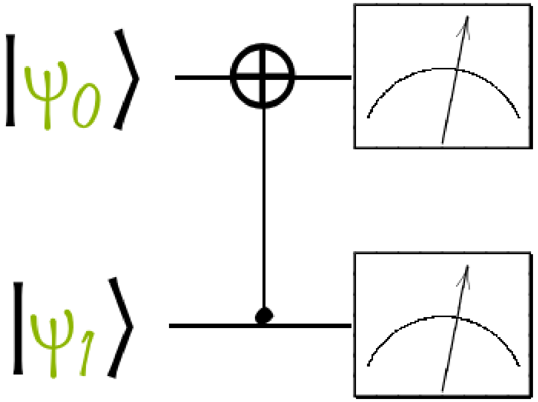
- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates



Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates
- In addition to the CNOT gate, we will see what form the reverse CNOT gate must have (i.e. with target and control qubits exchanged)

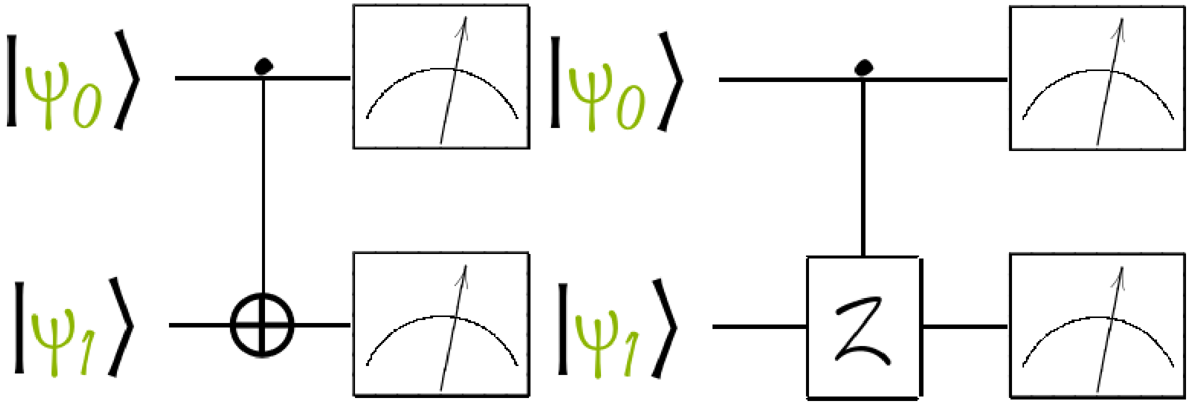


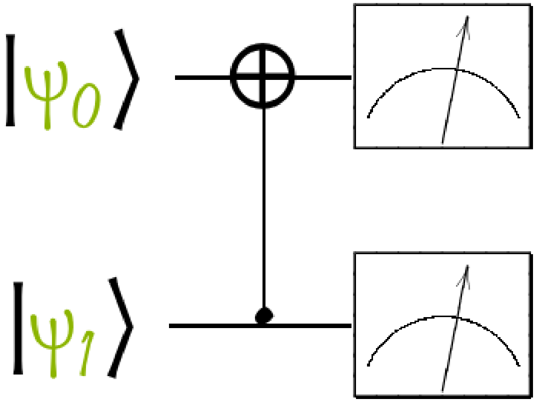


Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates
- In addition to the CNOT gate, we will see what form the reverse CNOT gate must have (i.e. with target and control qubits exchanged)
- This method is based on this alternative form for the CNOT gate

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$



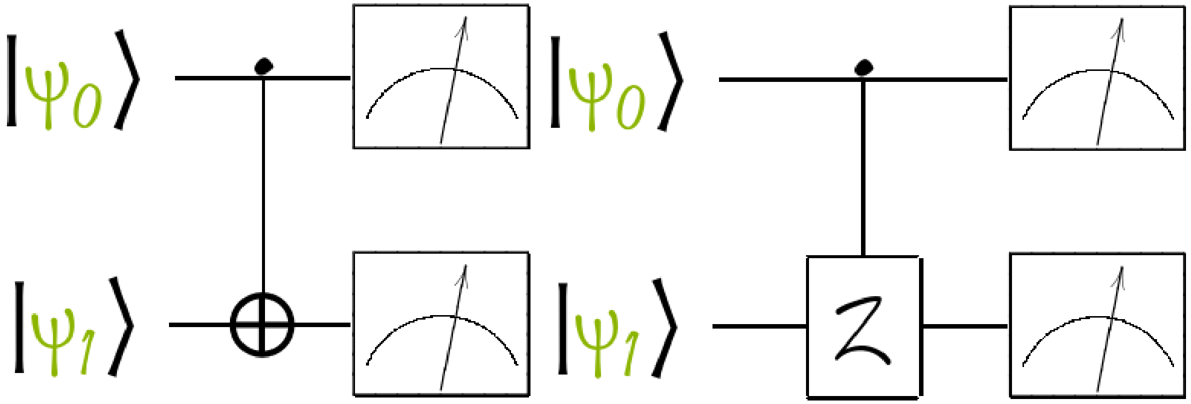


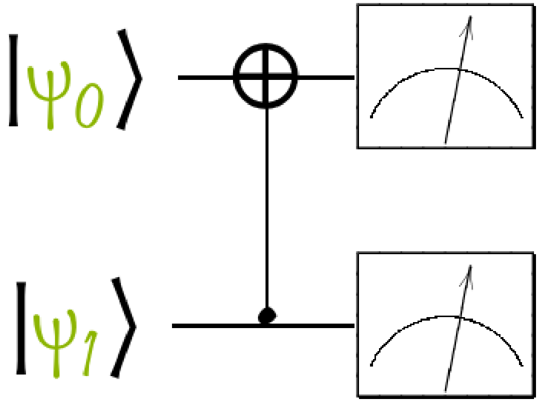
Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates
- In addition to the CNOT gate, we will see what form the reverse CNOT gate must have (i.e. with target and control qubits exchanged)
- This method is based on this alternative form for the CNOT gate

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

- That can be read as: "if the first qubit is 0, do nothing. If the first qubit is 1, apply X"





Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates
- In addition to the CNOT gate, we will see what form the reverse CNOT gate must have (i.e. with target and control qubits exchanged)
- This method is based on this alternative form for the CNOT gate

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

- That can be read as: "if the first qubit is 0, do nothing. If the first qubit is 1, apply X"

$$|0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1, 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0, 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates
- In addition to the CNOT gate, we will see what form the reverse CNOT gate must have (i.e. with target and control qubits exchanged)
- This method is based on this alternative form for the CNOT gate

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

- That can be read as: "if the first qubit is 0, do nothing. If the first qubit is 1, apply X"

$$|0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1,0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0,1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Alternative Controlled Gates

- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate
- In general, this method can be used to build generic controlled gates
- In addition to the CNOT gate, we will see what form the reverse CNOT gate must have (i.e. with target and control qubits exchanged)
- This method is based on this alternative form for the CNOT gate

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

- That can be read as: "if the first qubit is 0, do nothing. If the first qubit is 1, apply X"

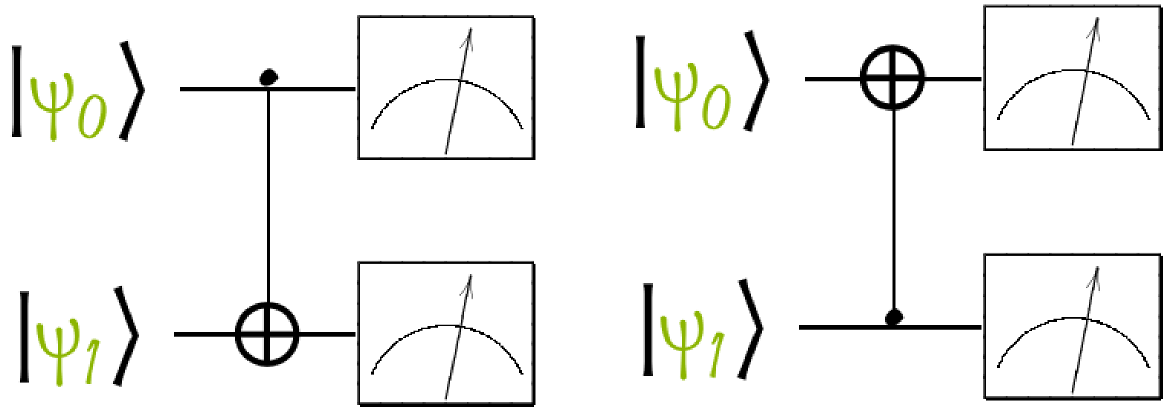
$$|0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1,0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0,1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

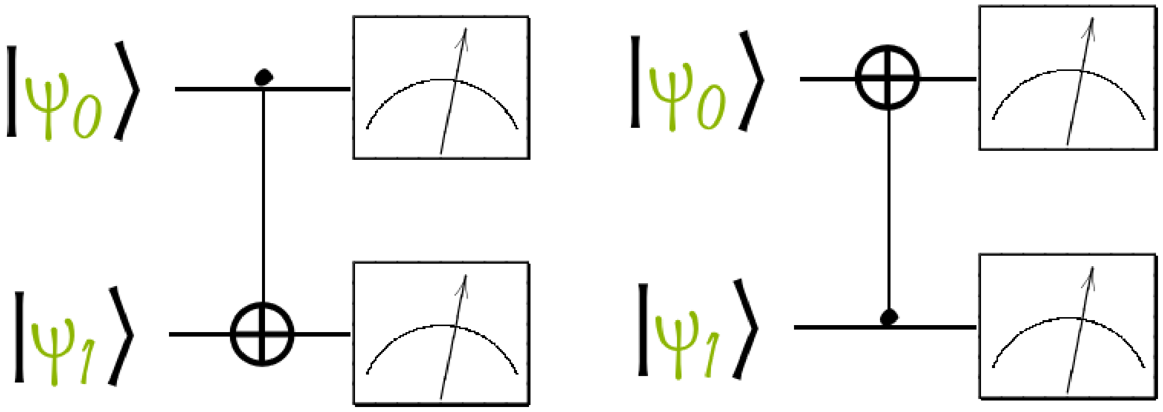
Alternative Controlled Gates

- Reverse CNOT gate



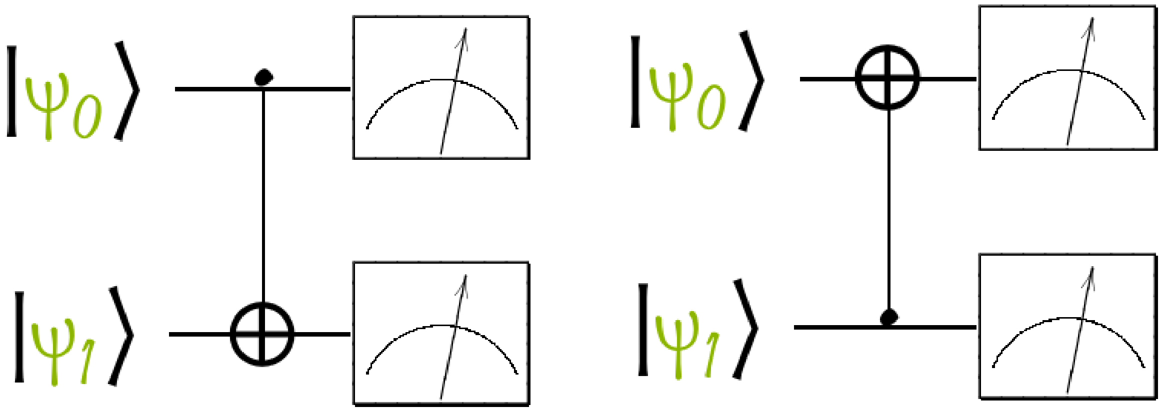
Alternative Controlled Gates

- Reverse CNOT gate
- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide



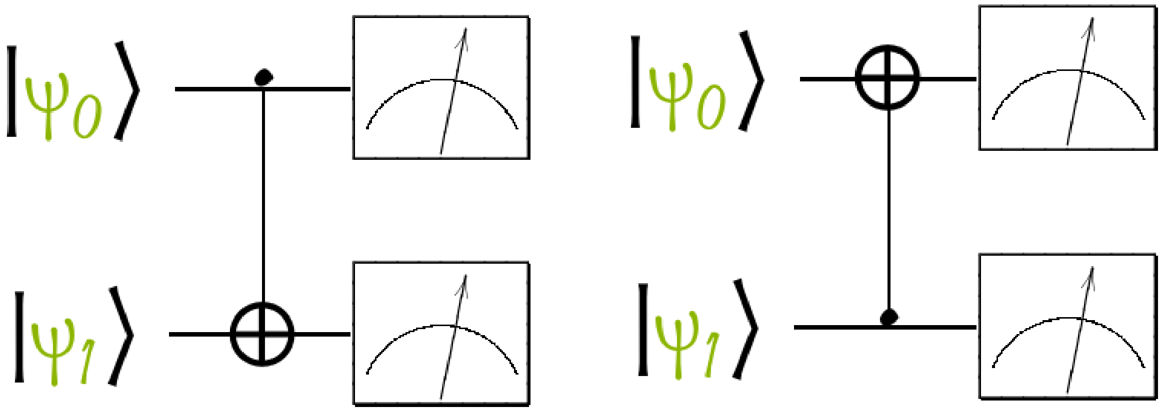
Alternative Controlled Gates

- Reverse CNOT gate
- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide
- The reverse CNOT reverses the roles of target qubit and control qubit from the original CNOT. Its description, therefore, is:



Alternative Controlled Gates

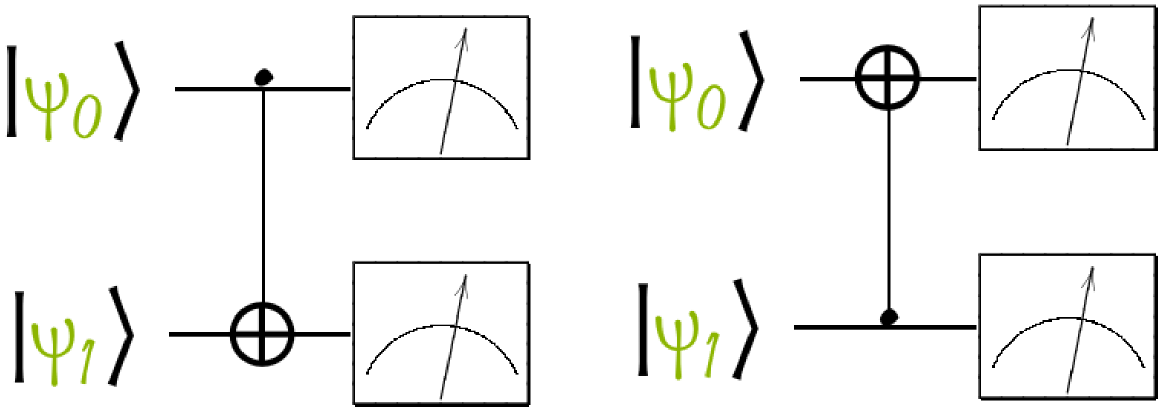
- Reverse CNOT gate
- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide
- The reverse CNOT reverses the roles of target qubit and control qubit from the original CNOT. Its description, therefore, is:
- If the second qubit is zero, do nothing. If the second qubit is one, apply the quantum gate X to the first qubit



Alternative Controlled Gates

- Reverse CNOT gate
- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide
- The reverse CNOT reverses the roles of target qubit and control qubit from the original CNOT. Its description, therefore, is:
- If the second qubit is zero, do nothing. If the second qubit is one, apply the quantum gate X to the first qubit

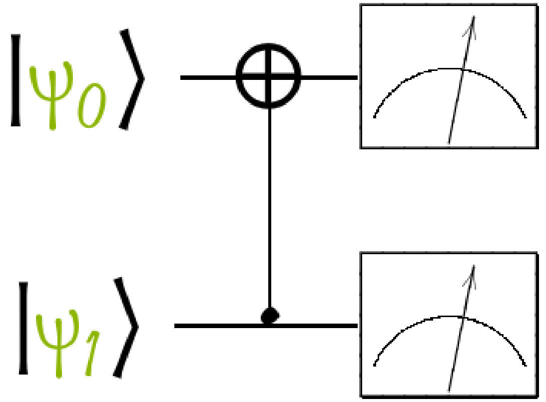
$$I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$$



Alternative Controlled Gates

- Reverse CNOT gate
- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide
- The reverse CNOT reverses the roles of target qubit and control qubit from the original CNOT. Its description, therefore, is:
- If the second qubit is zero, do nothing. If the second qubit is one, apply the quantum gate X to the first qubit

$$I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$$

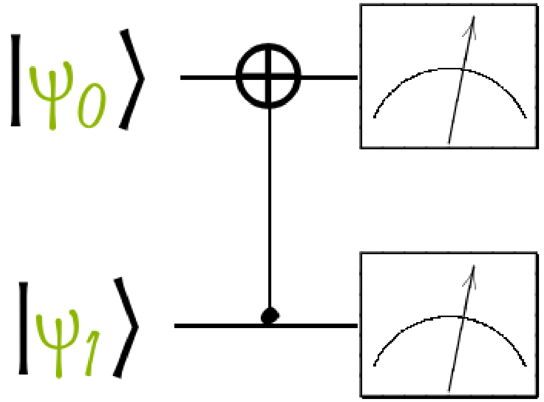


$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Alternative Controlled Gates

- Reverse CNOT gate
- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide
- The reverse CNOT reverses the roles of target qubit and control qubit from the original CNOT. Its description, therefore, is:
- If the second qubit is zero, do nothing. If the second qubit is one, apply the quantum gate X to the first qubit

$$I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$$

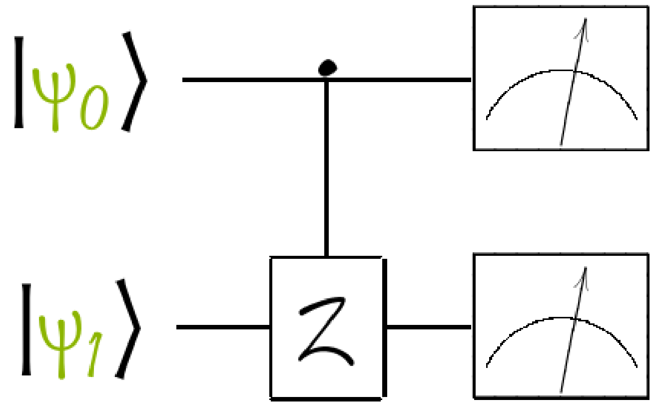


$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

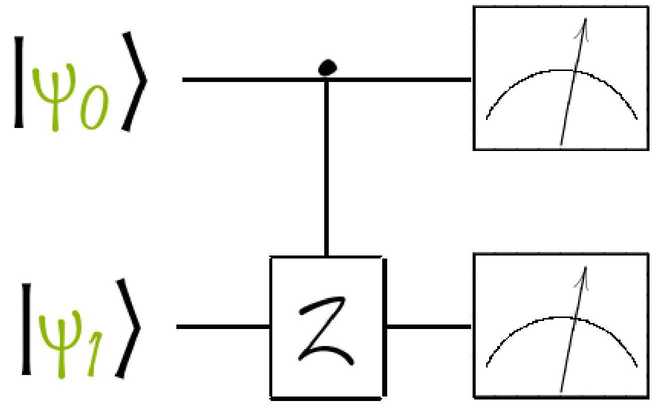
Alternative Controlled Gates

- Construction of generic controlled gates



Alternative Controlled Gates

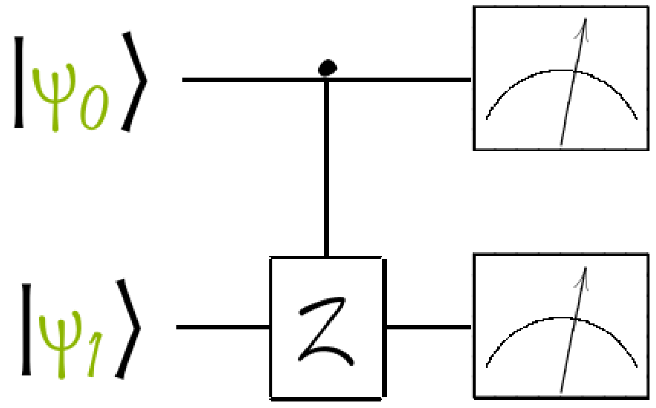
- Construction of generic controlled gates
- The idea is always the same: you think about the gate you want to use and it applies only when the conditions we want to impose are satisfied



Alternative Controlled Gates

- Construction of generic controlled gates
- The idea is always the same: you think about the gate you want to use and it applies only when the conditions we want to impose are satisfied
- Some examples are:

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

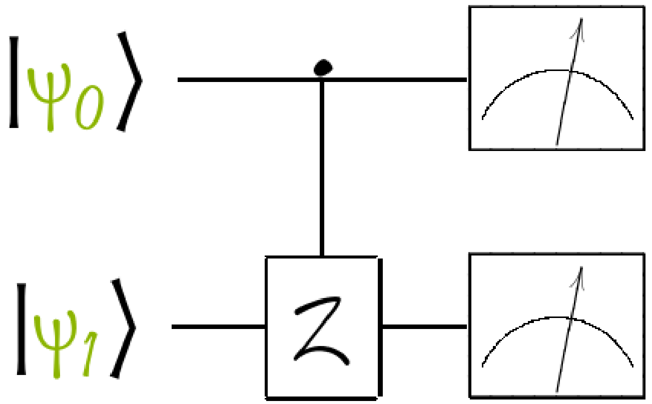


Alternative Controlled Gates

- Construction of generic controlled gates
- The idea is always the same: you think about the gate you want to use and it applies only when the conditions we want to impose are satisfied
- Some examples are:

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

$$I \otimes I \otimes |0\rangle\langle 0| \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I +$$
$$I \otimes I \otimes |1\rangle\langle 1| \otimes I \otimes I \otimes I \otimes I \otimes I \otimes X \otimes I$$



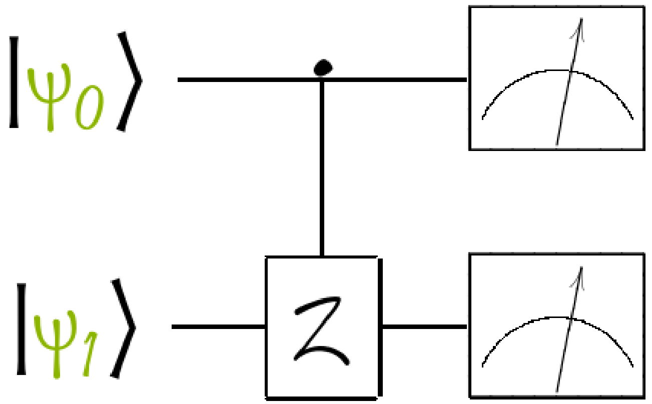
Alternative Controlled Gates

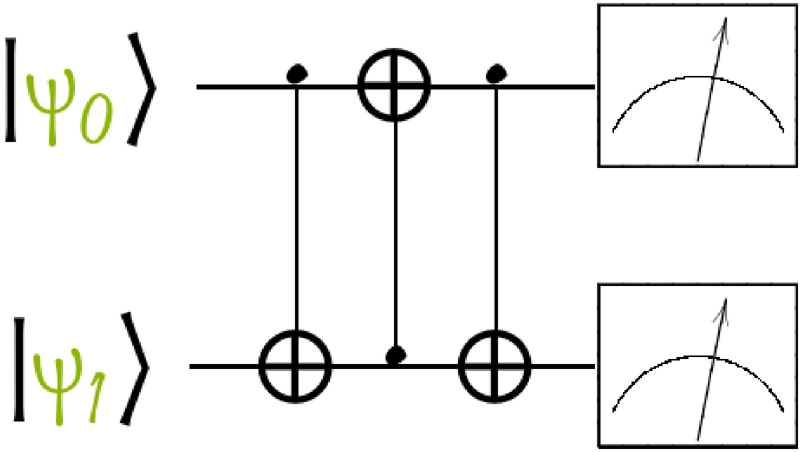
- Construction of generic controlled gates
- The idea is always the same: you think about the gate you want to use and it applies only when the conditions we want to impose are satisfied
- Some examples are:

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

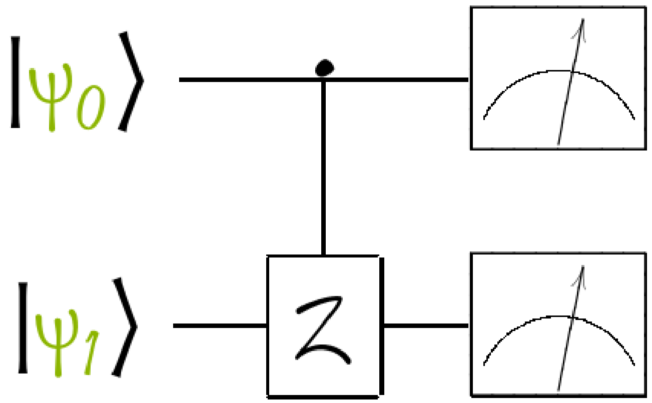
$$I \otimes I \otimes |0\rangle\langle 0| \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I +$$
$$I \otimes I \otimes |1\rangle\langle 1| \otimes I \otimes I \otimes I \otimes I \otimes I \otimes X \otimes I$$

- Exercise: find the gates



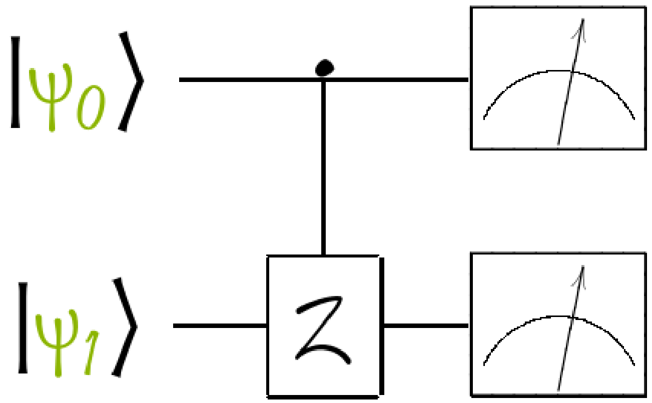


Alternative Controlled Gates



$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

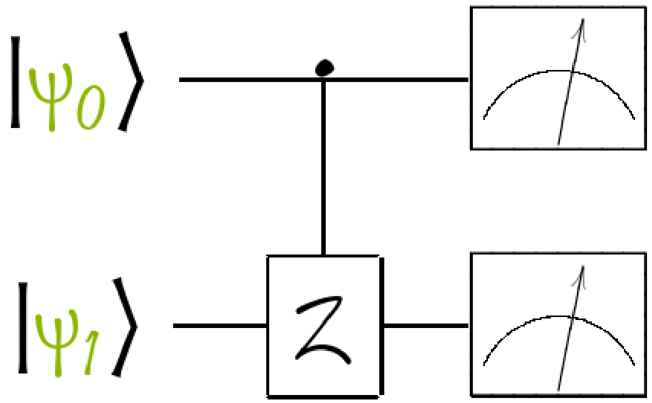
Alternative Controlled Gates



$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Alternative Controlled Gates

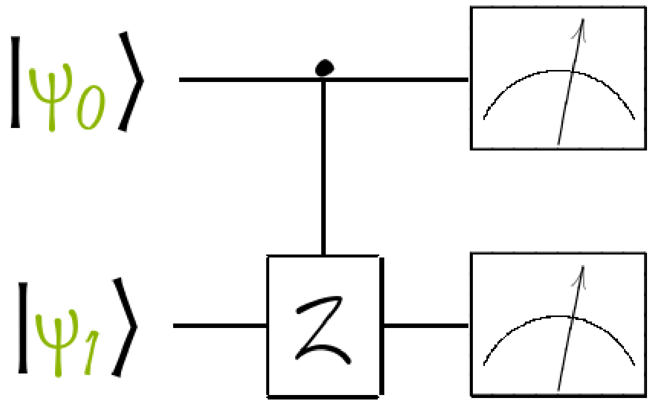


$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

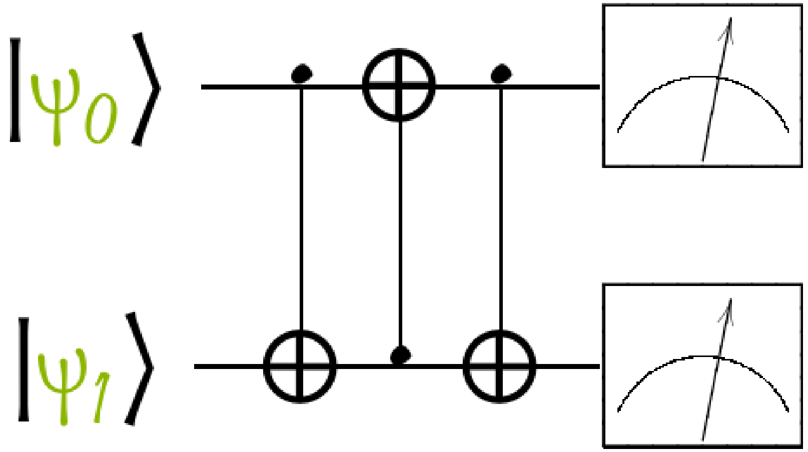
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

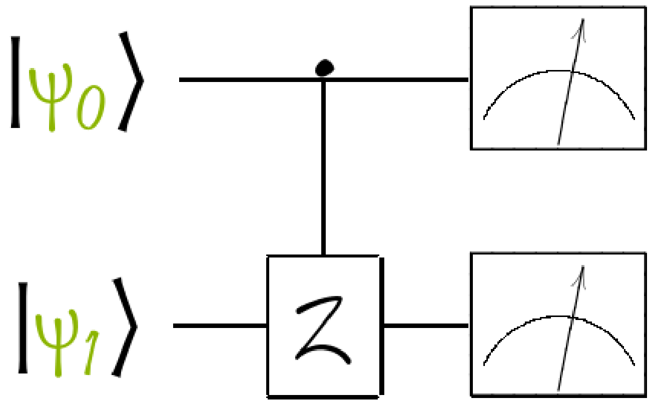
Alternative Controlled Gates



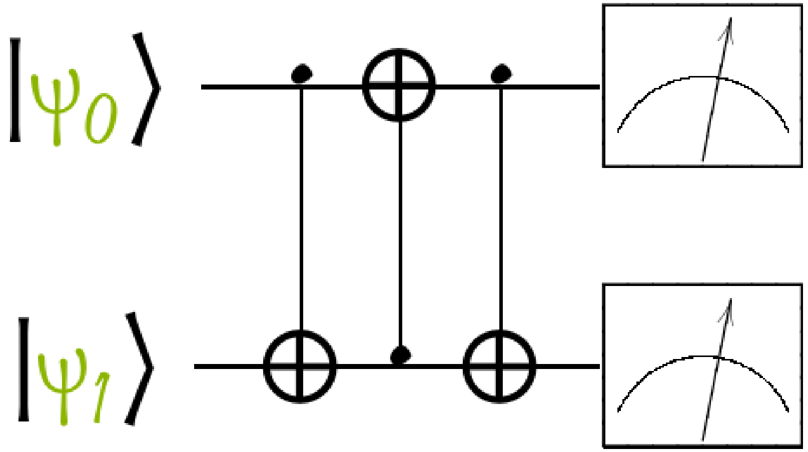
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



Alternative Controlled Gates

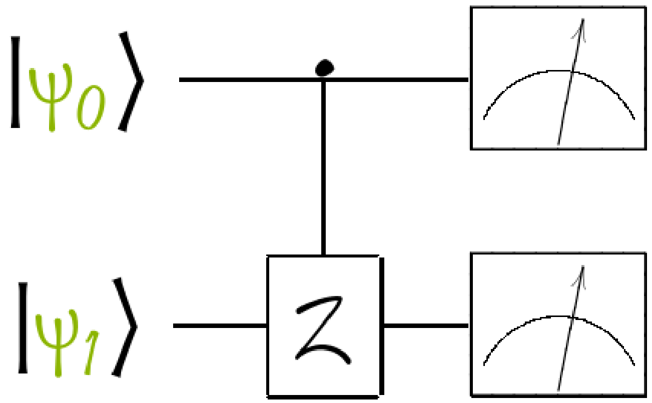


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

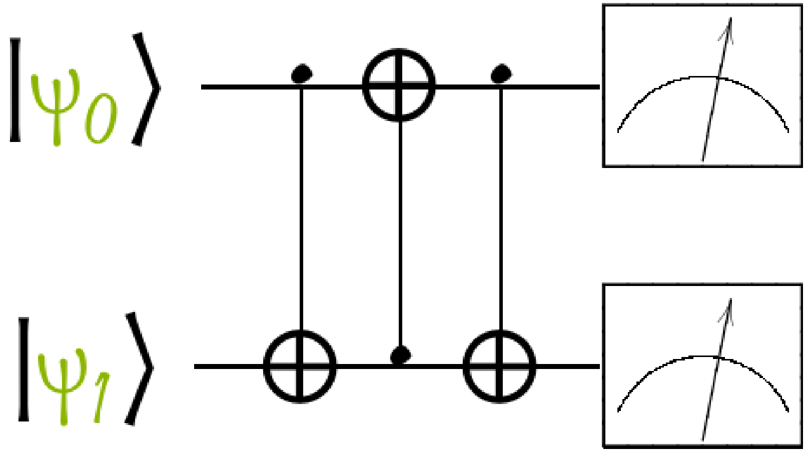


$$CNOT_1 \rightarrow RCNOT_1 \rightarrow CNOT_2$$

Alternative Controlled Gates



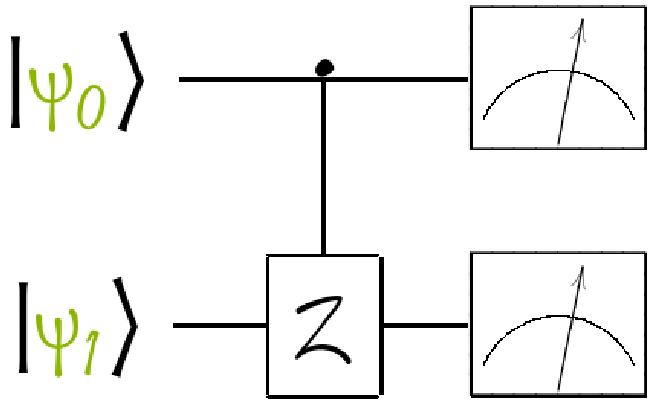
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



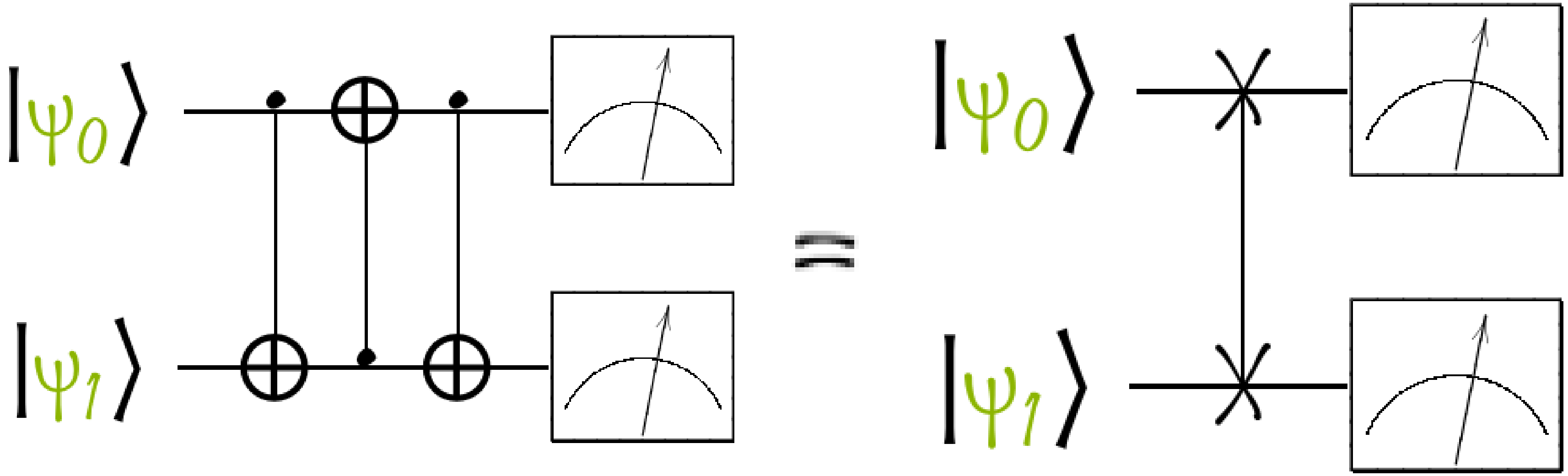
$$CNOT_1 \rightarrow RCNOT_1 \rightarrow CNOT_2$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

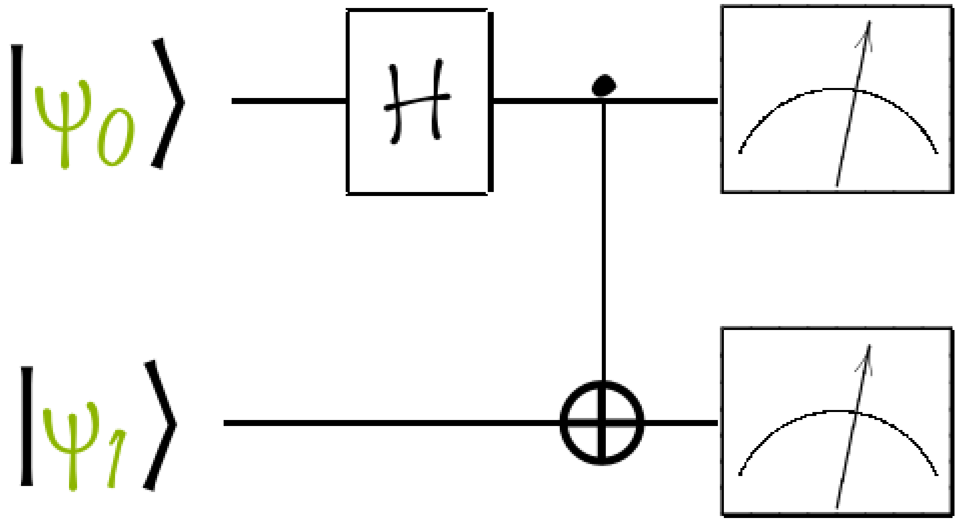
Alternative Controlled Gates



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



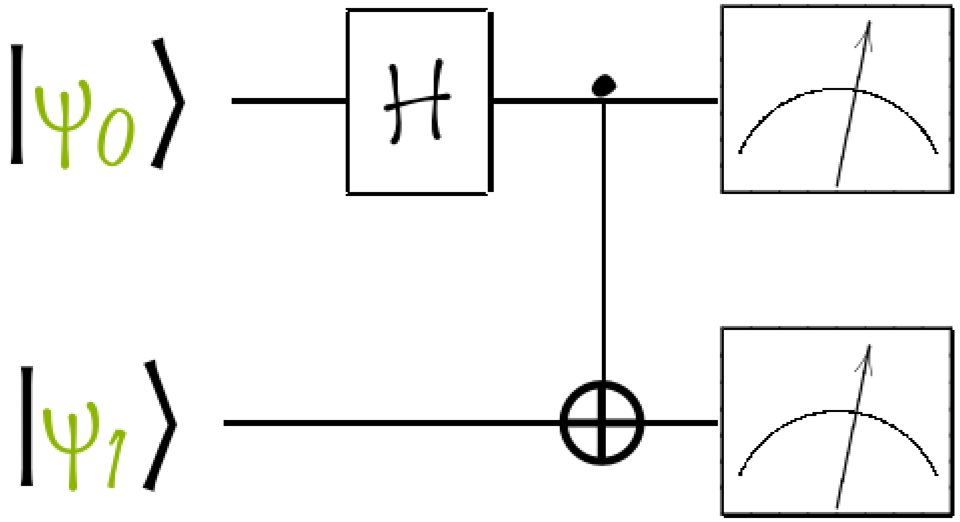
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

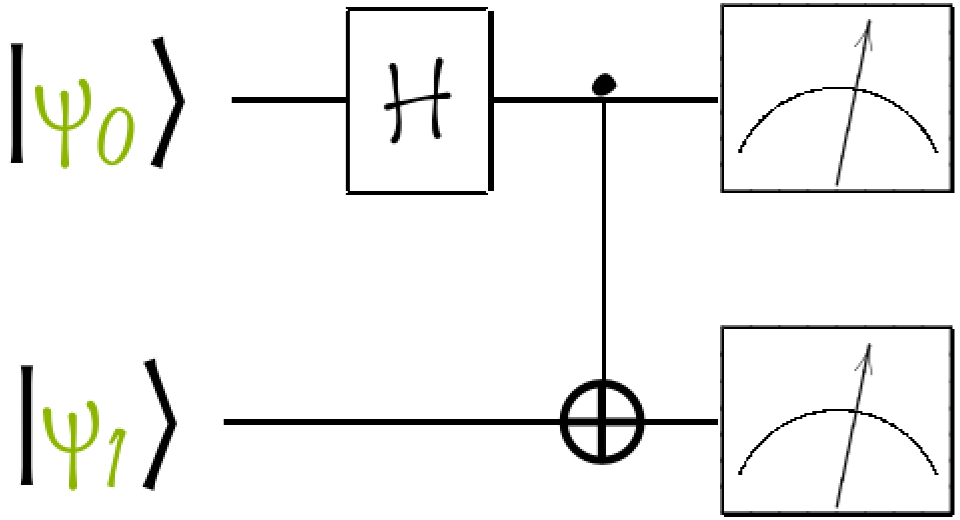
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

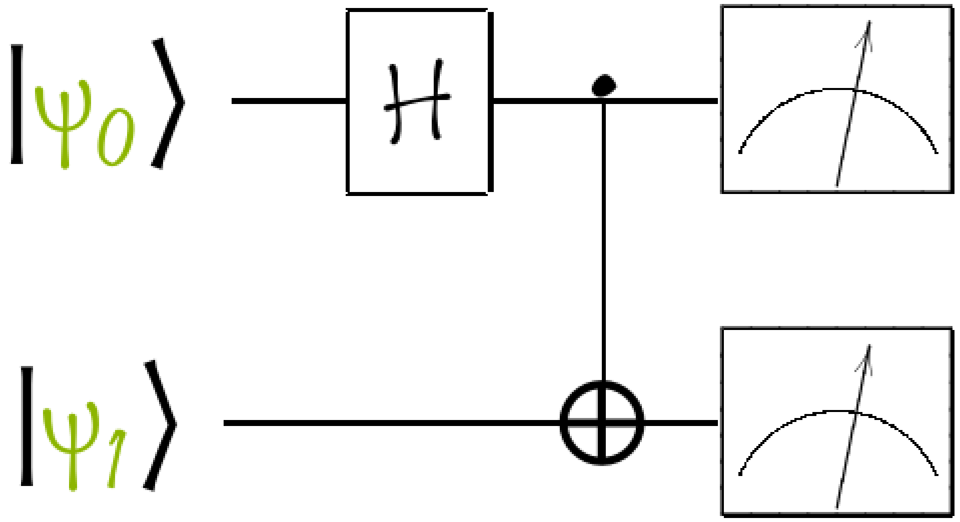
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

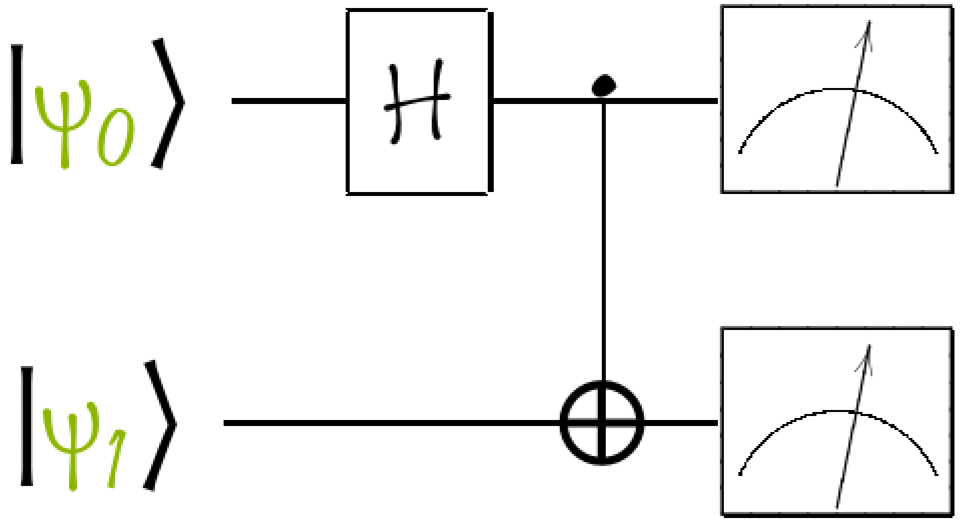
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

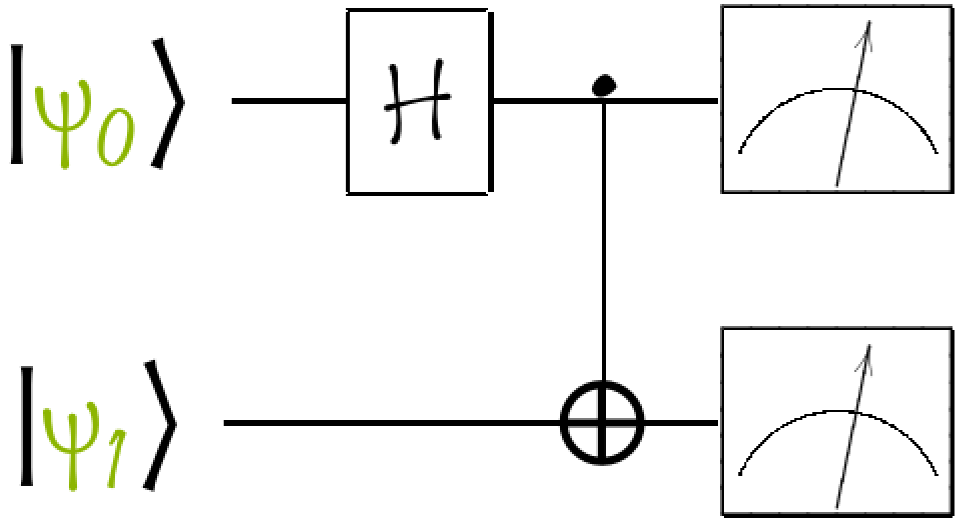
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

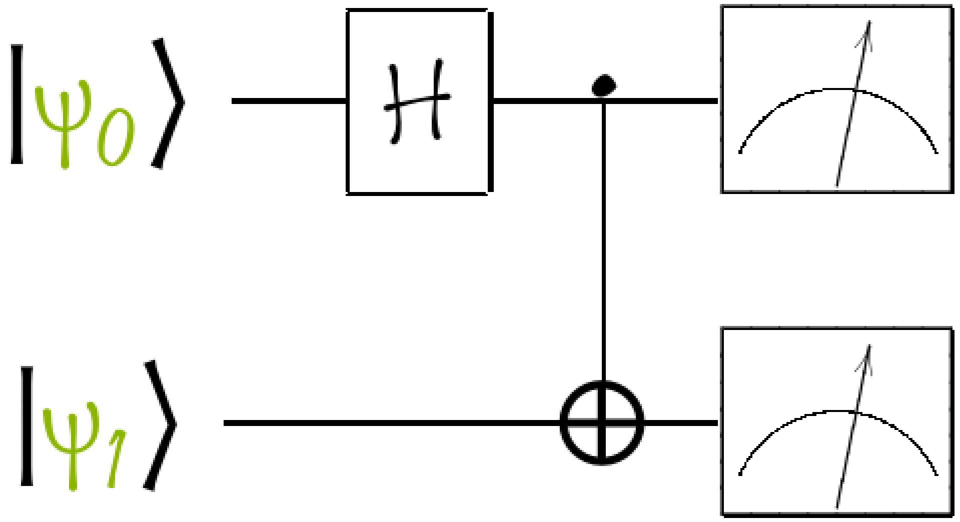
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

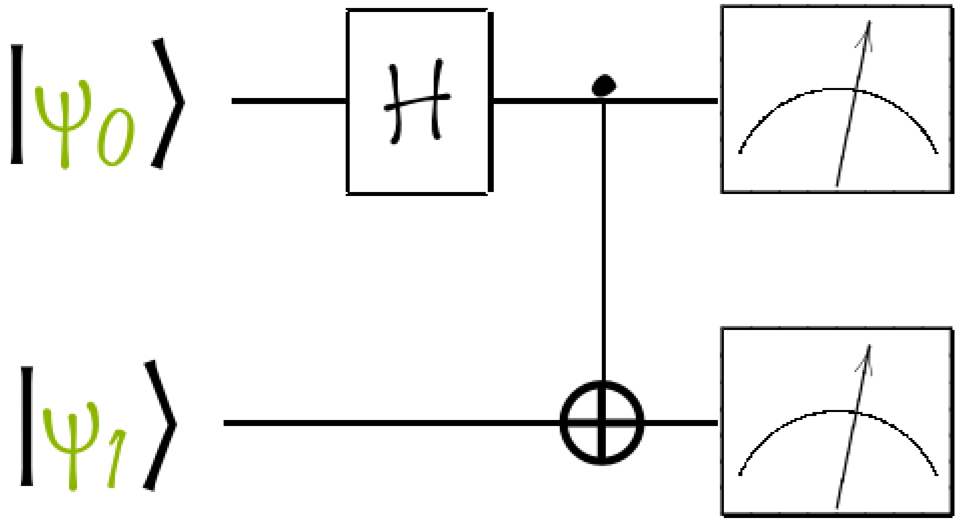
First Circuit example



$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

First Circuit example

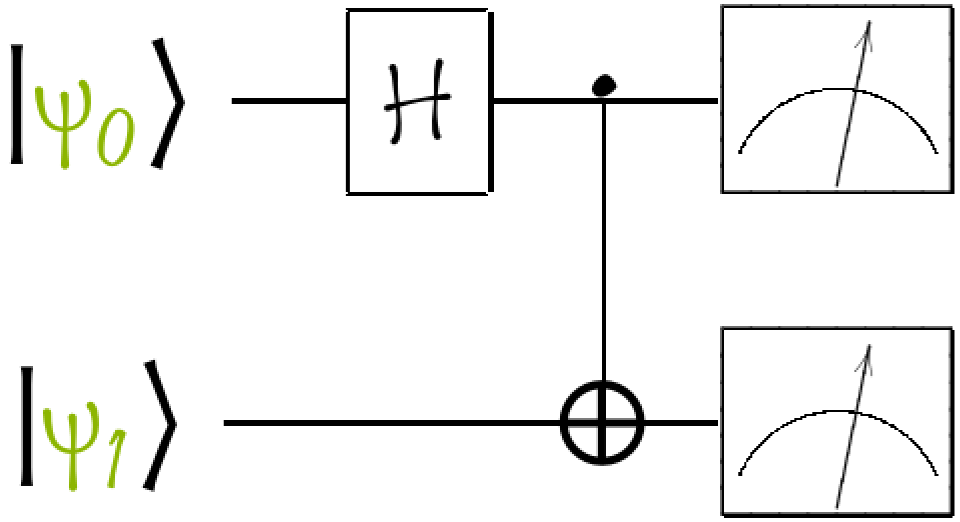


$$\begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

$\vert\psi_1\psi_2\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert 0\rangle$	50%	0%
$\vert 1\rangle$	0%	50%

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

First Circuit example

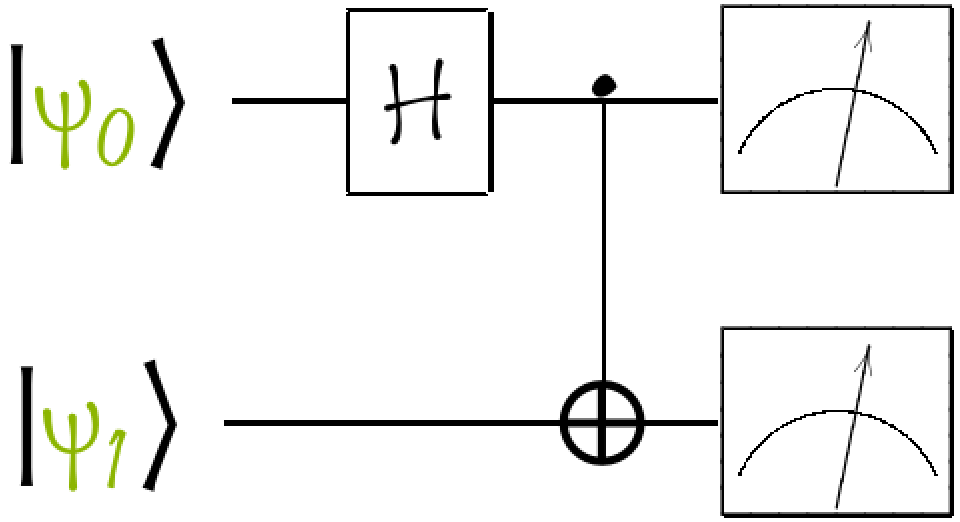


$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$\vert\psi_1\psi_2\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert 0\rangle$	50%	0%
$\vert 1\rangle$	0%	50%

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

First Circuit example



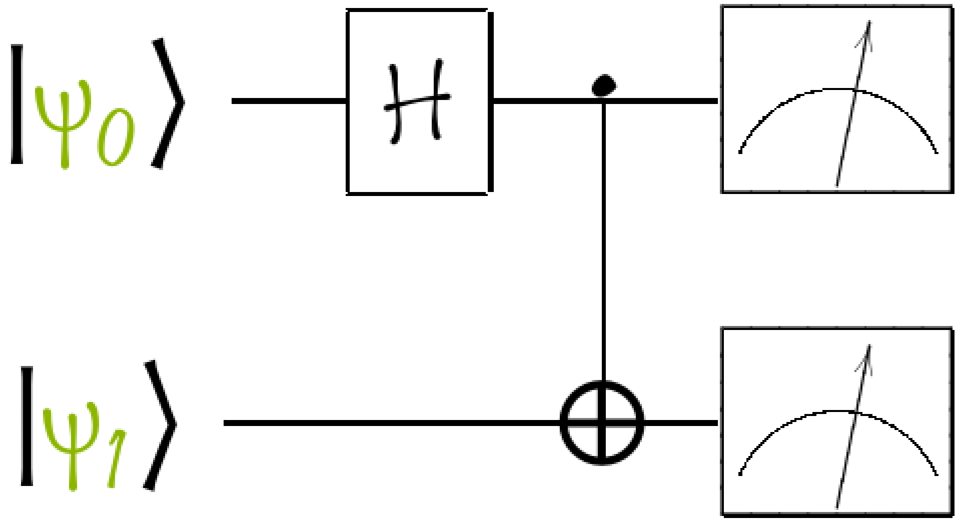
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$\vert\psi_1\psi_2\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert 0\rangle$	50%	0%
$\vert 1\rangle$	0%	50%

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = \frac{1}{\sqrt{2}}\cdot|00\rangle + 0\cdot|01\rangle + 0\cdot|10\rangle + \frac{1}{\sqrt{2}}\cdot|11\rangle$$

First Circuit example



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

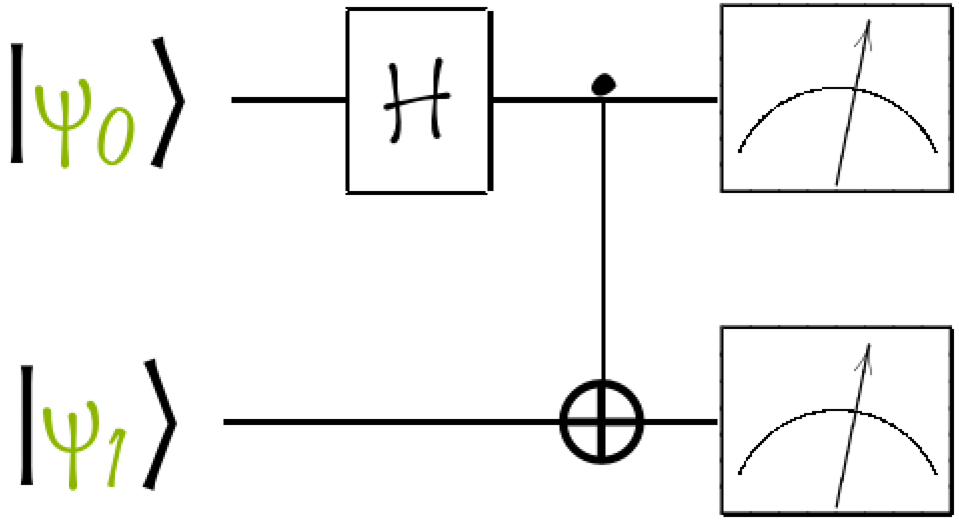
$\lvert\psi_1\psi_2\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 0\rangle$	50%	0%
$\lvert 1\rangle$	0%	50%

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi_1\psi_2\rangle = \frac{1}{\sqrt{2}}\cdot|00\rangle + 0\cdot|01\rangle + 0\cdot|10\rangle + \frac{1}{\sqrt{2}}\cdot|11\rangle$$

No Way!

First Circuit example



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$\|\psi_1\psi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	50%	0%
$\|1\rangle$	0%	50%

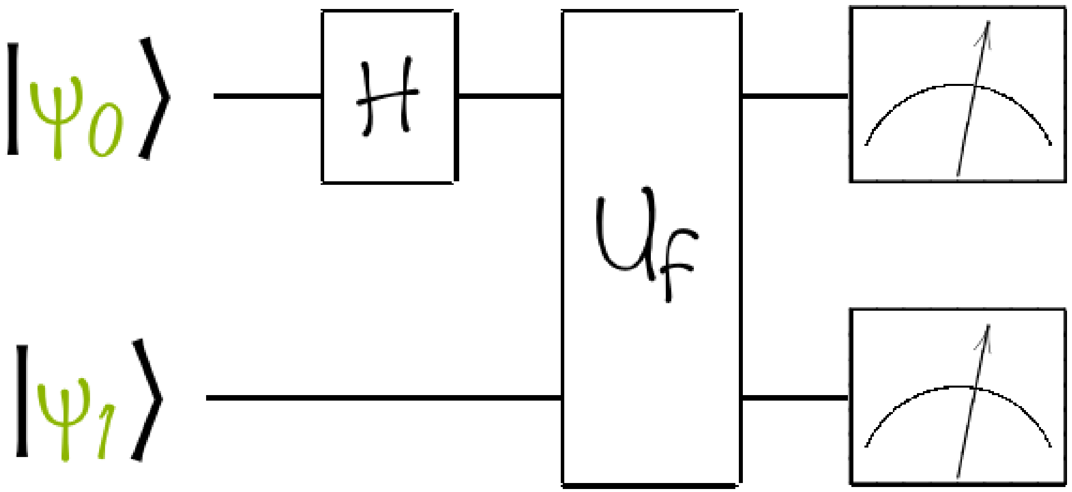
$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots$$

$$+ \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

$$\Downarrow \qquad \cancel{\Uparrow}$$

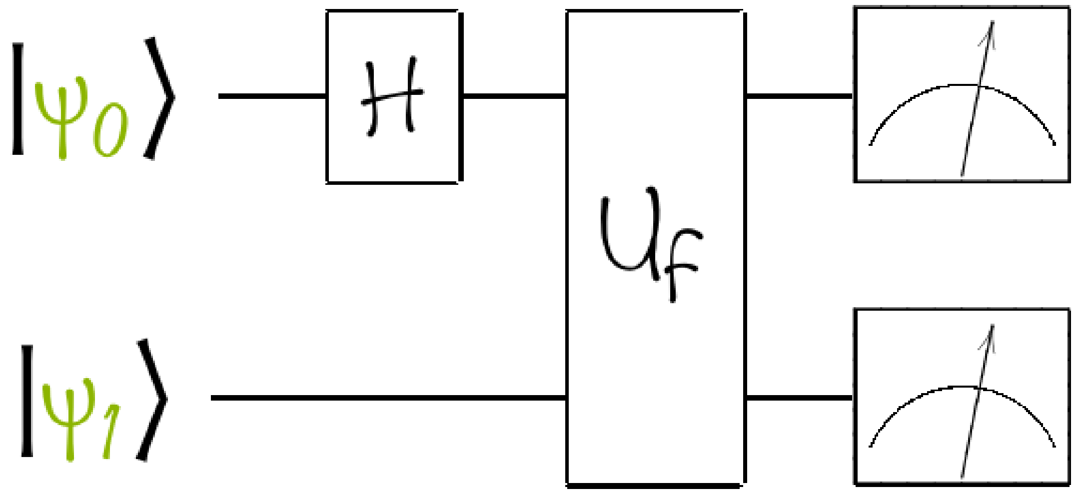
$$|\psi_1\psi_2 \dots \psi_N\rangle = \gamma_1|00 \dots 0\rangle + \gamma_2|00 \dots 01\rangle + \dots + \gamma_{N-1}|11 \dots 10\rangle + \gamma_N|11 \dots 1\rangle$$

Quantum Parallelism



- Let's now consider a circuit like the one in the figure

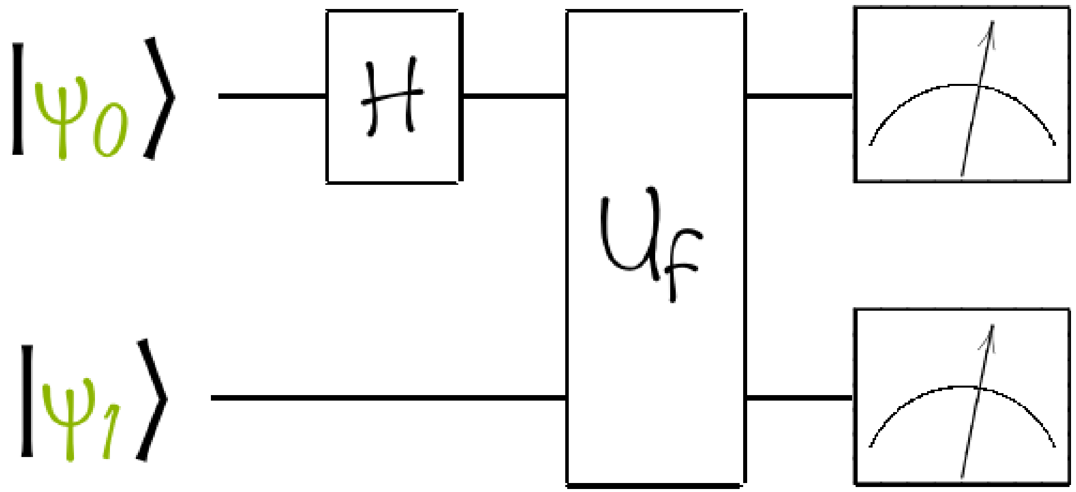
Quantum Parallelism



- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

$$f: \{0, 1\} \rightarrow \{0, 1\}$$

Quantum Parallelism



- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

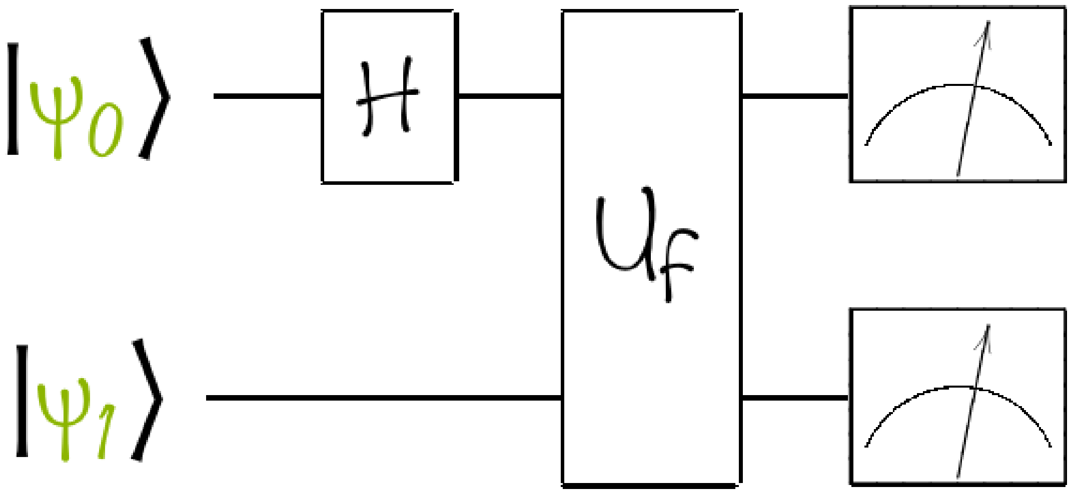
$$f : \{0, 1\} \rightarrow \{0, 1\}$$

- This particular two-qubits gate works as follows

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

Quantum Parallelism



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

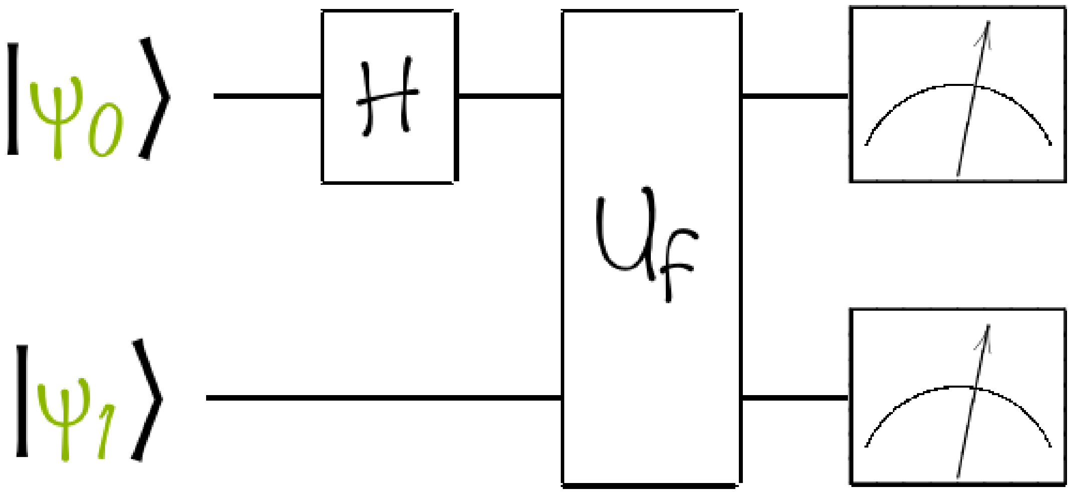
$$f : \{0, 1\} \rightarrow \{0, 1\}$$

- This particular two-qubits gate works as follows

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f\left(|\psi_1\rangle\right)$$

Quantum Parallelism



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\begin{bmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{bmatrix}$$

- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

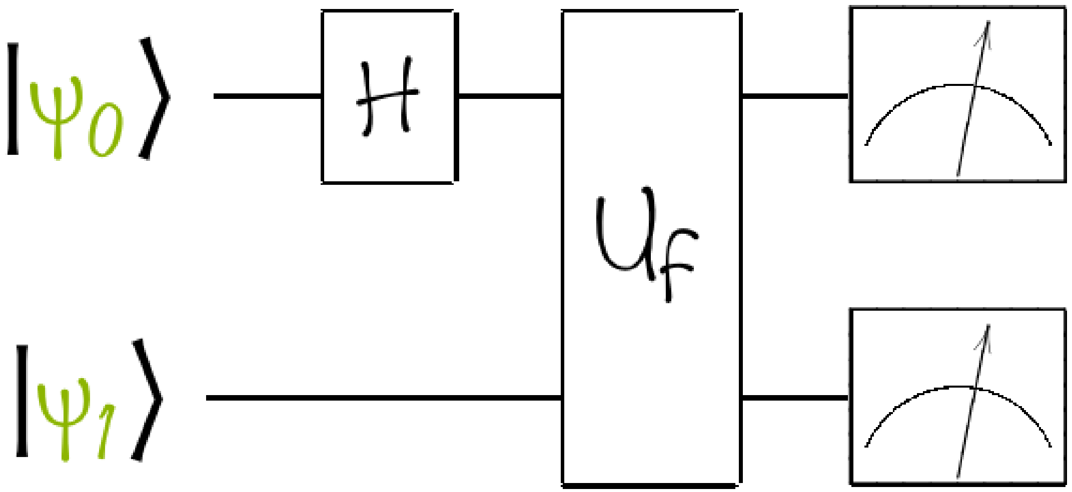
$$f:\{0,1\}\rightarrow\{0,1\}$$

- This particular two-qubits gate works as follows

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

Quantum Parallelism



- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

$$f: \{0, 1\} \rightarrow \{0, 1\}$$

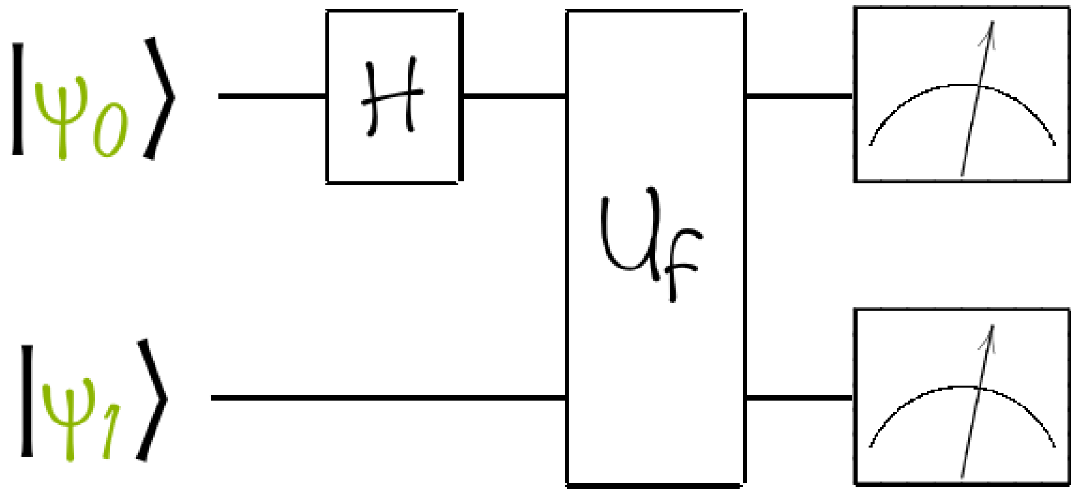
- This particular two-qubits gate works as follows

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\begin{bmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{bmatrix} \rightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

Quantum Parallelism



$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle \rightarrow$$

$$\frac{1}{\sqrt{2}}|0, 0 \oplus f(0)\rangle + \frac{1}{\sqrt{2}}|1, 0 \oplus f(1)\rangle$$

- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

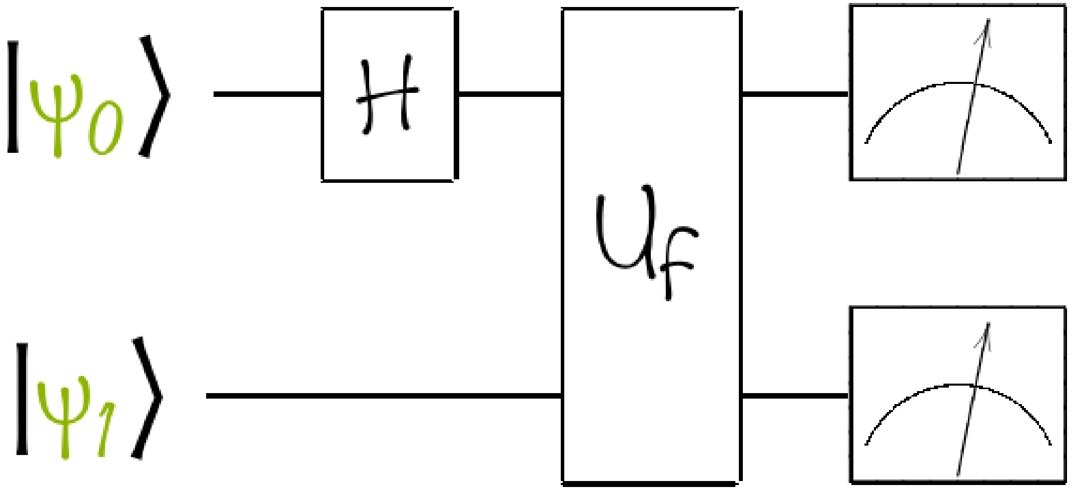
$$f: \{0, 1\} \rightarrow \{0, 1\}$$

- This particular two-qubits gate works as follows

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

Quantum Parallelism



$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle \rightarrow$$

$$\frac{1}{\sqrt{2}}|0, f(0)\rangle + \frac{1}{\sqrt{2}}|1, f(1)\rangle$$

- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

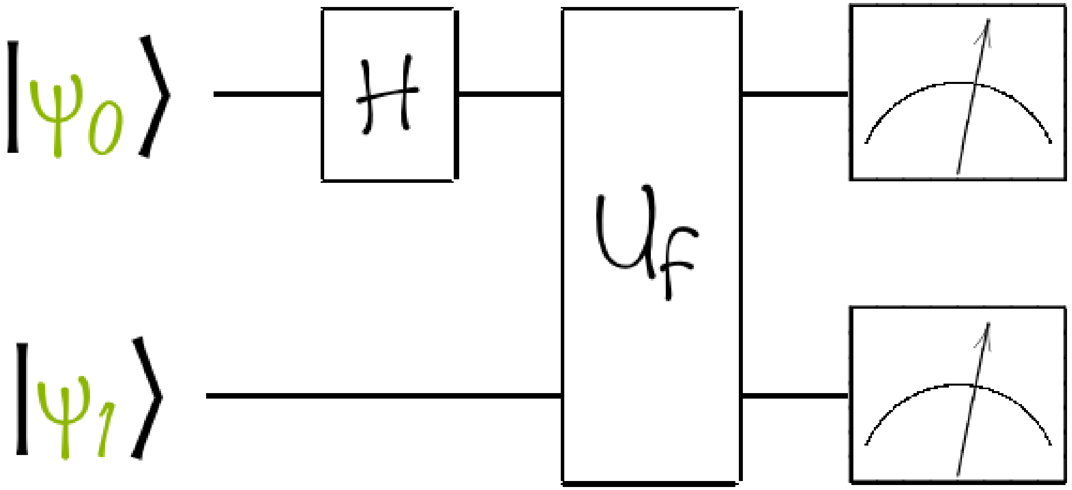
$$f : \{0, 1\} \rightarrow \{0, 1\}$$

- This particular two-qubits gate works as follows

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

Quantum Parallelism



$$|\psi_0\psi_1\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called U_f, is a gate associated with a certain function f

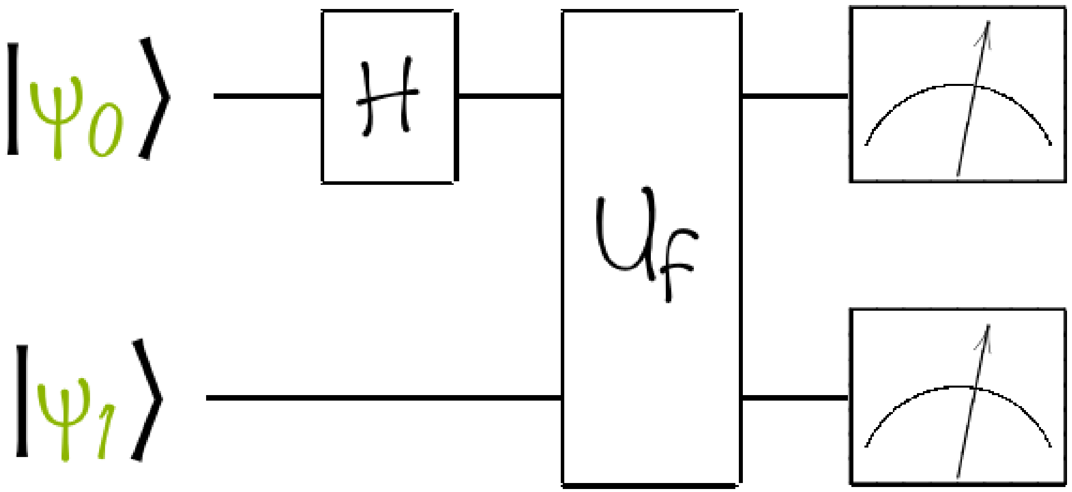
$$f : \{0, 1\} \rightarrow \{0, 1\}$$

- This particular two-qubits gate works as follows

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

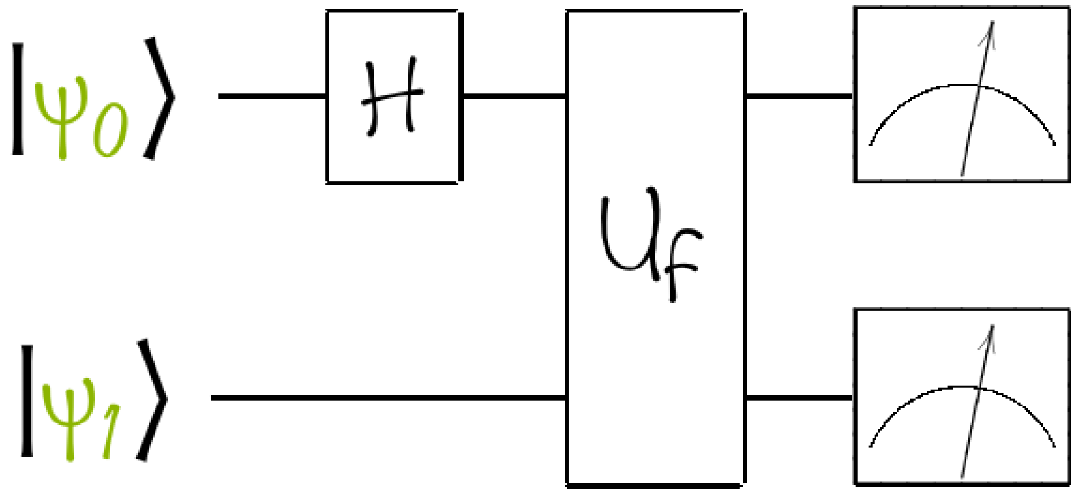
Quantum Parallelism



$$|\psi_0\psi_1\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

- Let us pause for a moment on the quantum state we have obtained

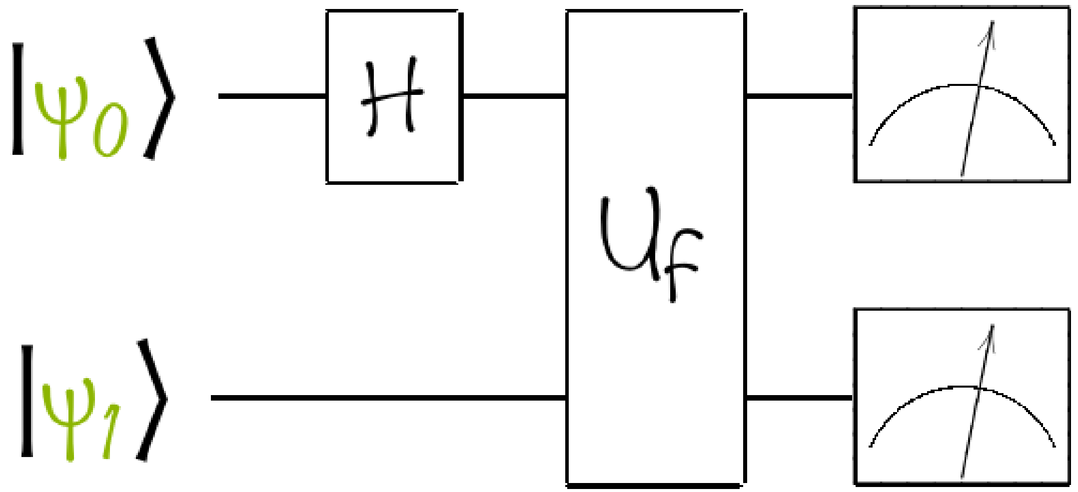
Quantum Parallelism



$$|\psi_0\psi_1\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

- Let us pause for a moment on the quantum state we have obtained
- It would appear that I have managed to create a superposition state that contains enough information to fully describe the function f

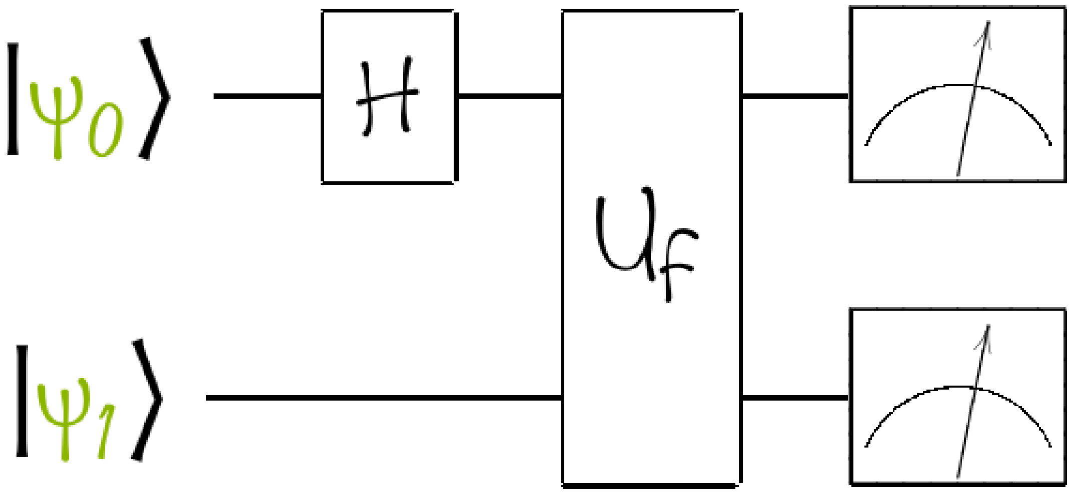
Quantum Parallelism



$$|\psi_0\psi_1\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

- Let us pause for a moment on the quantum state we have obtained
- It would appear that I have managed to create a superposition state that contains enough information to fully describe the function f
- This phenomenon is known by the name of quantum parallelism: unlike classical parallelism, which uses many cores, a quantum circuit is able to evaluate a function in several points at the same time

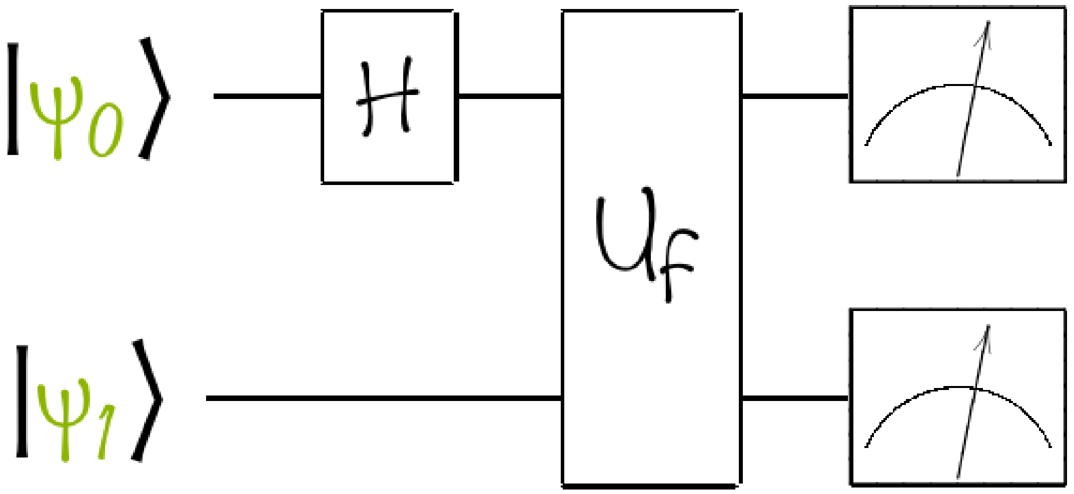
Quantum Parallelism



$$|\psi_0\psi_1\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

- Let us pause for a moment on the quantum state we have obtained
- It would appear that I have managed to create a superposition state that contains enough information to fully describe the function f
- This phenomenon is known by the name of quantum parallelism: unlike classical parallelism, which uses many cores, a quantum circuit is able to evaluate a function in several points at the same time
- But we must be very careful: quantum parallelism is a very powerful weapon, but it must be exploited with intelligence.

Quantum Parallelism



$$|\psi_0\psi_1\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

- Let us pause for a moment on the quantum state we have obtained
- It would appear that I have managed to create a superposition state that contains enough information to fully describe the function f
- This phenomenon is known by the name of quantum parallelism: unlike classical parallelism, which uses many cores, a quantum circuit is able to evaluate a function in several points at the same time
- But we must be very careful: quantum parallelism is a very powerful weapon, but it must be exploited with intelligence.
- In our case, for example, the state we have obtained is completely useless. Why?

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages
- The problem that the algorithm solves is the following:

$$f: \{0, 1\} \to \{0, 1\}$$

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages
- The problem that the algorithm solves is the following:

$$f: \{0, 1\} \rightarrow \{0, 1\}$$

we define f balanced if:

$$f(0) = 0, \quad f(1) = 1$$

$$f(0) = 1, \quad f(1) = 0$$

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages
- The problem that the algorithm solves is the following:

$$f : \{0, 1\} \rightarrow \{0, 1\}$$

we define f balanced if:

$$f(0) = 0, \quad f(1) = 1$$
$$f(0) = 1, \quad f(1) = 0$$

we define f constant if:

$$f(0) = 1, \quad f(1) = 1$$
$$f(0) = 0, \quad f(1) = 0$$

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages
- The problem that the algorithm solves is the following:

$$f : \{0, 1\} \rightarrow \{0, 1\}$$

we define f balanced if:

$$f(0) = 0, \quad f(1) = 1$$
$$f(0) = 1, \quad f(1) = 0$$

we define f constant if:

$$f(0) = 1, \quad f(1) = 1$$
$$f(0) = 0, \quad f(1) = 0$$

- Problem is: try to evaluate the nature of the function f using as few operations as possible

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages
- The problem that the algorithm solves is the following:

$$f: \{0,1\} \rightarrow \{0,1\}$$

we define f balanced if:

$$f(0) = 0, \quad f(1) = 1$$
$$f(0) = 1, \quad f(1) = 0$$

we define f constant if:

$$f(0) = 1, \quad f(1) = 1$$
$$f(0) = 0, \quad f(1) = 0$$

- Problem is: try to evaluate the nature of the function f using as few operations as possible
- Classically speaking, the most efficient algorithm to accomplish the task involves a minimum of two measurements: one for the value of f in point 0, the other for the value of f in point 1

Quantum Parallelism: Deutsch Algorithm

- Deutsch's algorithm is one of the simplest algorithms with which to show how to intelligently use quantum parallelism to obtain computational advantages
- The problem that the algorithm solves is the following:

$$f: \{0,1\} \rightarrow \{0,1\}$$

we define f balanced if: $f(0) = 0, \quad f(1) = 1$

$f(0) = 1, \quad f(1) = 0$

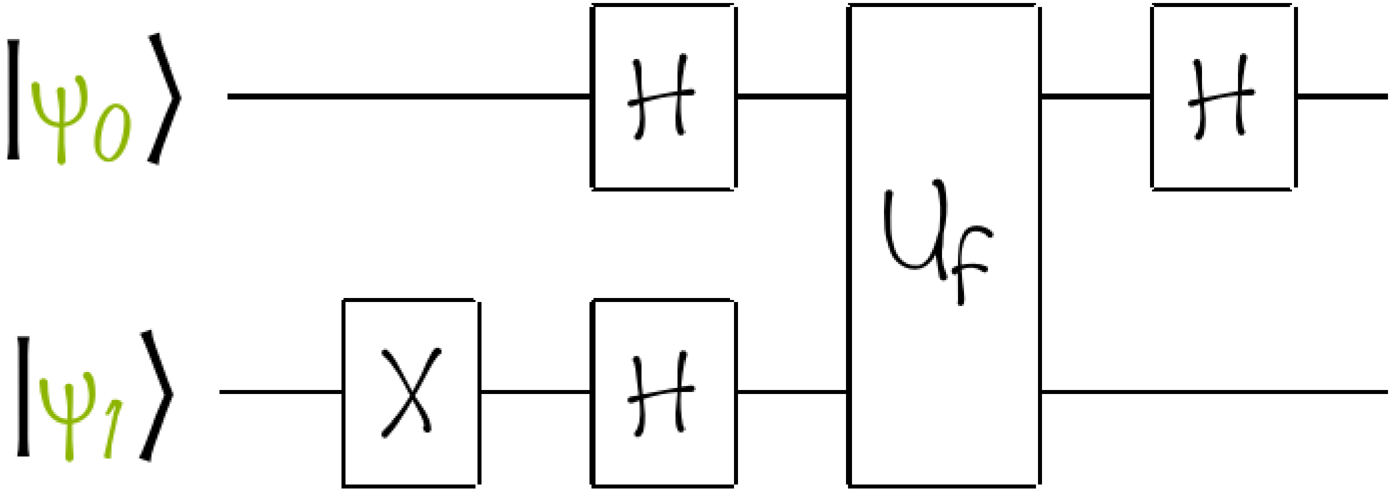
we define f constant if: $f(0) = 1, \quad f(1) = 1$

$f(0) = 0, \quad f(1) = 0$

- Problem is: try to evaluate the nature of the function f using as few operations as possible
- Classically speaking, the most efficient algorithm to accomplish the task involves a minimum of two measurements: one for the value of f in point 0, the other for the value of f in point 1

```
IF ( f(0) == 0 ):
    IF ( f(1) == 0 ):
        f is constant
    ELSE:
        f is balanced
ELSE:
    IF ( f(1) == 0 ):
        f is balanced
    ELSE:
        f is constant
```

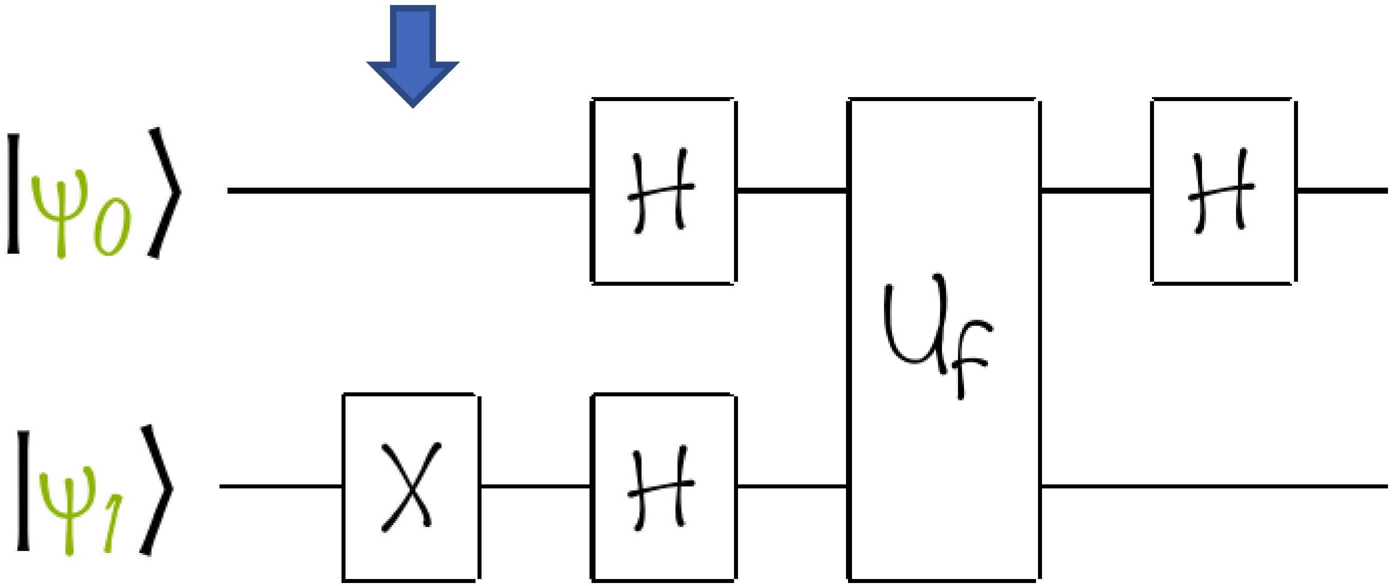
Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

Quantum Parallelism: Deutsch Algorithm



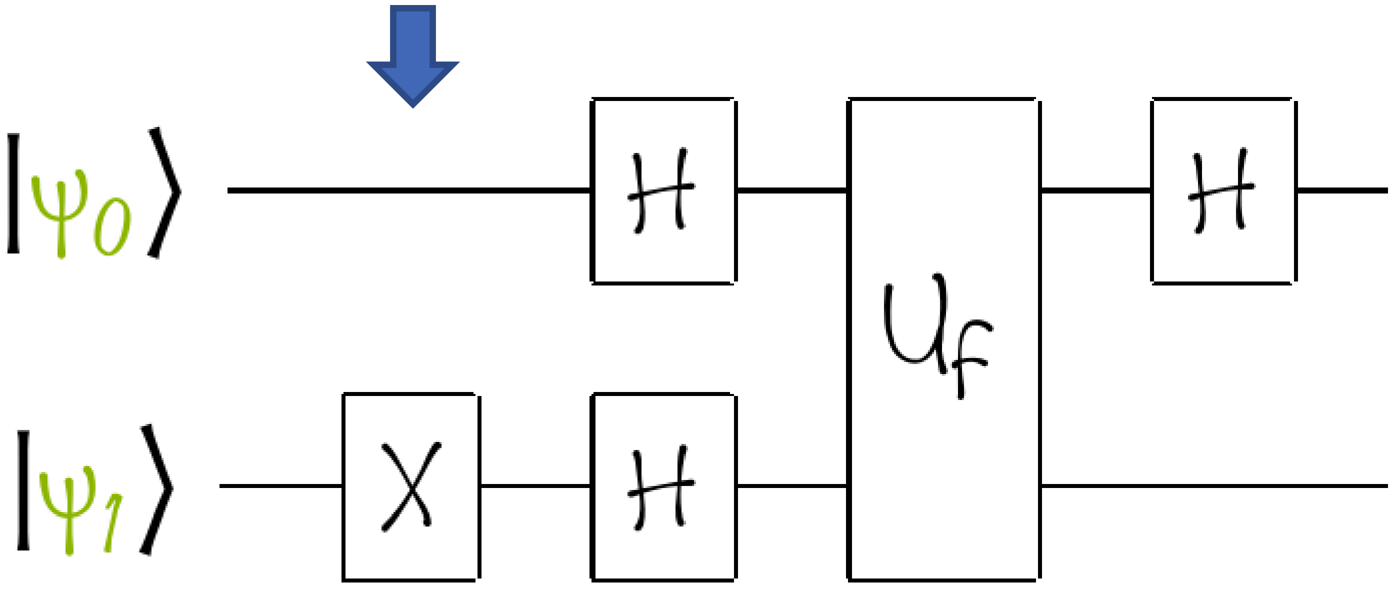
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

CINECA QUANTUM COMPUTING LAB

Quantum Parallelism: Deutsch Algorithm



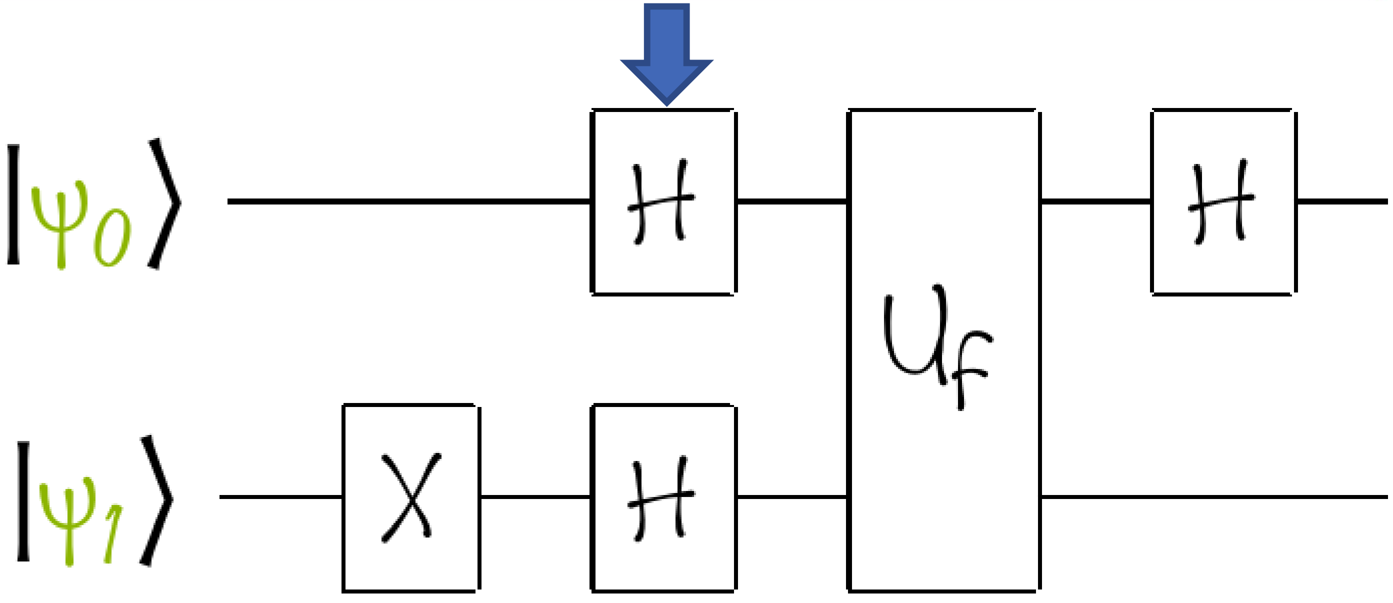
Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

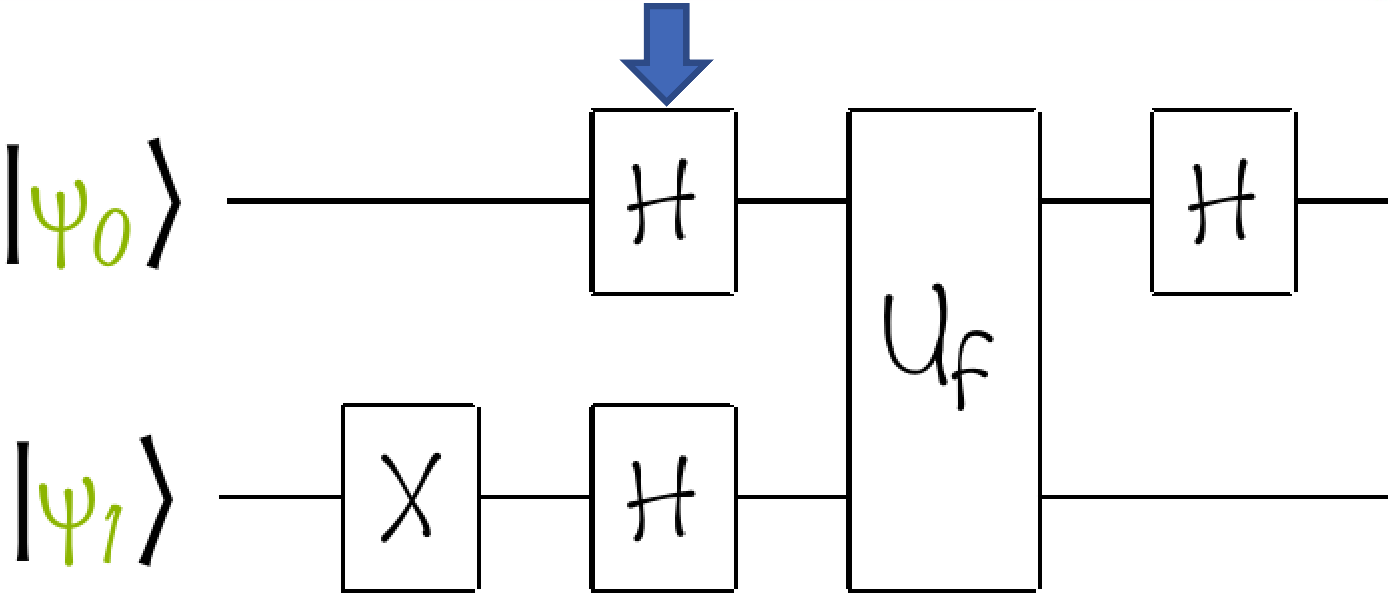
$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

CINECA QUANTUM COMPUTING LAB

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

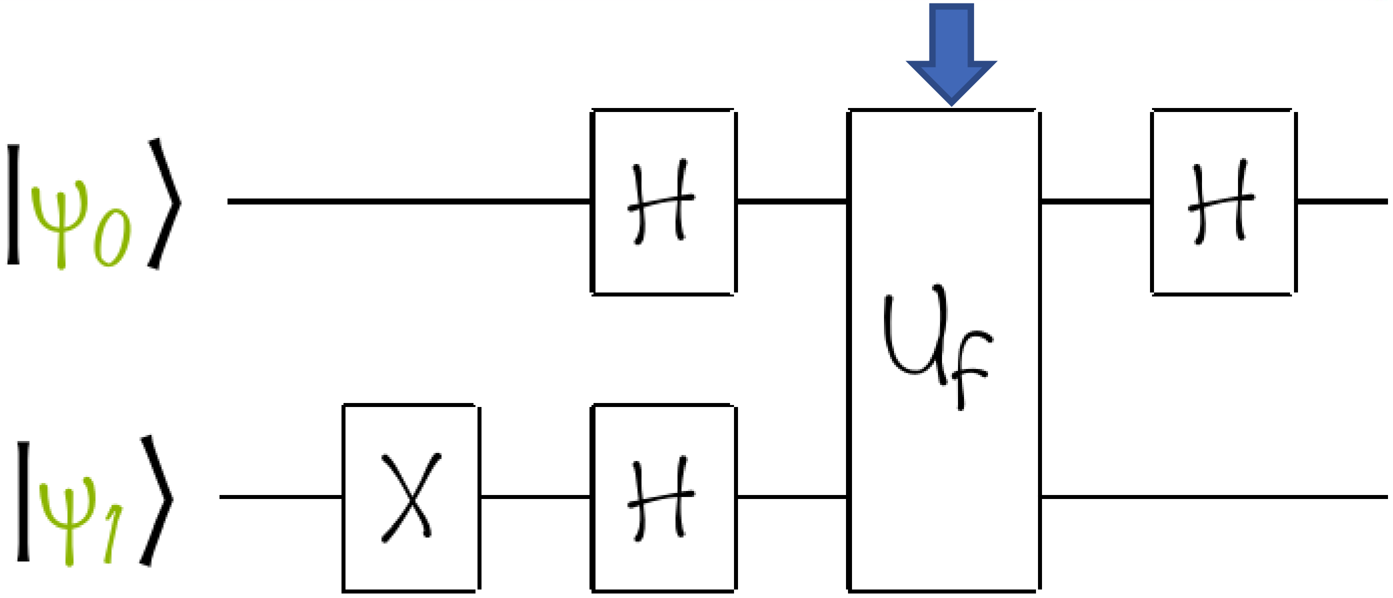
$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

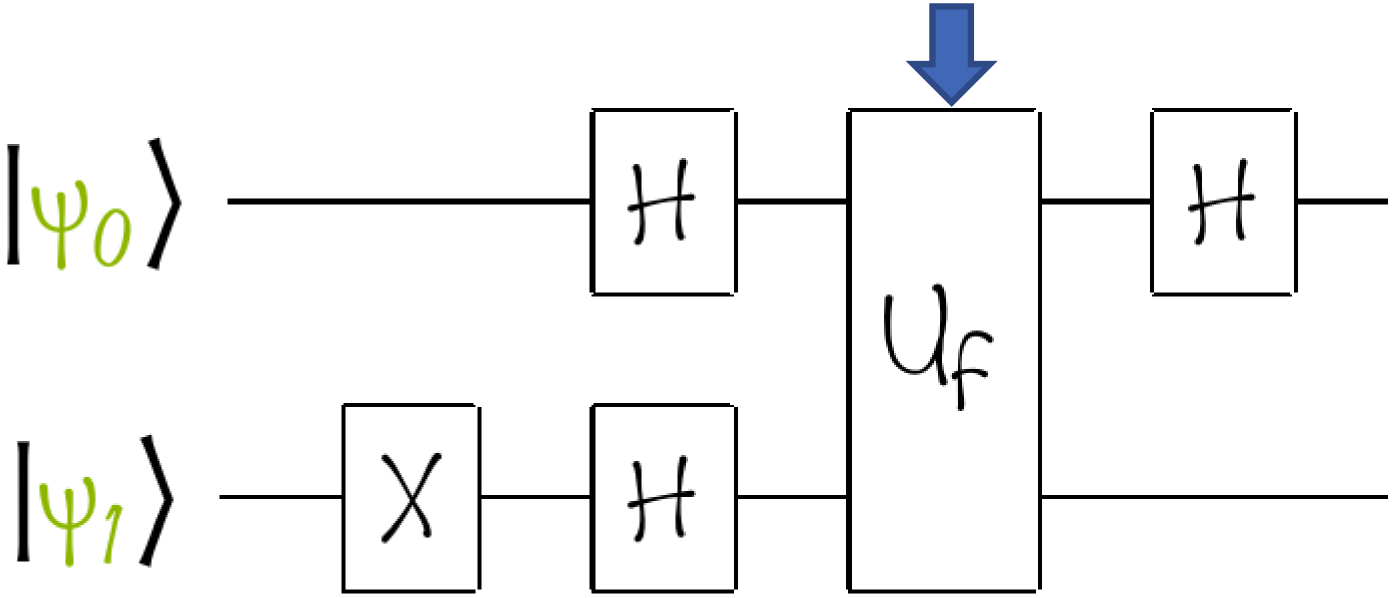
$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$|\psi_1\rangle \xrightarrow{U_f} |\psi_1\rangle$$

$$|\psi_2\rangle \xrightarrow{U_f} |\psi_2\rangle \oplus f(|\psi_1\rangle)$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

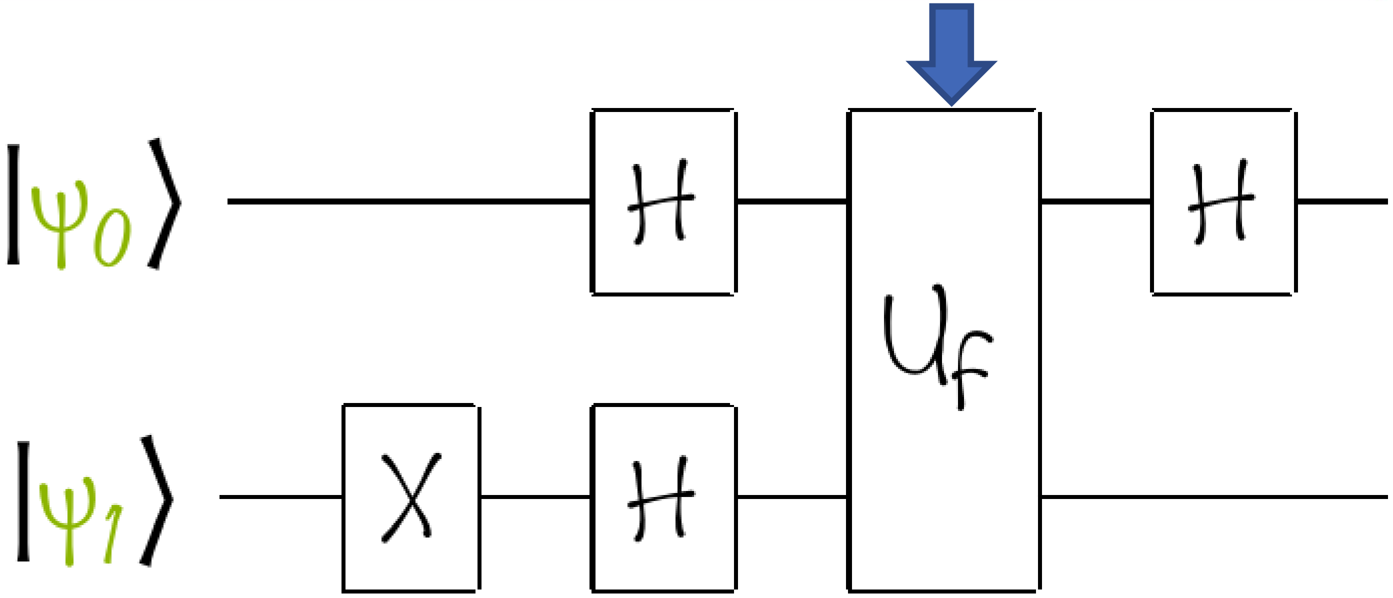
$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$|00\rangle \rightarrow |0, f(0)\rangle \qquad |01\rangle \rightarrow |0, 1 + f(0)\rangle$$

$$|10\rangle \rightarrow |1, f(1)\rangle \qquad |11\rangle \rightarrow |1, 1 + f(1)\rangle$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

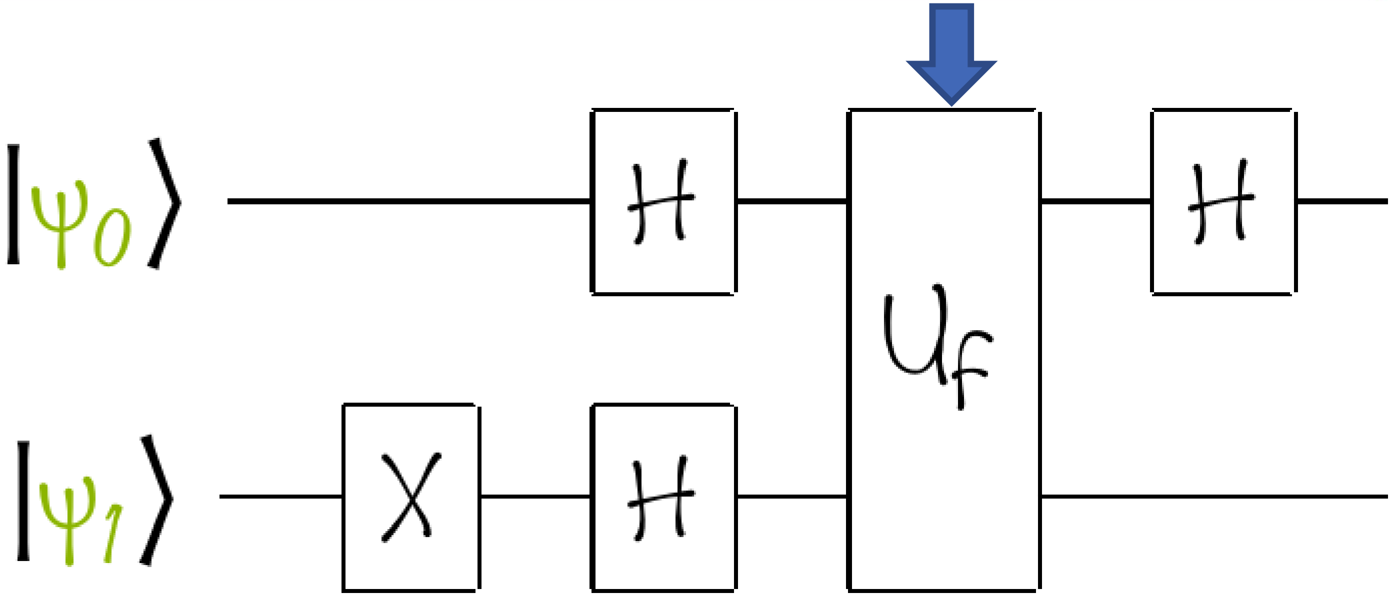
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

Case constant 0: f(0)=f(1)=0

$$|00\rangle \rightarrow |0,0\rangle \qquad |01\rangle \rightarrow |0,1\rangle$$

$$|10\rangle \rightarrow |1,0\rangle \qquad |11\rangle \rightarrow |1,1\rangle$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

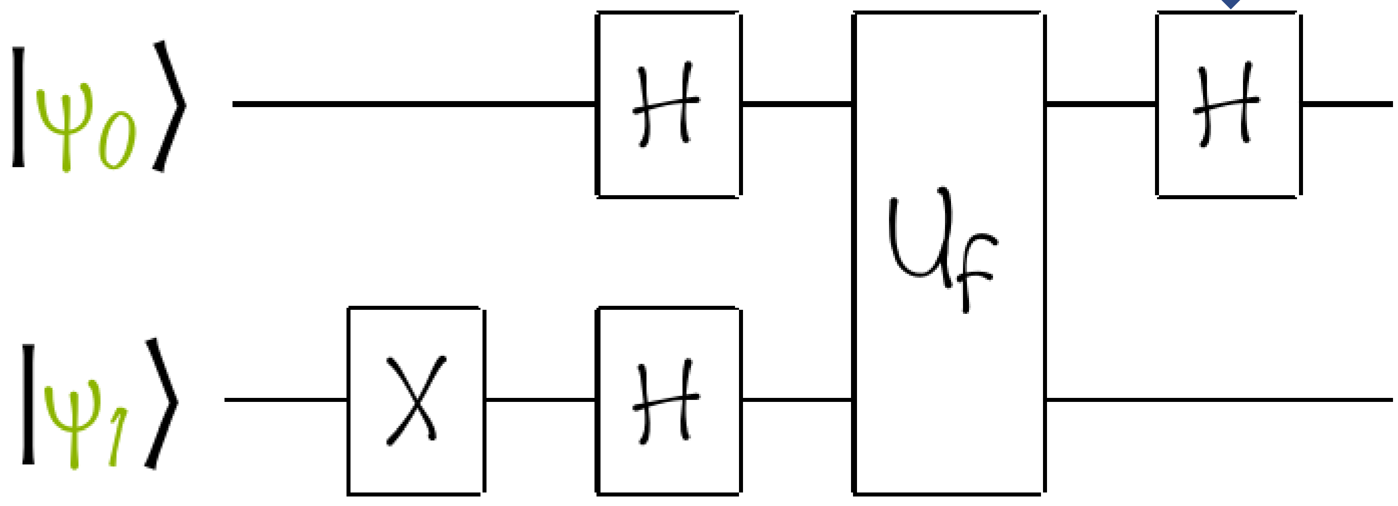
Case constant 0: f(0)=f(1)=0

$$|00\rangle \rightarrow |0,0\rangle \qquad |01\rangle \rightarrow |0,1\rangle$$

$$|10\rangle \rightarrow |1,0\rangle \qquad |11\rangle \rightarrow |1,1\rangle$$

Quantum Parallelism: Deutsch Algorithm

Case constant 0: f(0)=f(1)=0



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

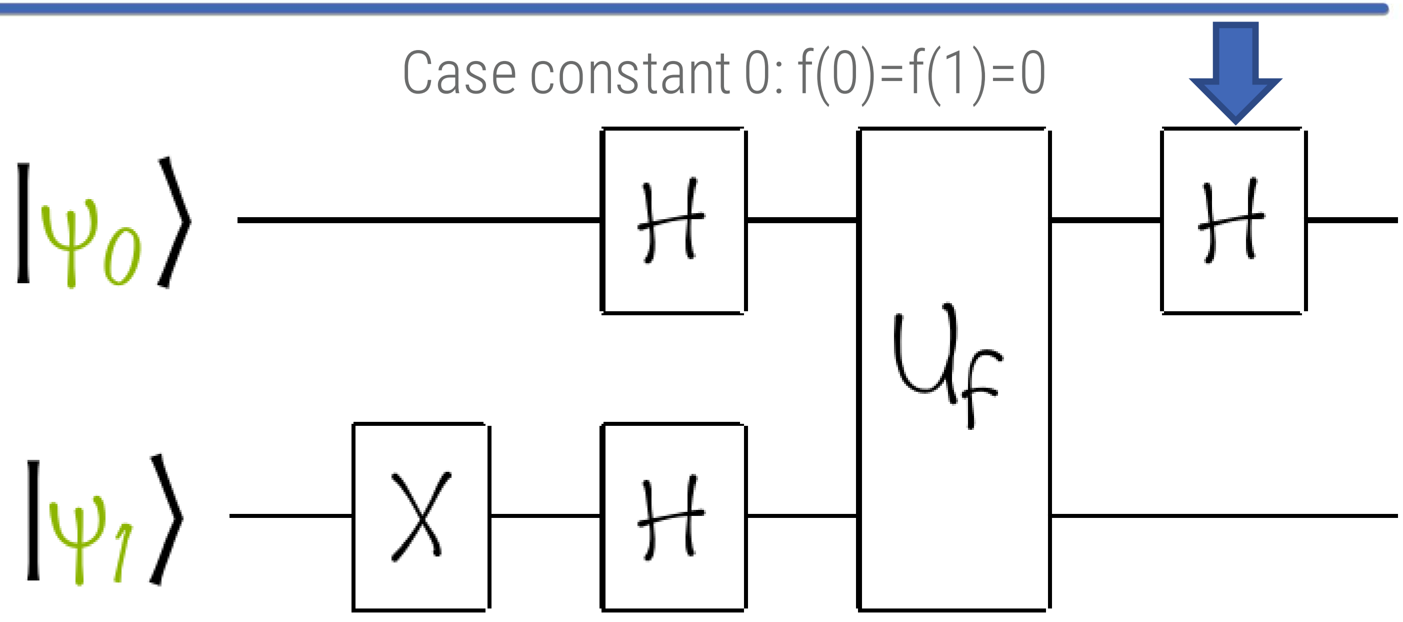
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

Quantum Parallelism: Deutsch Algorithm

Case constant 0: f(0)=f(1)=0



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

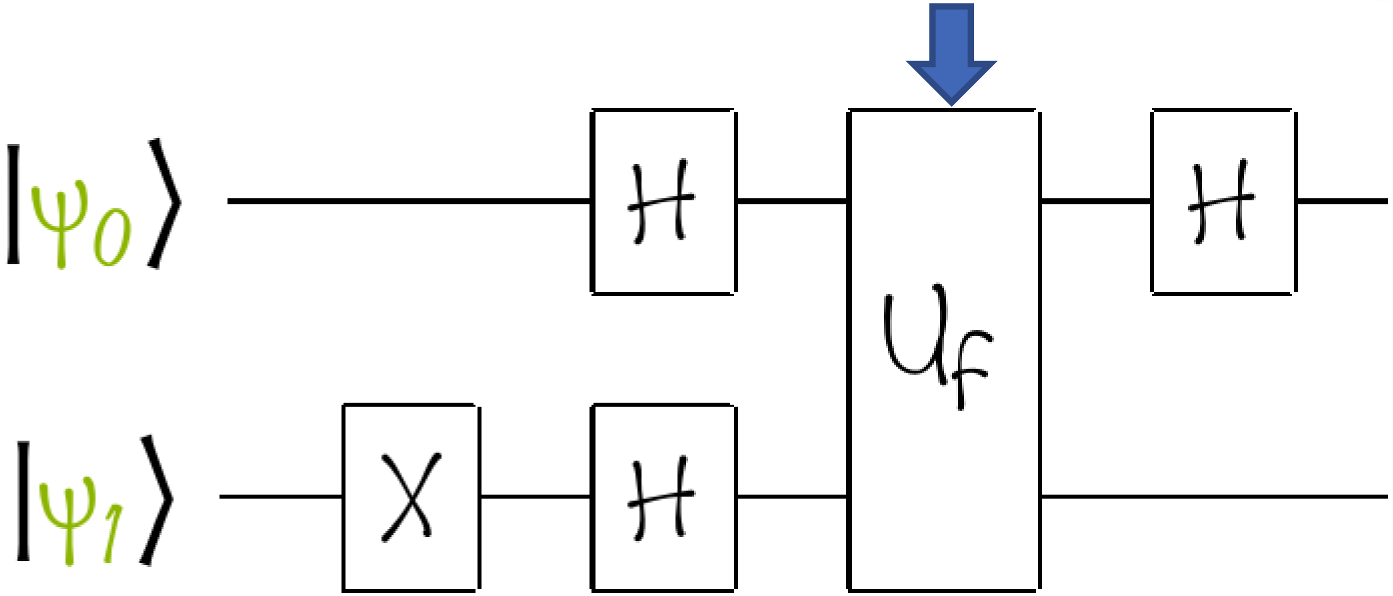
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

Case constant 0: f(0)=f(1)=0

$$\boxed{\frac{1}{\sqrt{2}} \cdot |00\rangle - \frac{1}{\sqrt{2}} \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle}$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

Case constant 1: f(0)=f(1)=1

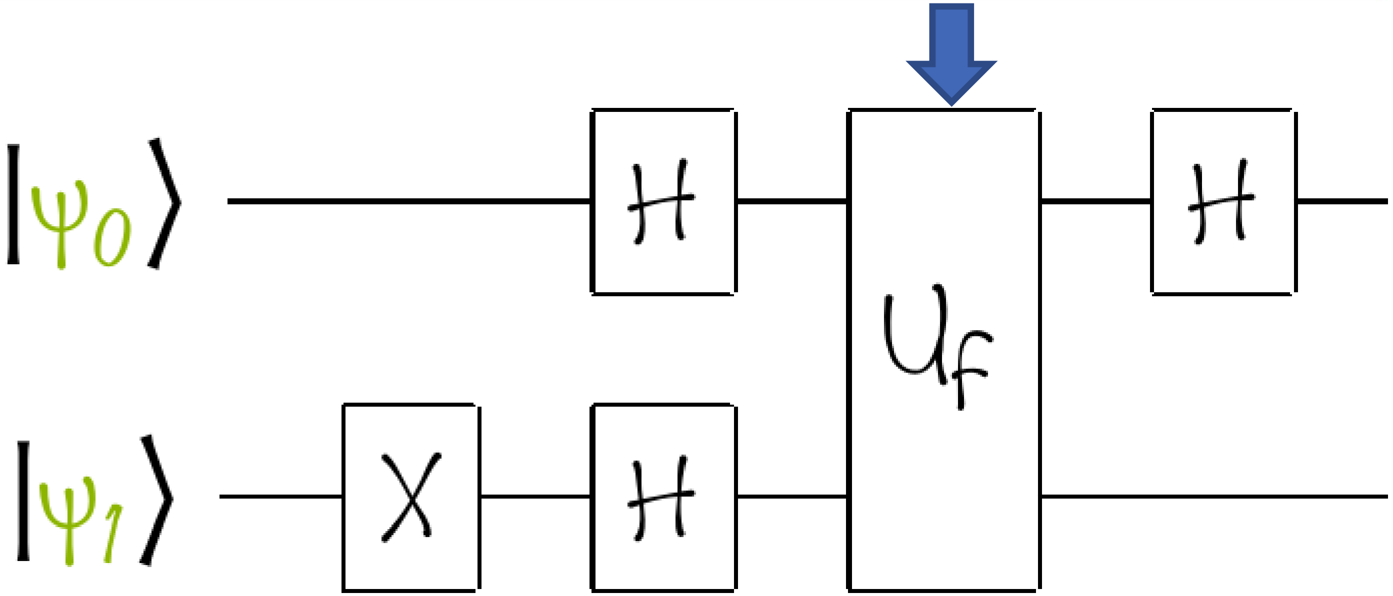
$$|00\rangle \rightarrow |0,1\rangle \qquad |01\rangle \rightarrow |0,0\rangle$$

$$|10\rangle \rightarrow |1,1\rangle \qquad |11\rangle \rightarrow |1,0\rangle$$

$$|00\rangle \rightarrow |0,f(0)\rangle \qquad |01\rangle \rightarrow |0,1+f(0)\rangle$$

$$|10\rangle \rightarrow |1,f(1)\rangle \qquad |11\rangle \rightarrow |1,1+f(1)\rangle$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$-\frac{1}{2} \cdot |00\rangle + \frac{1}{2} \cdot |01\rangle - \frac{1}{2} \cdot |10\rangle + \frac{1}{2} \cdot |11\rangle$$

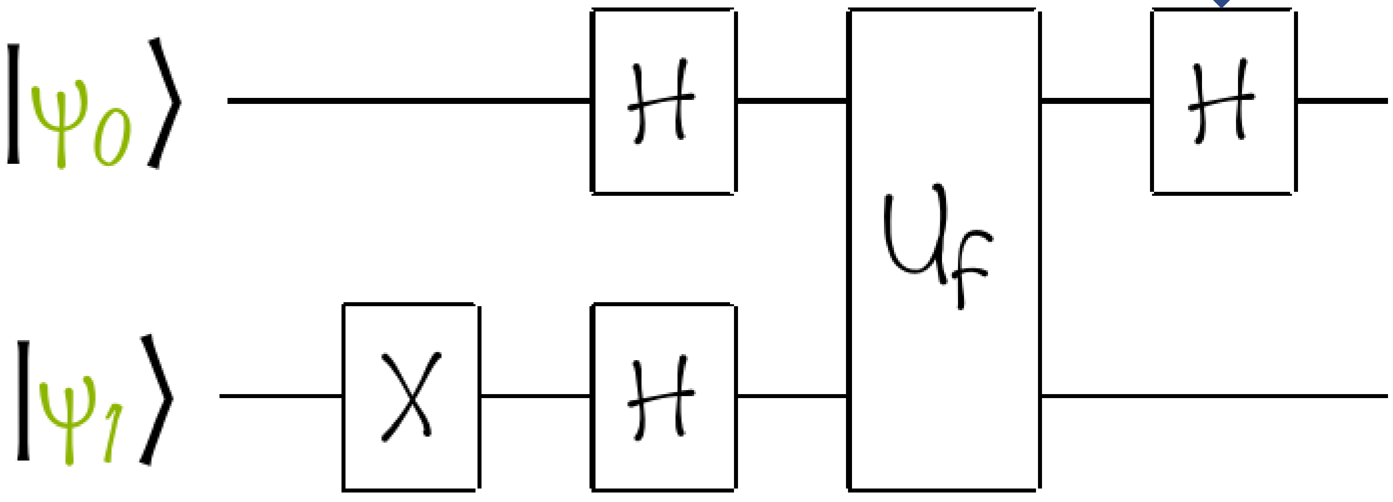
Case constant 1: f(0)=f(1)=1

$$|00\rangle \rightarrow |0,1\rangle \qquad |01\rangle \rightarrow |0,0\rangle$$

$$|10\rangle \rightarrow |1,1\rangle \qquad |11\rangle \rightarrow |1,0\rangle$$

Quantum Parallelism: Deutsch Algorithm

Case constant 1: f(0)=f(1)=1



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

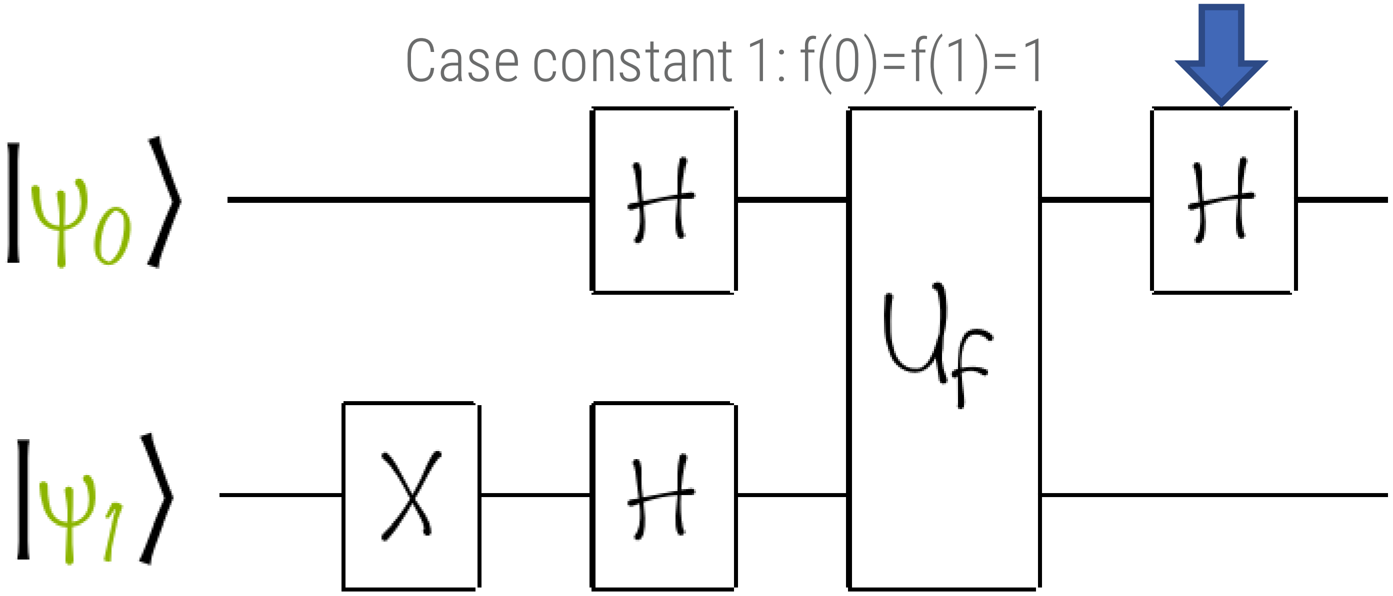
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$-\frac{1}{2} \cdot |00\rangle + \frac{1}{2} \cdot |01\rangle - \frac{1}{2} \cdot |10\rangle + \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Quantum Parallelism: Deutsch Algorithm

Case constant 1: f(0)=f(1)=1



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

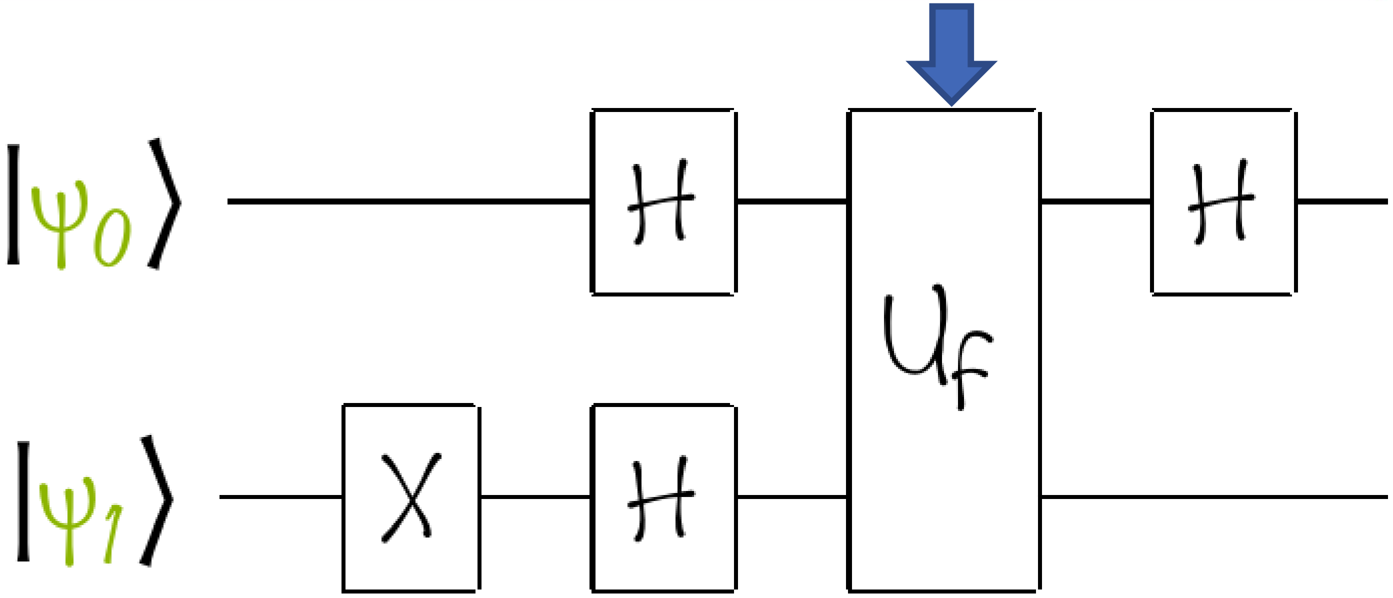
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$-\frac{1}{2} \cdot |00\rangle + \frac{1}{2} \cdot |01\rangle - \frac{1}{2} \cdot |10\rangle + \frac{1}{2} \cdot |11\rangle$$

Case constant 1: f(0)=f(1)=1

$$\boxed{-\frac{1}{\sqrt{2}} \cdot |00\rangle + \frac{1}{\sqrt{2}} \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle}$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

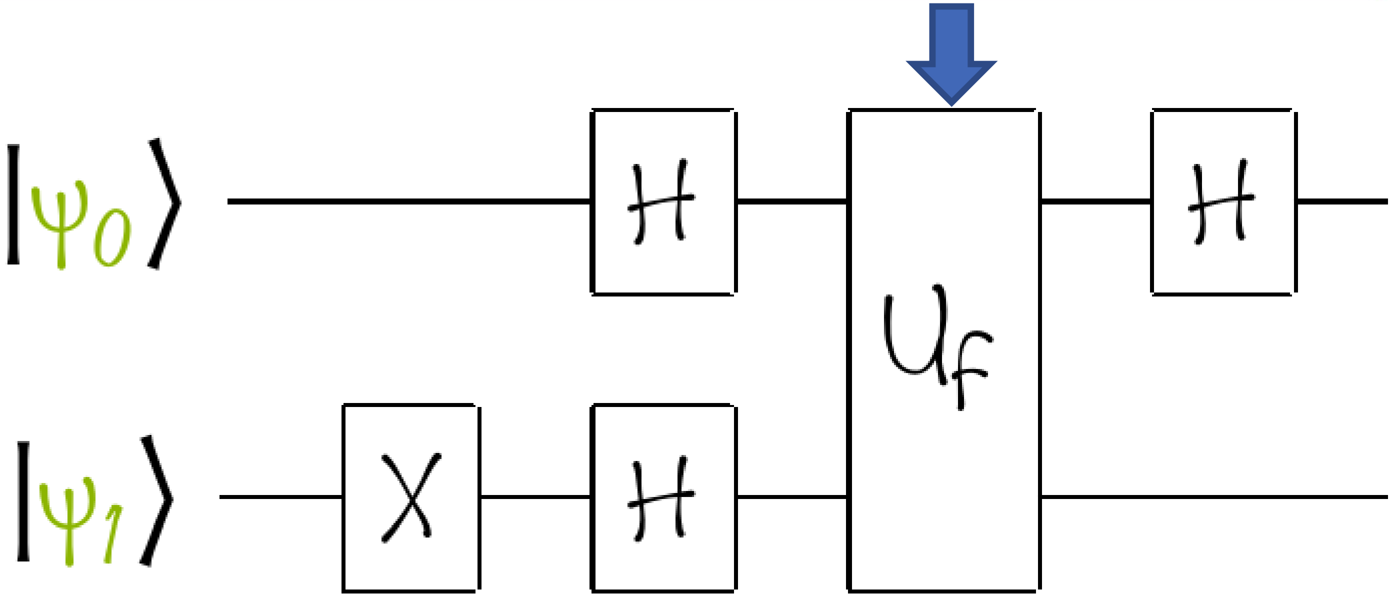
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

Case balanced 0: f(0)=0, f(1)=1

$$|00\rangle \rightarrow |0,0\rangle \qquad |01\rangle \rightarrow |0,1\rangle$$
$$|10\rangle \rightarrow |1,1\rangle \qquad |11\rangle \rightarrow |1,0\rangle$$

$$|00\rangle \rightarrow |0,f(0)\rangle \qquad |01\rangle \rightarrow |0,1+f(0)\rangle$$
$$|10\rangle \rightarrow |1,f(1)\rangle \qquad |11\rangle \rightarrow |1,1+f(1)\rangle$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle - \frac{1}{2} \cdot |10\rangle + \frac{1}{2} \cdot |11\rangle$$

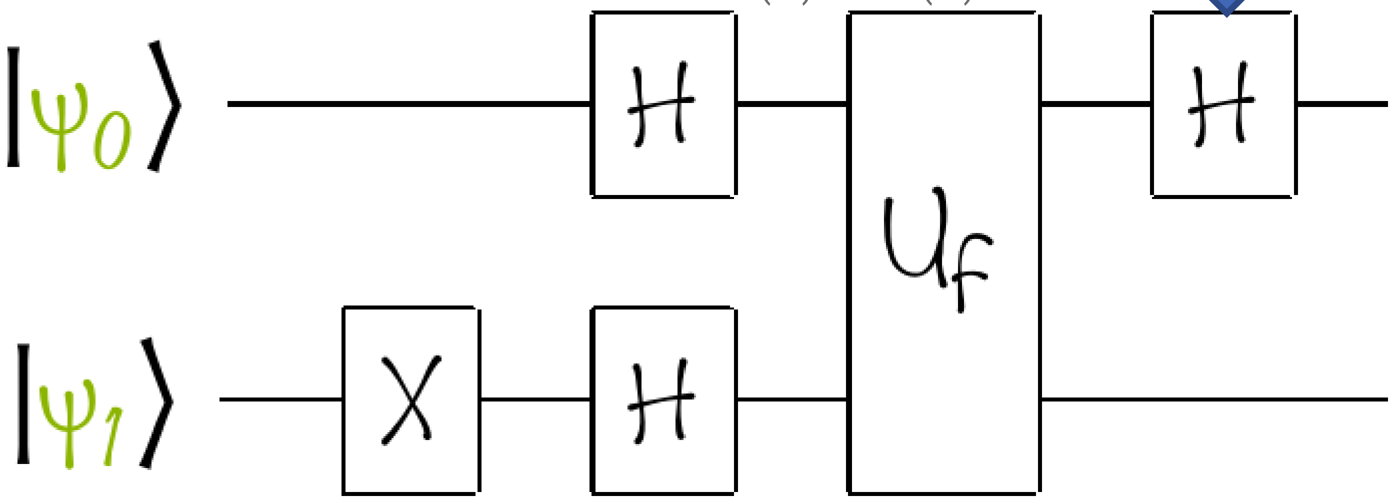
Case balanced 0: f(0)=0, f(1)=1

$$|00\rangle \rightarrow |0,0\rangle \qquad |01\rangle \rightarrow |0,1\rangle$$

$$|10\rangle \rightarrow |1,1\rangle \qquad |11\rangle \rightarrow |1,0\rangle$$

Quantum Parallelism: Deutsch Algorithm

Case balanced 0: f(0)=0, f(1)=1



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

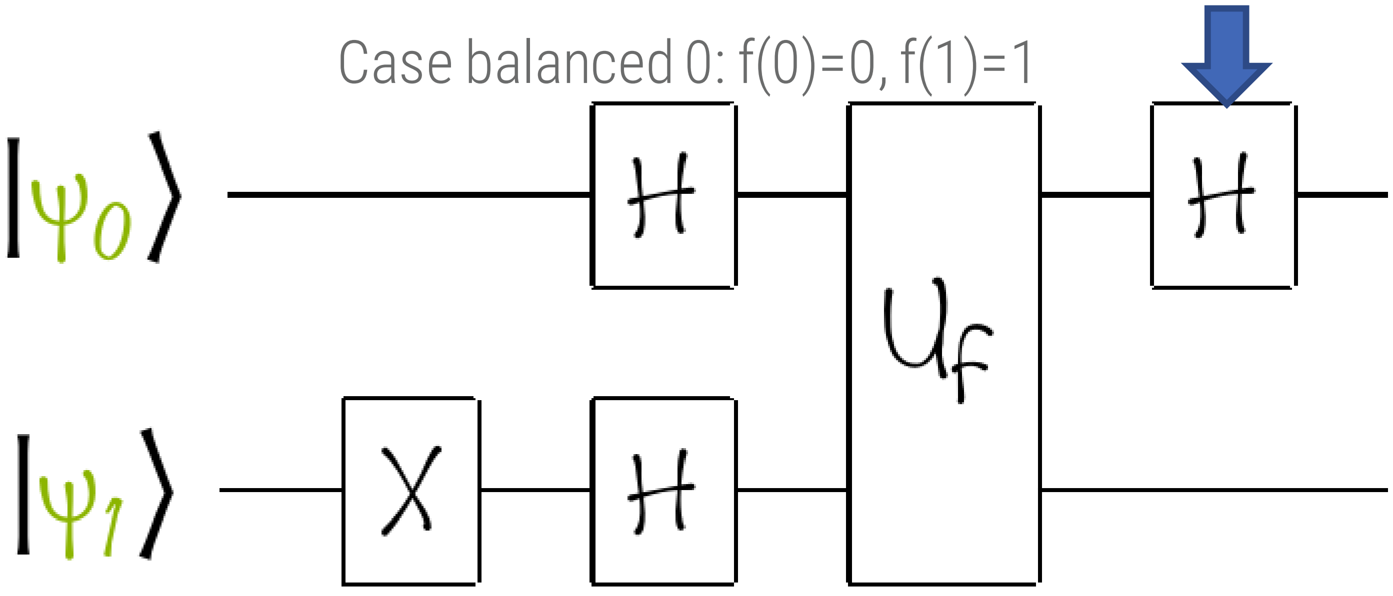
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle - \frac{1}{2} \cdot |10\rangle + \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

Quantum Parallelism: Deutsch Algorithm

Case balanced 0: f(0)=0, f(1)=1



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

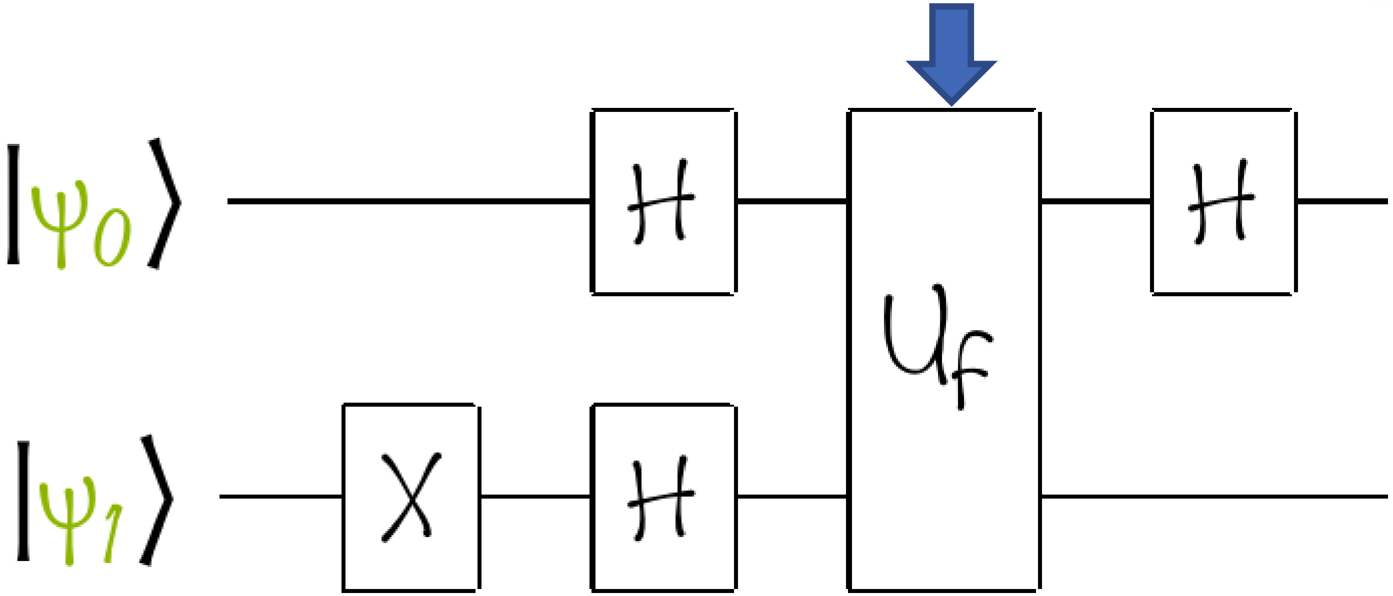
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle - \frac{1}{2} \cdot |10\rangle + \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

Case balanced 0: f(0)=0, f(1)=1

$$\boxed{0 \cdot |00\rangle + 0 \cdot |01\rangle + \frac{1}{\sqrt{2}} \cdot |10\rangle - \frac{1}{\sqrt{2}} \cdot |11\rangle}$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

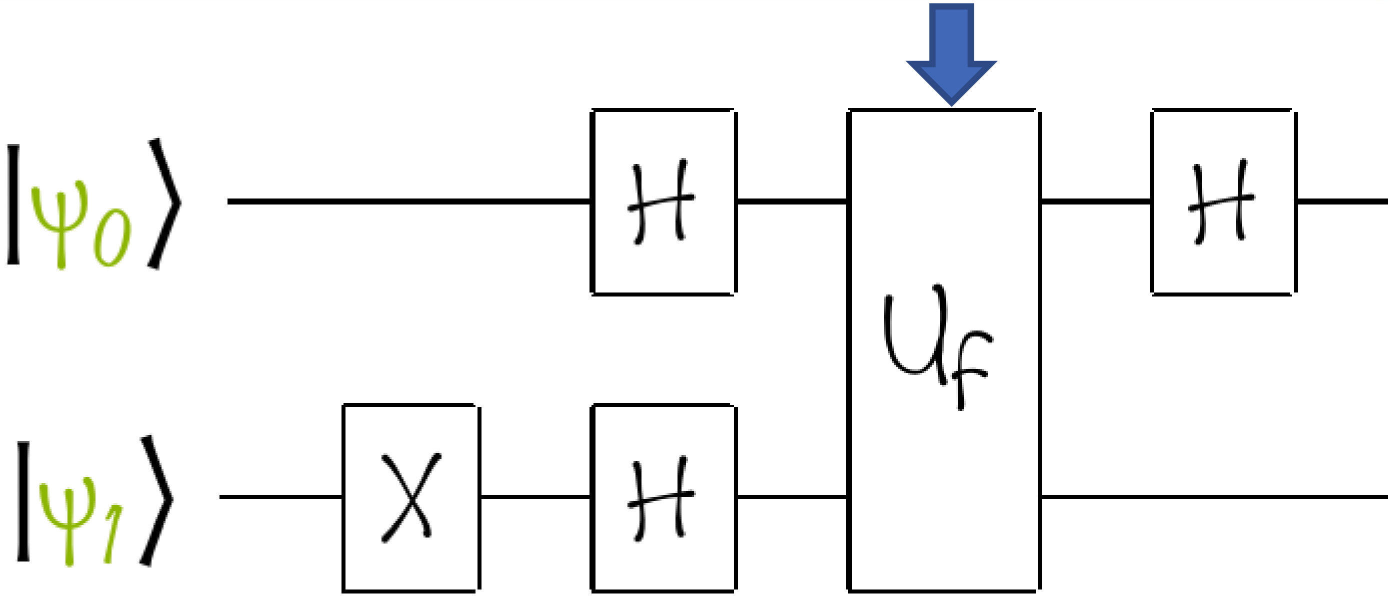
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

Case balanced 1: f(0)=1, f(1)=0

$$|00\rangle \rightarrow |0,1\rangle \quad |01\rangle \rightarrow |0,0\rangle$$
$$|10\rangle \rightarrow |1,0\rangle \quad |11\rangle \rightarrow |1,1\rangle$$

$$|00\rangle \rightarrow |0,f(0)\rangle \quad |01\rangle \rightarrow |0,1+f(0)\rangle$$
$$|10\rangle \rightarrow |1,f(1)\rangle \quad |11\rangle \rightarrow |1,1+f(1)\rangle$$

Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$-\frac{1}{2} \cdot |00\rangle + \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

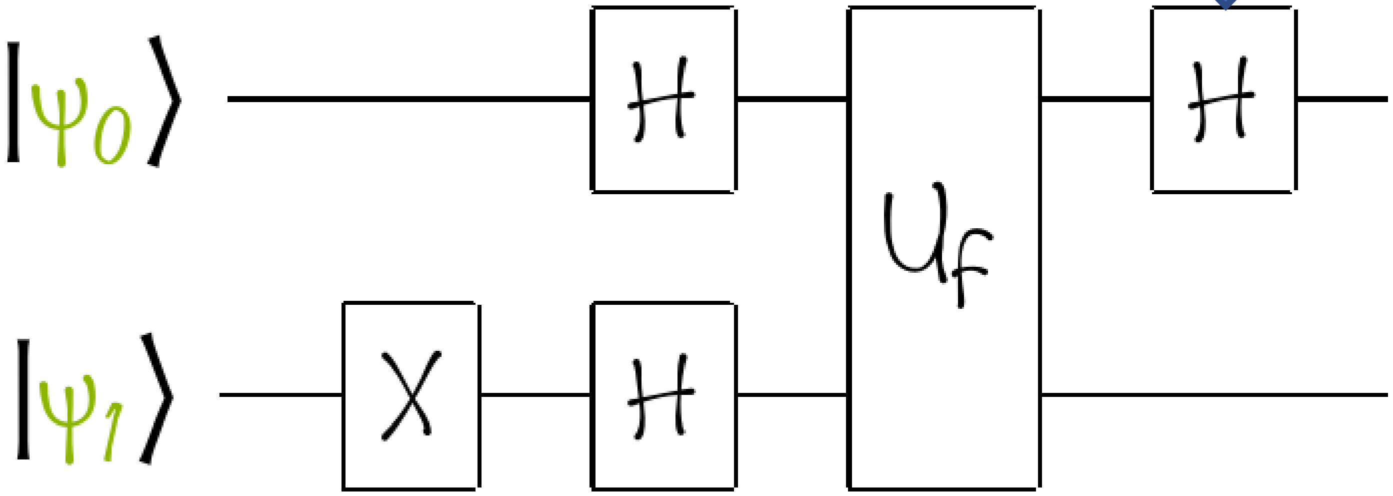
Case balanced 1: f(0)=1, f(1)=0

$$|00\rangle \rightarrow |0,1\rangle \qquad |01\rangle \rightarrow |0,0\rangle$$

$$|10\rangle \rightarrow |1,0\rangle \qquad |11\rangle \rightarrow |1,1\rangle$$

Quantum Parallelism: Deutsch Algorithm

Case balanced 1: f(0)=1, f(1)=0



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

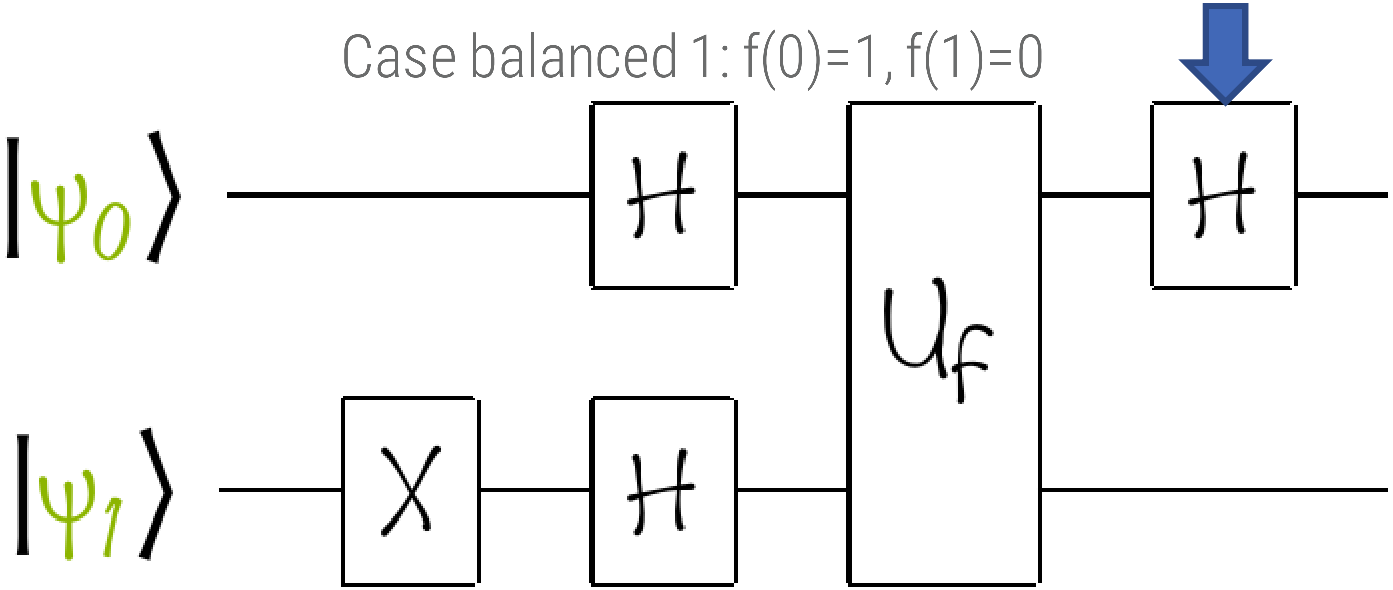
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$-\frac{1}{2} \cdot |00\rangle + \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

Quantum Parallelism: Deutsch Algorithm

Case balanced 1: f(0)=1, f(1)=0



Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$0 \cdot |00\rangle + 1 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

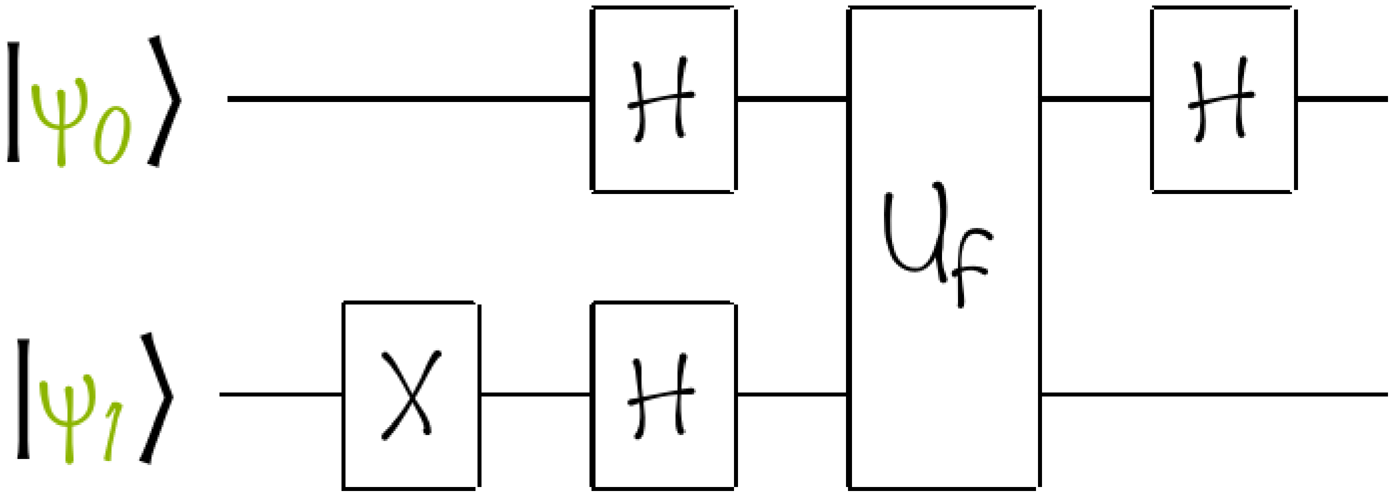
$$-\frac{1}{2} \cdot |00\rangle + \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

Case balanced 1: f(0)=1, f(1)=0

$$\boxed{0 \cdot |00\rangle + 0 \cdot |01\rangle - \frac{1}{\sqrt{2}} \cdot |10\rangle + \frac{1}{\sqrt{2}} \cdot |11\rangle}$$

Quantum Parallelism: Deutsch Algorithm



Case constant 0: f(0)=f(1)=0

$$\frac{1}{\sqrt{2}} \cdot |00\rangle - \frac{1}{\sqrt{2}} \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

Case balanced 0: f(0)=0, f(1)=1

$$0 \cdot |00\rangle + 0 \cdot |01\rangle + \frac{1}{\sqrt{2}} \cdot |10\rangle - \frac{1}{\sqrt{2}} \cdot |11\rangle$$

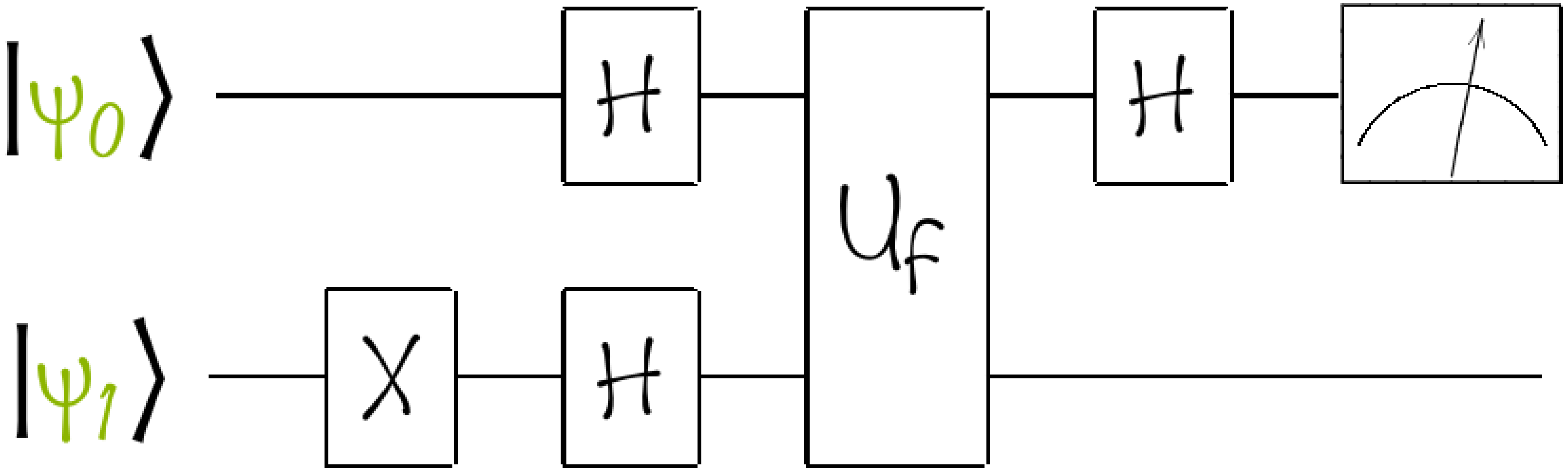
Case constant 1: f(0)=f(1)=1

$$-\frac{1}{\sqrt{2}} \cdot |00\rangle + \frac{1}{\sqrt{2}} \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

Case balanced 1: f(0)=1, f(1)=0

$$0 \cdot |00\rangle + 0 \cdot |01\rangle - \frac{1}{\sqrt{2}} \cdot |10\rangle + \frac{1}{\sqrt{2}} \cdot |11\rangle$$

Quantum Parallelism: Deutsch Algorithm



Case constant 0: f(0)=f(1)=0

$$\frac{1}{\sqrt{2}} \cdot |00\rangle - \frac{1}{\sqrt{2}} \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

Case balanced 0: f(0)=0, f(1)=1

$$0 \cdot |00\rangle + 0 \cdot |01\rangle + \frac{1}{\sqrt{2}} \cdot |10\rangle - \frac{1}{\sqrt{2}} \cdot |11\rangle$$

Case constant 1: f(0)=f(1)=1

$$-\frac{1}{\sqrt{2}} \cdot |00\rangle + \frac{1}{\sqrt{2}} \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

Case balanced 1: f(0)=1, f(1)=0

$$0 \cdot |00\rangle + 0 \cdot |01\rangle - \frac{1}{\sqrt{2}} \cdot |10\rangle + \frac{1}{\sqrt{2}} \cdot |11\rangle$$

Exponential and Polynomial Speed-up

Computational order of an algorithm: measures the number of operations required to solve a given problem. It is usually expressed as a function of the size of the problem to be solved (parameter N)

Exponential and Polynomial Speed-up

Computational order of an algorithm: measures the number of operations required to solve a given problem. It is usually expressed as a function of the size of the problem to be solved (parameter N)

Polynomial Speed-Up

Exponential and Polynomial Speed-up

Computational order of an algorithm: measures the number of operations required to solve a given problem. It is usually expressed as a function of the size of the problem to be solved (parameter N)

Polynomial Speed-Up

Exponential Speed-Up

Exponential and Polynomial Speed-up

Computational order of an algorithm: measures the number of operations required to solve a given problem. It is usually expressed as a function of the size of the problem to be solved (parameter N)

Polynomial Speed-Up

We pass from one computational order to another polynomially smaller

Example: N to $\sqrt{N}$

Exponential Speed-Up

Exponential and Polynomial Speed-up

Computational order of an algorithm: measures the number of operations required to solve a given problem. It is usually expressed as a function of the size of the problem to be solved (parameter N)

Polynomial Speed-Up

We pass from one computational order to another polynomially smaller

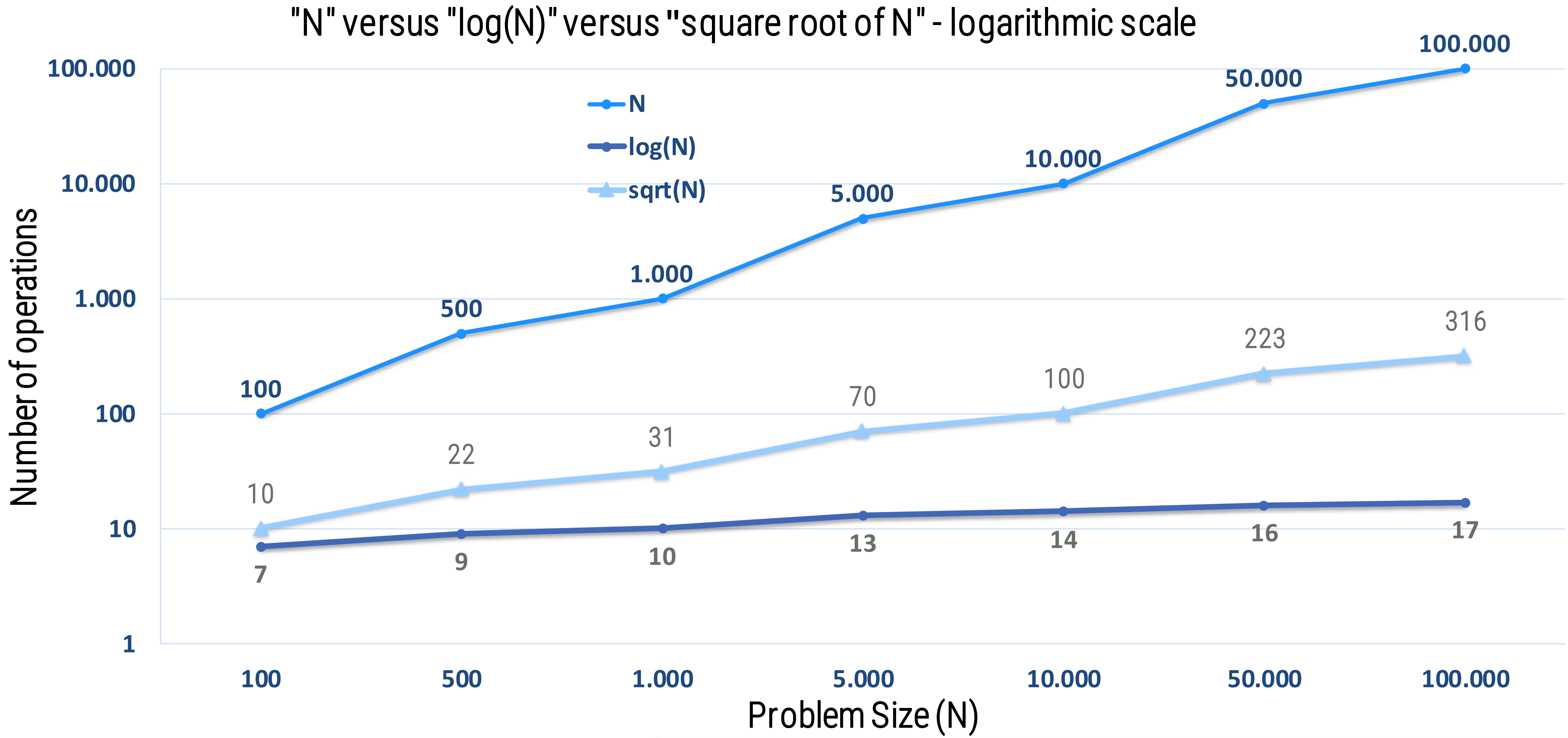
Example: N to $\sqrt{\boldsymbol{N}}$

Exponential Speed-Up

We move from one computational order to another exponentially smaller

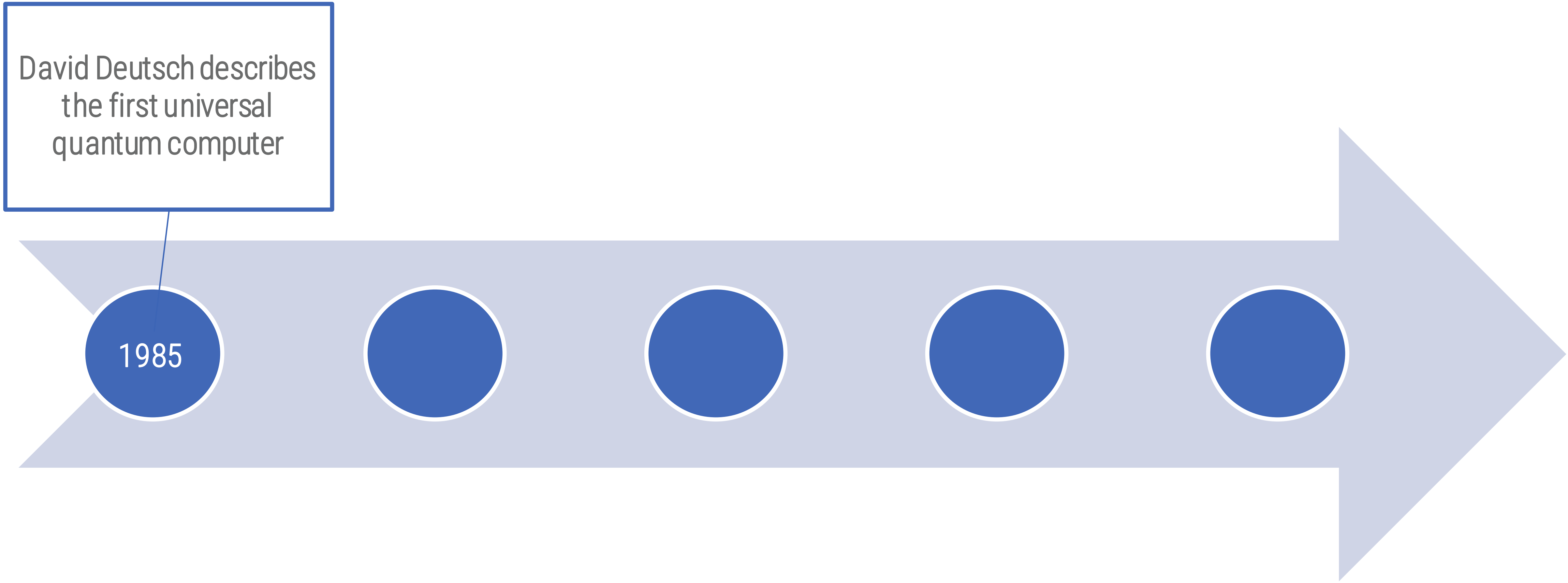
Example: N to $\boldsymbol{log}\ (\boldsymbol{N})$

Exponential and Polynomial Speed-up

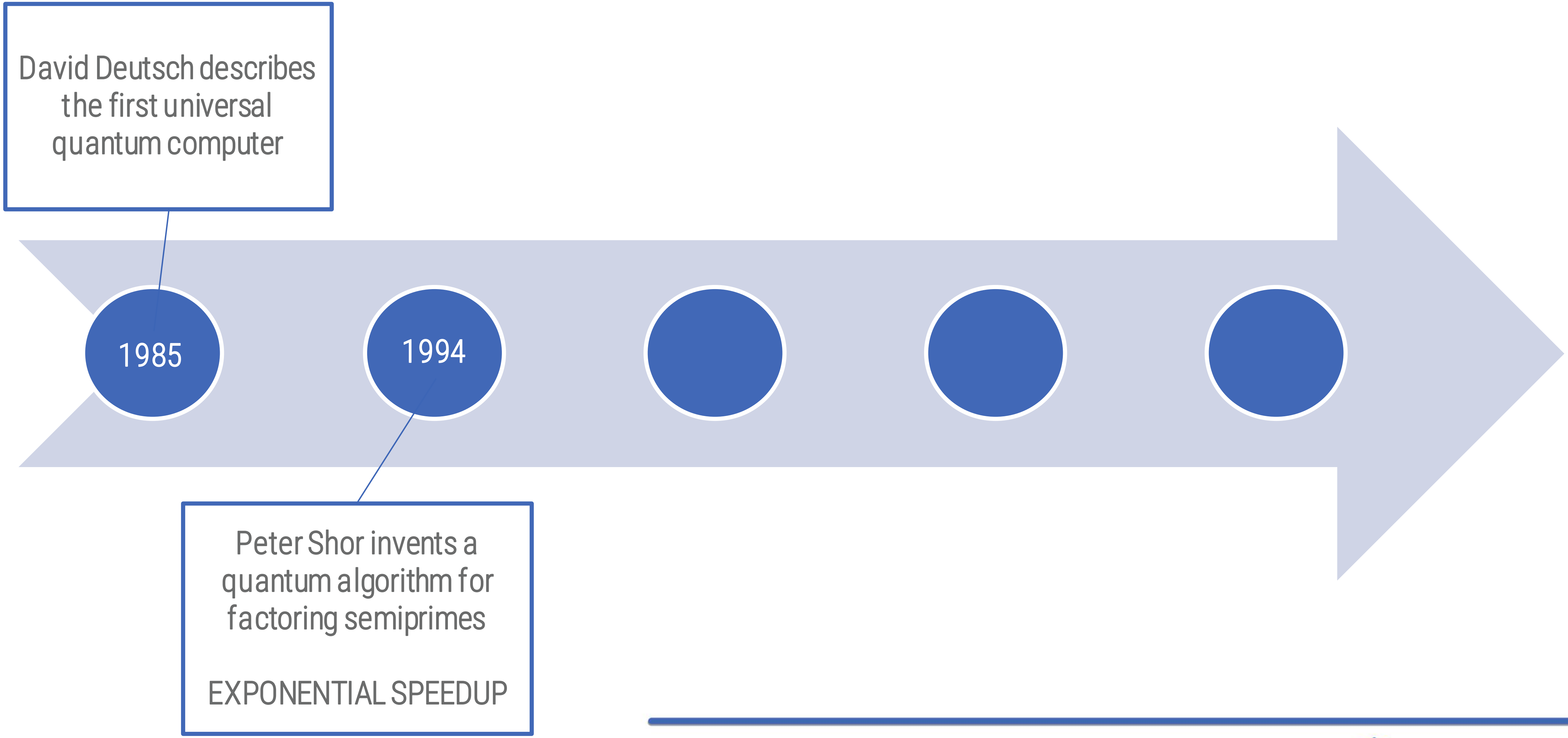


CINECA QUANTUM COMPUTING LAB

Brief Timeline of Quantum Algorithms



Brief Timeline of Quantum Algorithms



Brief Timeline of Quantum Algorithms

1985

David Deutsch describes the first universal quantum computer

1994

Peter Shor invents a quantum algorithm for factoring semiprimes

EXPONENTIAL SPEEDUP

1996

Lov Grover invents the quantum search algorithm in a database

POLYNOMIAL SPEEDUP

Brief Timeline of Quantum Algorithms

1985

David Deutsch describes the first universal quantum computer

1994

Peter Shor invents a quantum algorithm for factoring semiprimes

EXPONENTIAL SPEEDUP

1996

Lov Grover invents the quantum search algorithm in a database

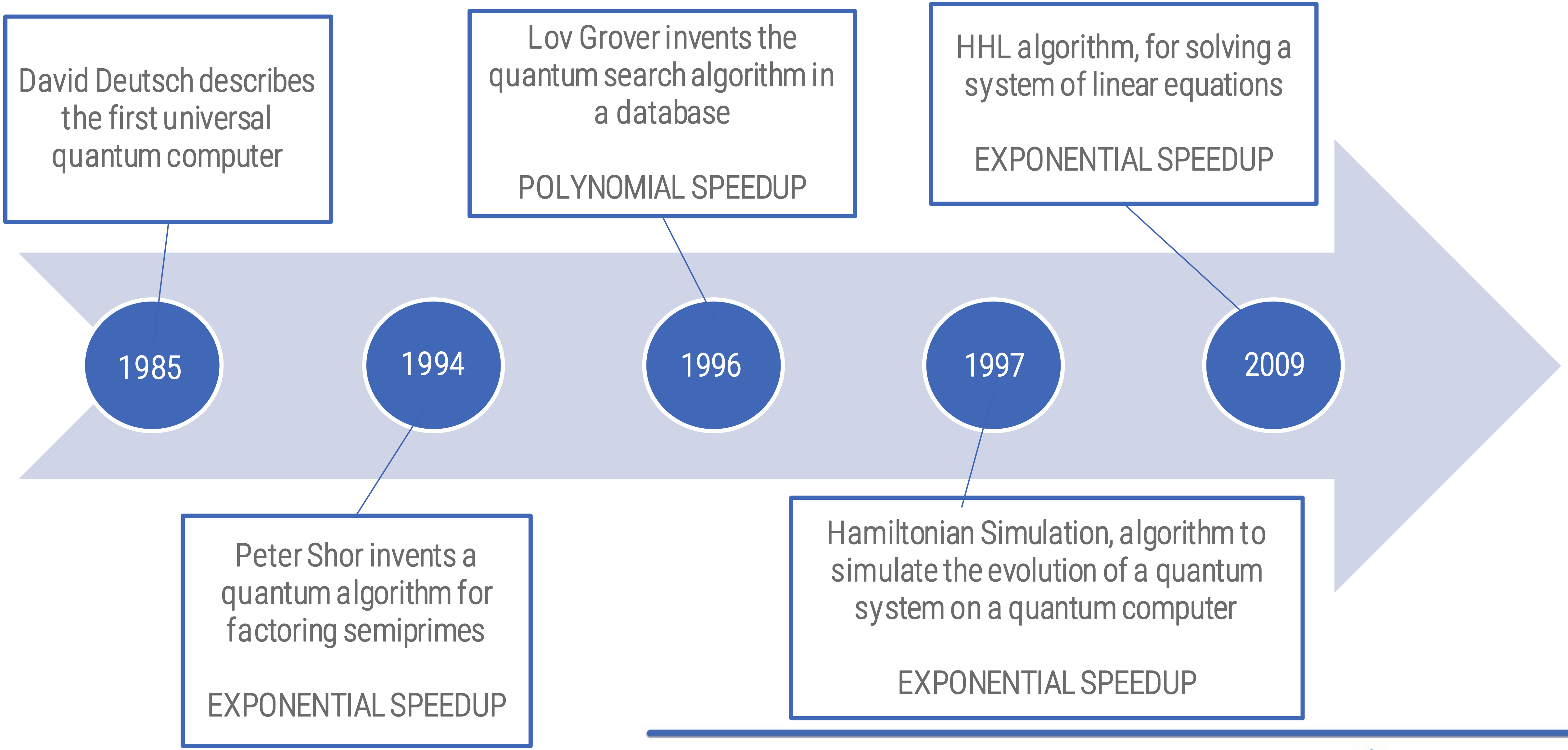
POLYNOMIAL SPEEDUP

1997

Hamiltonian Simulation, algorithm to simulate the evolution of a quantum system on a quantum computer

EXPONENTIAL SPEEDUP

Brief Timeline of Quantum Algorithms

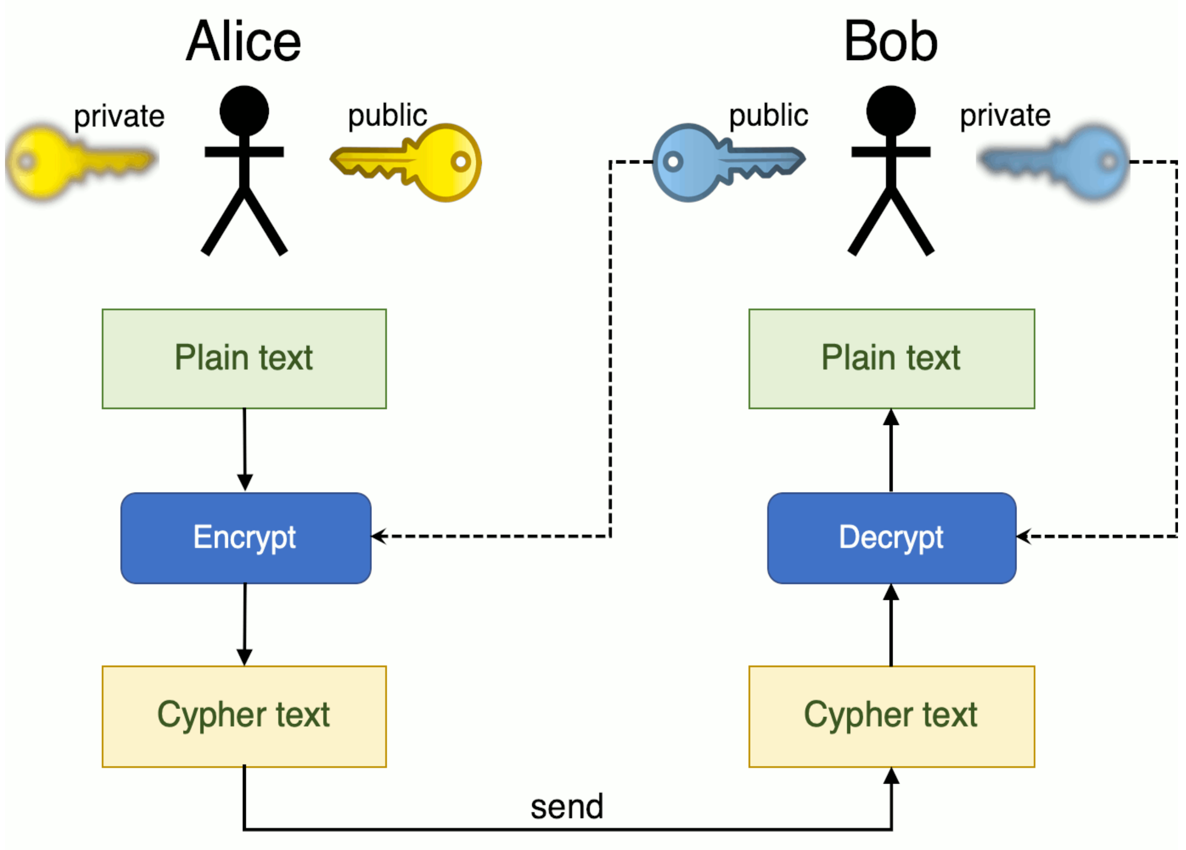


The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

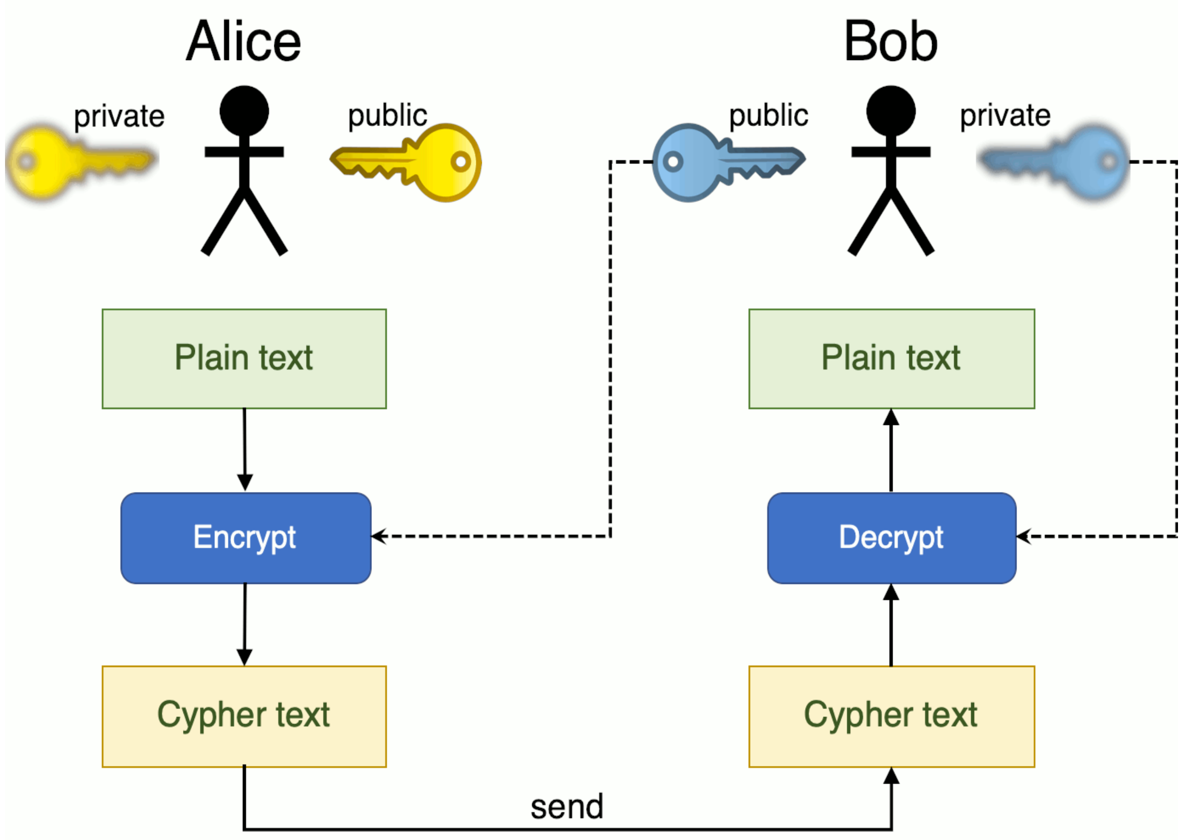
The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data



The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data



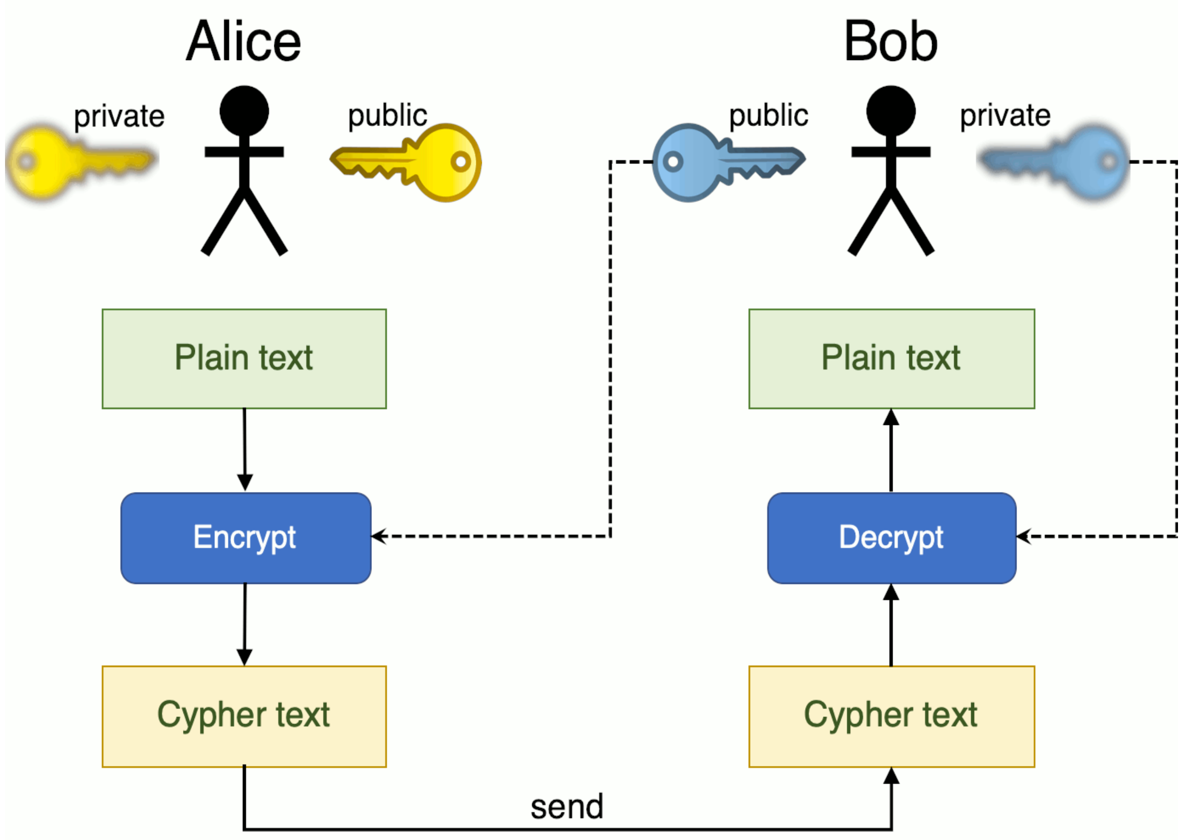
Public Key

Known by everybody

Used by the sender to encrypt a secret message

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data



Public Key

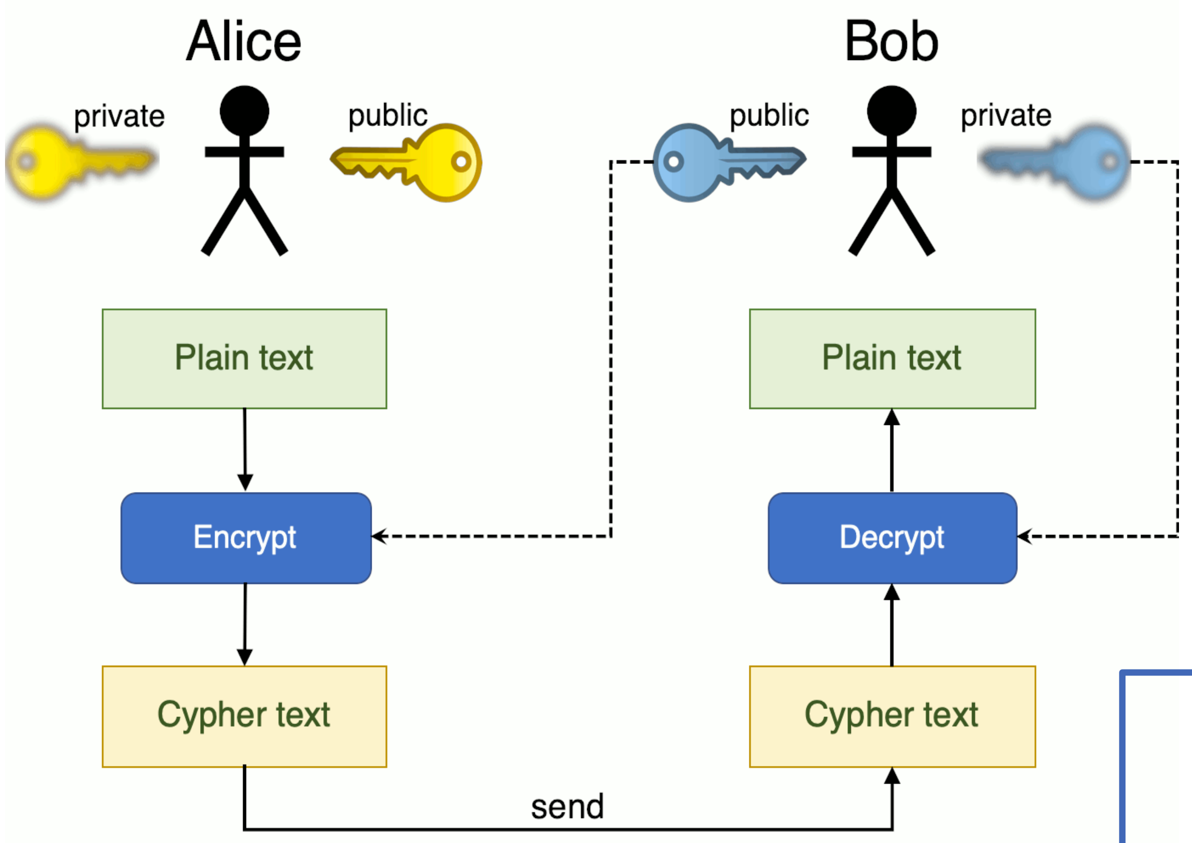
Known by everybody
Used by the sender to encrypt a secret message

Private Key

Known by the owner only
Used by the recipient to decrypt a secret message

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data



Public Key

Known by everybody
Used by the sender to encrypt a secret message

Private Key

Known by the owner only
Used by the recipient to decrypt a secret message

Theoretically, it is possible to extrapolate the private key from the public key

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In practice, it's not that simple

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In practice, it's not that simple

The protection of the private key, in fact, is entrusted to a class of mathematical problems that are very difficult to solve

An example is given by the factoring of semi-prime numbers: given a number N, find two prime numbers p and q such that

$$N = p \times q$$

They are all problems of an exponential computational order

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

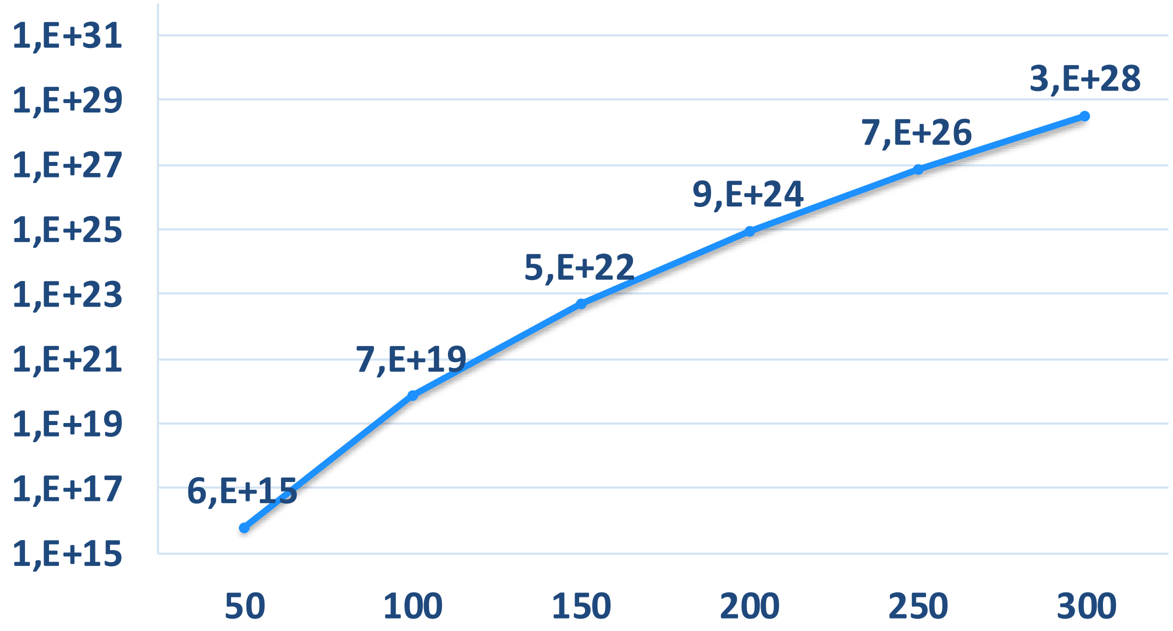
In practice, it's not that simple

The protection of the private key, in fact, is entrusted to a class of mathematical problems that are very difficult to solve

An example is given by the factoring of semi-prime numbers: given a number N, find two prime numbers p and q such that

$$N = p \times q$$

They are all problems of an exponential computational order



The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

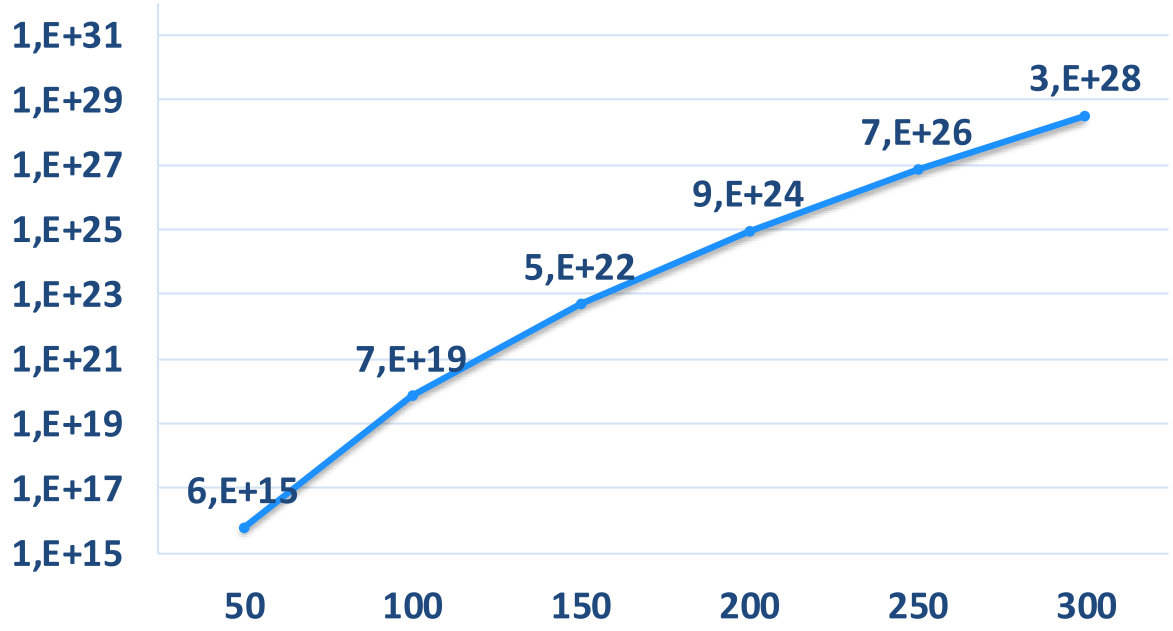
In practice, it's not that simple

The protection of the private key, in fact, is entrusted to a class of mathematical problems that are very difficult to solve

An example is given by the factoring of semi-prime numbers: given a number N, find two prime numbers p and q such that

$$N = p \times q$$

They are all problems of an exponential computational order



Summit, the most powerful supercomputer in the world, performs 2E+17 operations per second.
Making 3E+28 would cost about 4,700 years

CINECA QUANTUM COMPUTING LAB

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In 1994, Peter Shor invents the algorithm that today has his name

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In 1994, Peter Shor invents the algorithm that today has his name

Shor's algorithm is able to solve all the problems used to protect private keys

Including the factoring of semi-prime numbers

The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In 1994, Peter Shor invents the algorithm that today has his name

Shor's algorithm is able to solve all the problems used to protect private keys

Including the factoring of semi-prime numbers

It belongs to the category of quantum algorithms that feature one
EXPONENTIAL SPEED-UP

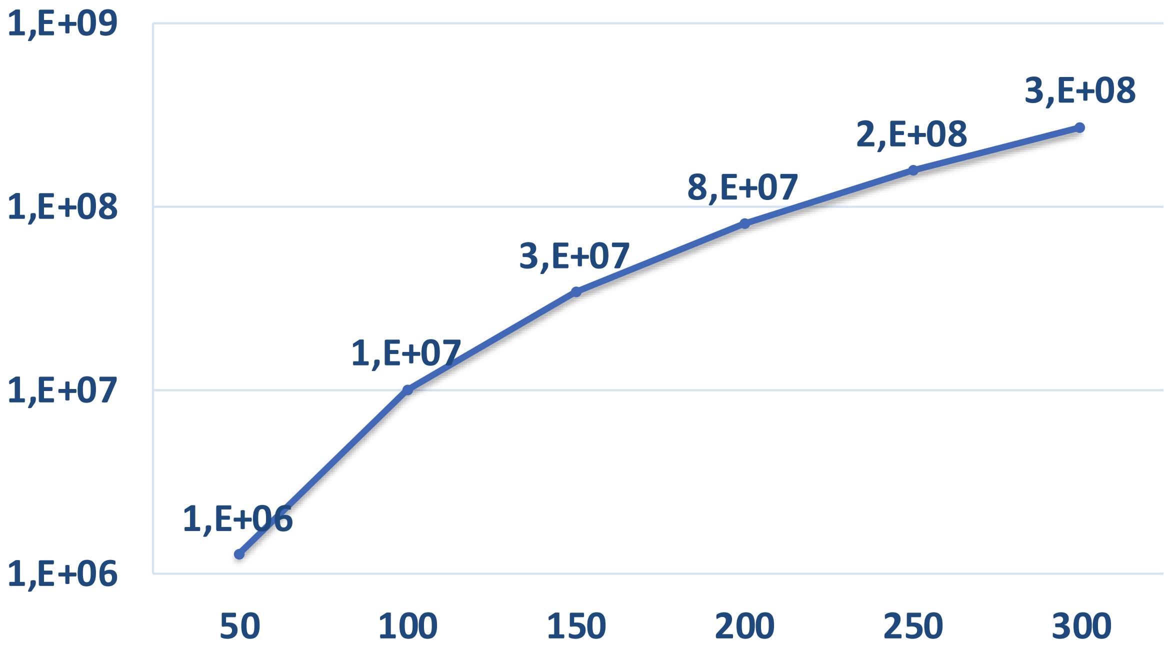
The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In 1994, Peter Shor invents the algorithm that today has his name

Shor's algorithm is able to solve all the problems used to protect private keys

Including the factoring of semi-prime numbers



It belongs to the category of quantum algorithms that feature one EXPONENTIAL SPEED-UP

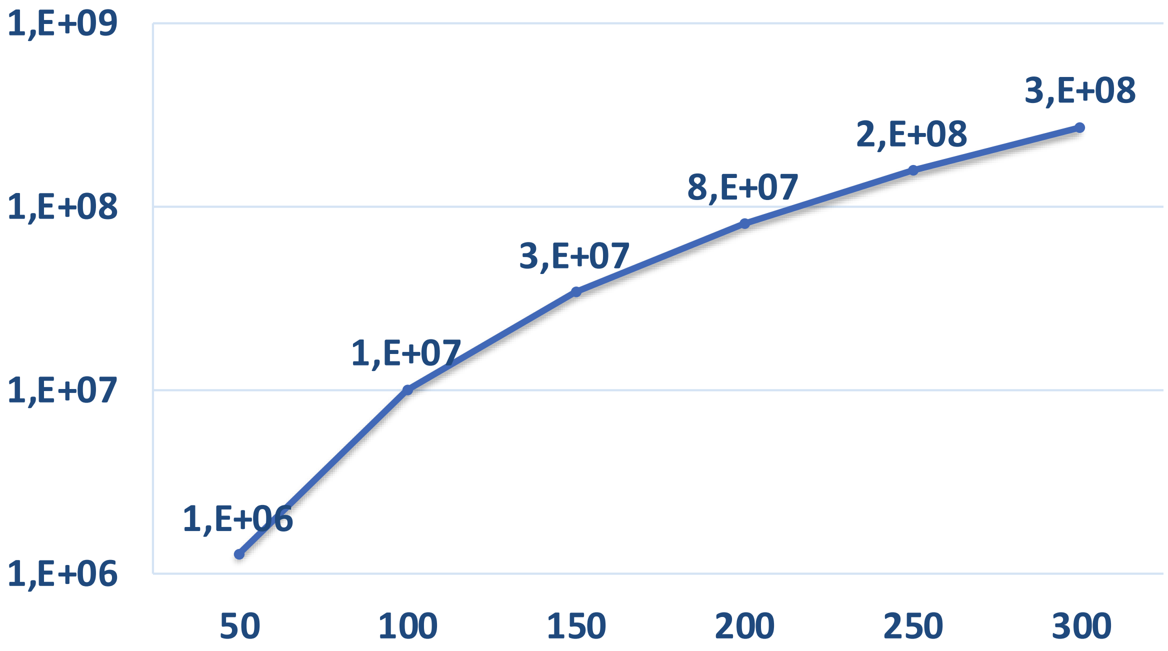
The impact of QC in the world - Shor's algorithm

Asymmetric encryption: how we protect our data

In 1994, Peter Shor invents the algorithm that today has his name

Shor's algorithm is able to solve all the problems used to protect private keys

Including the factoring of semi-prime numbers

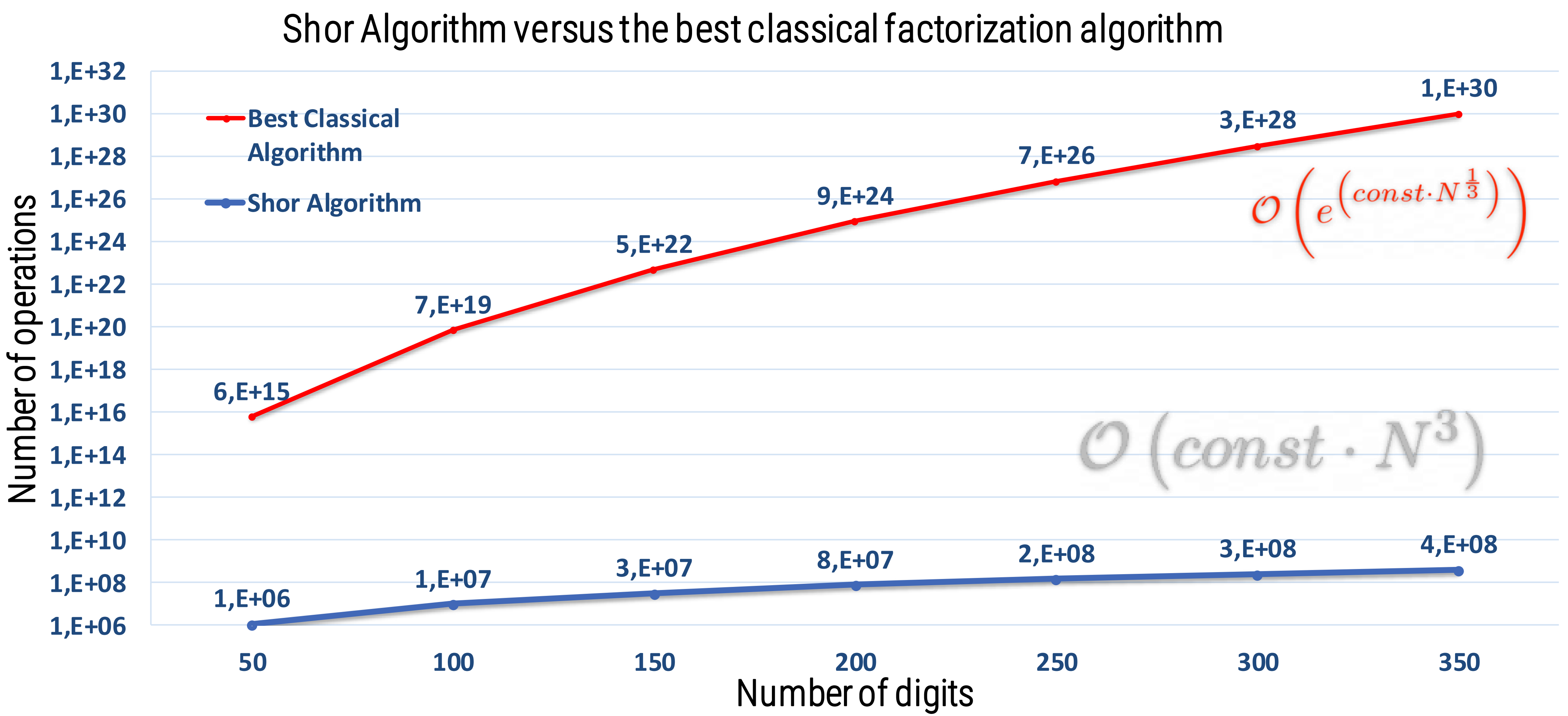


It belongs to the category of quantum algorithms that feature one
EXPONENTIAL SPEED-UP

If Summit could run Shor's algorithm, it would take just 1.5 nanoseconds to factor a number consisting of 300 digits

CINECA QUANTUM COMPUTING LAB

The impact of QC in the world - Shor's algorithm



Quantum computers: State of the Art

- So far we have seen how quantum computing works mathematically.

Quantum computers: State of the Art

- So far we have seen how quantum computing works mathematically.
- We have discovered that it is possible to exploit certain phenomena, in particular quantum parallelism, to build quantum algorithms capable of solving some types of problems exponentially faster.

IBM Q

PASQAL

rigetti

IONQ

D-Wave
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- So far we have seen how quantum computing works mathematically.
- We have discovered that it is possible to exploit certain phenomena, in particular quantum parallelism, to build quantum algorithms capable of solving some types of problems exponentially faster.
- The example of Deutsch's algorithm made us begin to understand the potential of quantum computers

IBM Q

PASQAL

rigetti

IONQ

D:wave
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- So far we have seen how quantum computing works mathematically.
- We have discovered that it is possible to exploit certain phenomena, in particular quantum parallelism, to build quantum algorithms capable of solving some types of problems exponentially faster.
- The example of Deutsch's algorithm made us begin to understand the potential of quantum computers
- We then briefly mentioned the existence of other very powerful quantum algorithms, such as the Shor algorithm

IBM Q

PASQAL

rigetti

IONQ

D:wave
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- So far we have seen how quantum computing works mathematically.
- We have discovered that it is possible to exploit certain phenomena, in particular quantum parallelism, to build quantum algorithms capable of solving some types of problems exponentially faster.
- The example of Deutsch's algorithm made us begin to understand the potential of quantum computers
- We then briefly mentioned the existence of other very powerful quantum algorithms, such as the Shor algorithm
- A question now arises: why today's quantum computers are still not used to implement the most powerful algorithms?

IBM Q

PASQAL

rigetti

IONQ

D:wave
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- Short answer: today's quantum computers are not advanced enough. They are much more like advanced prototypes than production machines

Quantum computers: State of the Art

- Short answer: today's quantum computers are not advanced enough. They are much more like advanced prototypes than production machines
- First of all, current machines don't have enough qubits to implement meaningful problems using the algorithms mentioned above

IBM Q

PASQAL

rigetti

IONQ

D:wave
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- Short answer: today's quantum computers are not advanced enough. They are much more like advanced prototypes than production machines
- First of all, current machines don't have enough qubits to implement meaningful problems using the algorithms mentioned above
- But the number of qubits is not the only problem that limits the use of current quantum computers

IBM Q

PASQAL

rigetti

IONQ

D:WAVE
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- Short answer: today's quantum computers are not advanced enough. They are much more like advanced prototypes than production machines
- First of all, current machines don't have enough qubits to implement meaningful problems using the algorithms mentioned above
- But the number of qubits is not the only problem that limits the use of current quantum computers
- As you can imagine, building a quantum computer is not an easy task. There are many problems to overcome.

IBM Q

PASQAL

rigetti

IONQ

D:wave
The Quantum Computing Company™

XANADU

Quantum computers: State of the Art

- Short answer: today's quantum computers are not advanced enough. They are much more like advanced prototypes than production machines
- First of all, current machines don't have enough qubits to implement meaningful problems using the algorithms mentioned above
- But the number of qubits is not the only problem that limits the use of current quantum computers
- As you can imagine, building a quantum computer is not an easy task. There are many problems to overcome.
- Let's see how to get an idea of the actual power of a quantum computer

IBM Q

PASQAL

rigetti

IONQ

D:Wave The Quantum Computing Company™

XANADU

Building a Quantum Computer

- The first thing that, trivially, we have to look at when we try to evaluate the effectiveness of a real quantum computer is the number of qubits.

- Number of Qubits

Building a Quantum Computer

- The first thing that, trivially, we have to look at when we try to evaluate the effectiveness of a real quantum computer is the number of qubits.
- As we will see shortly, so far the most powerful General Purpose quantum computer models do not exceed one hundred qubits

- Number of Qubits

Building a Quantum Computer

- The first thing that, trivially, we have to look at when we try to evaluate the effectiveness of a real quantum computer is the number of qubits.
- As we will see shortly, so far the most powerful General Purpose quantum computer models do not exceed one hundred qubits
- Few for a universal quantum computer, close in number for other applications

- Number of Qubits

Building a Quantum Computer

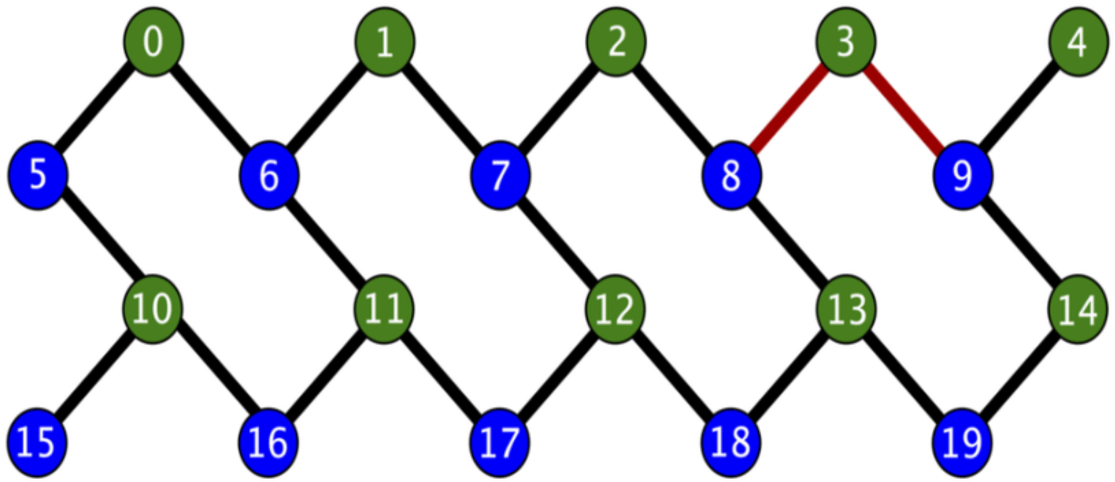
- The first thing that, trivially, we have to look at when we try to evaluate the effectiveness of a real quantum computer is the number of qubits.
- As we will see shortly, so far the most powerful General Purpose quantum computer models do not exceed one hundred qubits
- Few for a universal quantum computer, close in number for other applications
- Very important thing: bigger and bigger models keep coming out. The next milestone, which seems to apply to everyone, is to reach a thousand qubits by 2023

- Number of Qubits

Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.

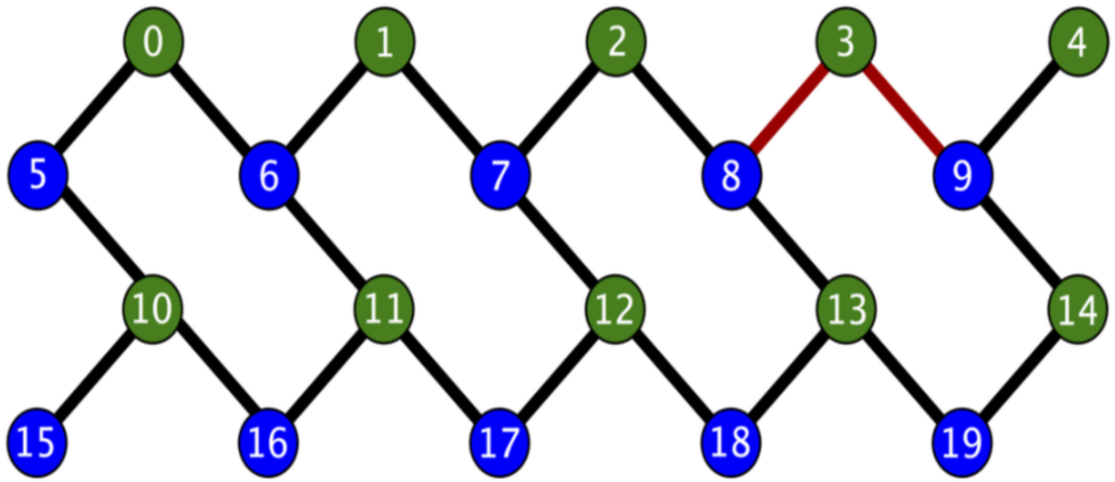
- Number of Qubits
- Connectivity of Qubits



Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.
- In quantum computing as a mathematical science, we never talk about the possibility that two qubits are not connected to each other.

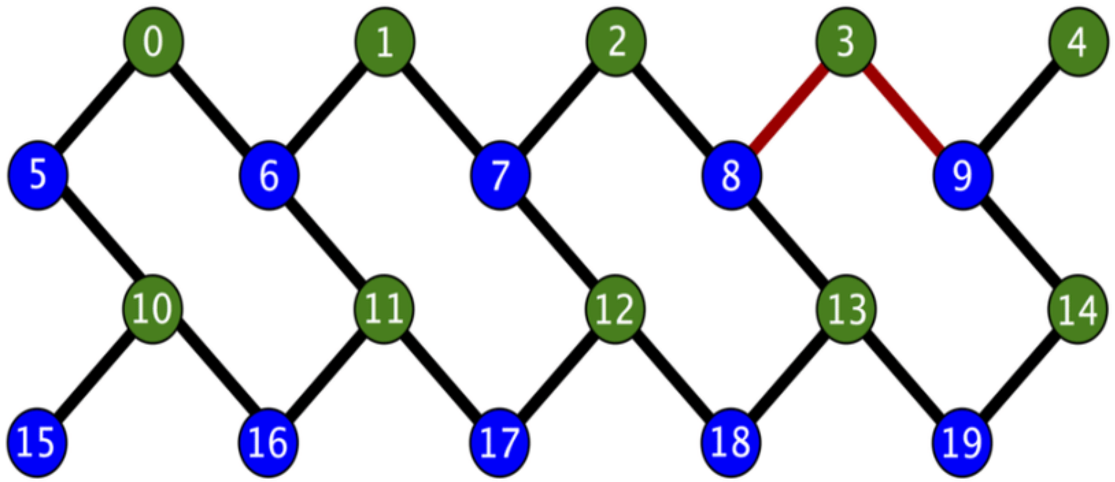
- Number of Qubits
- Connectivity of Qubits



Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.
- In quantum computing as a mathematical science, we never talk about the possibility that two qubits are not connected to each other.
- In practice, if two or more qubits are not connected to each other, it is not possible to perform quantum gates involving them together.

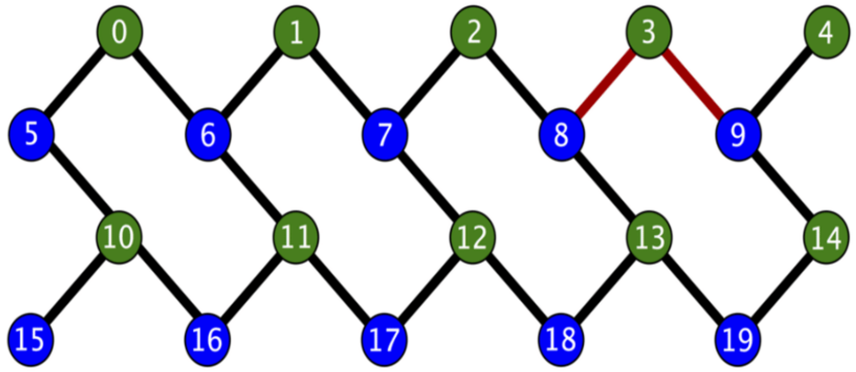
- Number of Qubits
- Connectivity of Qubits

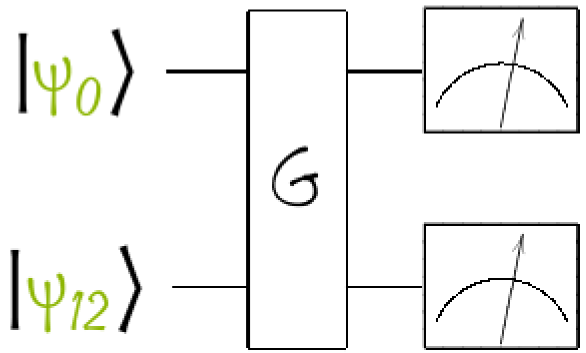


Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.
- In quantum computing as a mathematical science, we never talk about the possibility that two qubits are not connected to each other.
- In practice, if two or more qubits are not connected to each other, it is not possible to perform quantum gates involving them together.
- In these cases, it is necessary to alter the quantum circuit to adapt it to the needs of the machine being used (quantum compilation)

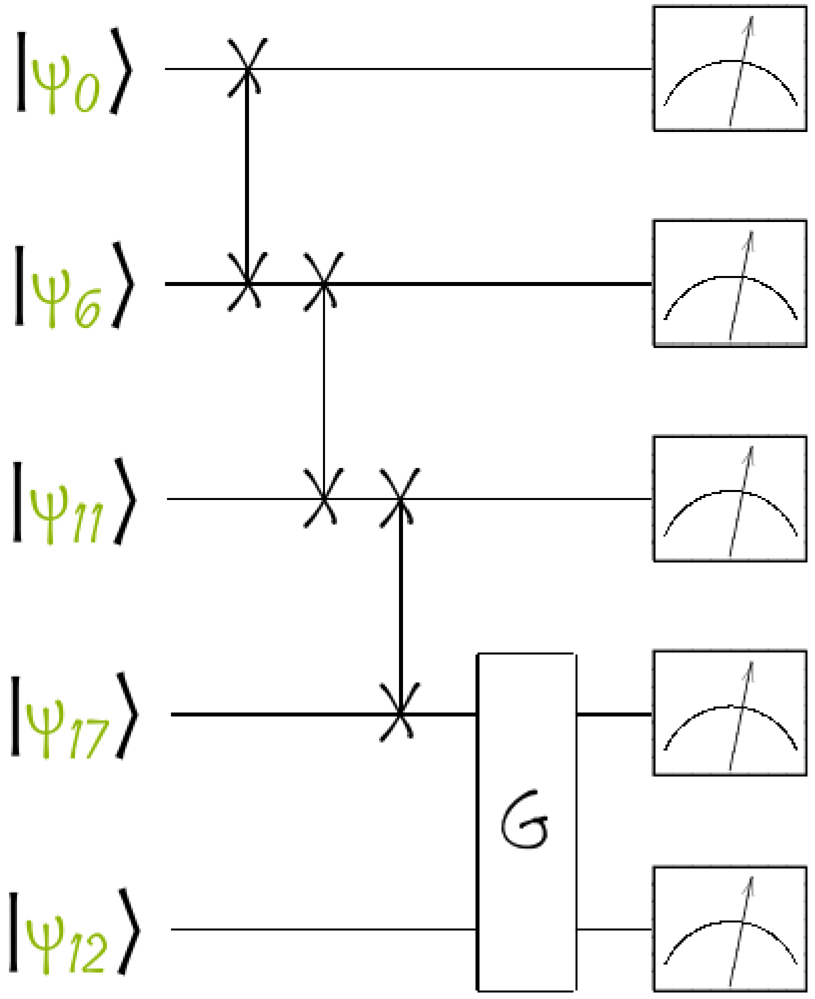
- Number of Qubits
- Connectivity of Qubits





Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.
- In quantum computing as a mathematical science, we never talk about the possibility that two qubits are not connected to each other.
- In practice, if two or more qubits are not connected to each other, it is not possible to perform quantum gates involving them together.
- In these cases, it is necessary to alter the quantum circuit to adapt it to the needs of the machine being used (quantum compilation)



Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.
- In quantum computing as a mathematical science, we never talk about the possibility that two qubits are not connected to each other.
- In practice, if two or more qubits are not connected to each other, it is not possible to perform quantum gates involving them together.
- In these cases, it is necessary to alter the quantum circuit to adapt it to the needs of the machine being used (quantum compilation)

- Number of Qubits
- Connectivity of Qubits

Building a Quantum Computer

- The third problem that we must take into consideration, which is also totally absent on paper, is the problem of the decoherence of the superposition states

- Number of Qubits
- Connectivity of Qubits
- Coherence Time

Building a Quantum Computer

- The third problem that we must take into consideration, which is also totally absent on paper, is the problem of the decoherence of the superposition states
- Quantum superposition states are extremely fragile states.

- Number of Qubits
- Connectivity of Qubits
- Coherence Time

Building a Quantum Computer

- The third problem that we must take into consideration, which is also totally absent on paper, is the problem of the decoherence of the superposition states
- Quantum superposition states are extremely fragile states.
- In order to be maintained for a sufficiently long time it is necessary that the quantum computer is protected from practically any form of noise

- Number of Qubits
- Connectivity of Qubits
- Coherence Time

Building a Quantum Computer

- The third problem that we must take into consideration, which is also totally absent on paper, is the problem of the decoherence of the superposition states
- Quantum superposition states are extremely fragile states.
- In order to be maintained for a sufficiently long time it is necessary that the quantum computer is protected from practically any form of noise
- Even the qubits themselves contribute to producing noise capable of destroying quantum states: this is one of the reasons why it is difficult to make large chipsets.

- Number of Qubits
- Connectivity of Qubits
- Coherence Time

Building a Quantum Computer

- The third problem that we must take into consideration, which is also totally absent on paper, is the problem of the decoherence of the superposition states
- Quantum superposition states are extremely fragile states.
- In order to be maintained for a sufficiently long time it is necessary that the quantum computer is protected from practically any form of noise
- Even the qubits themselves contribute to producing noise capable of destroying quantum states: this is one of the reasons why it is difficult to make large chipsets.
- How can the problem be addressed? In addition to isolating the QPU from the outside world as much as possible, it is necessary to improve the technology of the qubits themselves

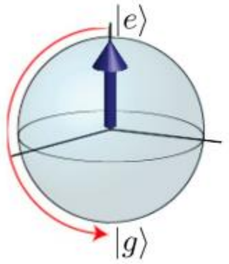
- Number of Qubits
- Connectivity of Qubits
- Coherence Time

Building a Quantum Computer

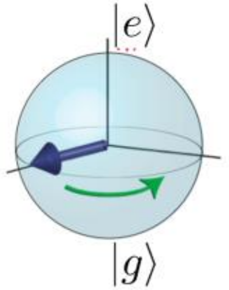
- The third problem that we must take into consideration, which is also totally absent on paper, is the problem of the decoherence of the superposition states
- Quantum superposition states are extremely fragile states.
- In order to be maintained for a sufficiently long time it is necessary that the quantum computer is protected from practically any form of noise
- Even the qubits themselves contribute to producing noise capable of destroying quantum states: this is one of the reasons why it is difficult to make large chipsets.
- How can the problem be addressed? In addition to isolating the QPU from the outside world as much as possible, it is necessary to improve the technology of the qubits themselves

- Number of Qubits
- Connectivity of Qubits
- Coherence Time

- T_1: energy relaxation time



- T_2: dephasing time



Building a Quantum Computer

- The fourth thing to consider is the application of quantum gates to qubits circuits.

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits

Building a Quantum Computer

- The fourth thing to consider is the application of quantum gates to qubits circuits.
- Several techniques are used to physically build a quantum gate

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits

Building a Quantum Computer

- The fourth thing to consider is the application of quantum gates to qubits circuits.
- Several techniques are used to physically build a quantum gate
- What must be taken into account when thinking about a real quantum computer is that not all quantum gates may be available for a certain hardware.

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits
  - Not every gate is available

Building a Quantum Computer

- The fourth thing to consider is the application of quantum gates to qubits circuits.
- Several techniques are used to physically build a quantum gate
- What must be taken into account when thinking about a real quantum computer is that not all quantum gates may be available for a certain hardware.
- In addition, they have an application time that affects the maximum time available to complete a circuit

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits
  - Not every gate is available
  - Not instantaneous

Building a Quantum Computer

- The fourth thing to consider is the application of quantum gates to qubits circuits.
- Several techniques are used to physically build a quantum gate
- What must be taken into account when thinking about a real quantum computer is that not all quantum gates may be available for a certain hardware.
- In addition, they have an application time that affects the maximum time available to complete a circuit
- And remember: the time available to complete a quantum circuit is not infinite, quite the opposite. The quantum computer works as long as it can maintain its superposition state

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits
    - Not every gate is available
    - Not instantaneous

Building a Quantum Computer

- Last but not least, the applications of quantum gates are not free from errors

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits
  - Not every gate is available
  - Not instantaneous
  - Errors on applications (gate fidelity)

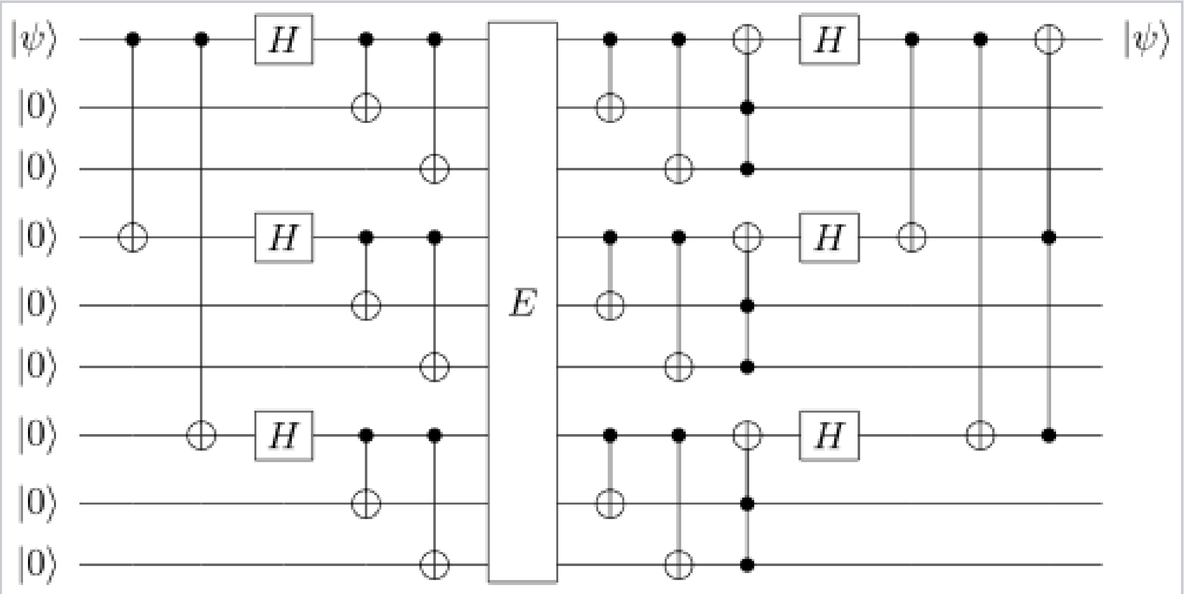
Building a Quantum Computer

- Last but not least, the applications of quantum gates are not free from errors
- This is a very important problem: to have a reliable quantum computer, the error percentage on the application of the gates must be zero

- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits
  - Not every gate is available
  - Not instantaneous
  - Errors on applications (gate fidelity)

Building a Quantum Computer

- Last but not least, the applications of quantum gates are not free from errors
- This is a very important problem: to have a reliable quantum computer, the error percentage on the application of the gates must be zero
- This is not true in reality, unfortunately. There are error mitigation techniques but they require an exaggerated number of additional qubits per hour (example of Shor's error correction code)



Circuito quantistico del codice Shor

Building a Quantum Computer

- On May 14, 2021, Google made public the technical characteristics of its latest quantum computer, Sycamore

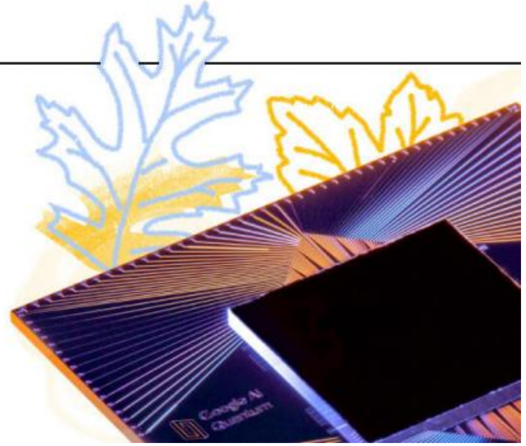
Quantum AI

Quantum Computer Datasheet

Published May 14, 2021

Description

At the core of Google's Quantum Computing Service is the Sycamore processor. It has up to 54 superconducting qubits in a square grid lattice suitable for general Noisy Intermediate Scale Quantum (NISQ) algorithms like Hartree-Fock (chemistry), QAOA (optimization), and machine learning. Standard single- and two-qubit gates are



Building a Quantum Computer

- On May 14, 2021, Google made public the technical characteristics of its latest quantum computer, Sycamore
- Sycamore is a fairly important machine in the history of quantum computers: it was the first computer to achieve an early form of quantum supremacy (October 2019)

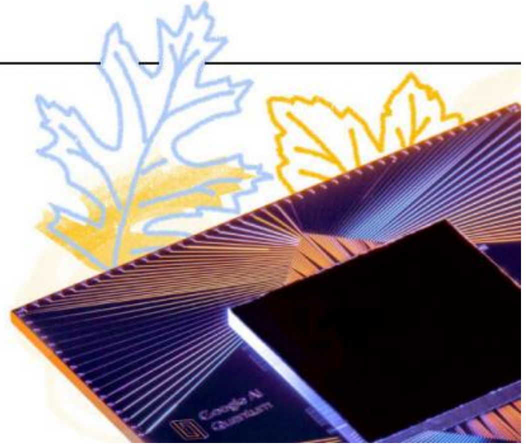
Quantum AI

Quantum Computer Datasheet

Published May 14, 2021

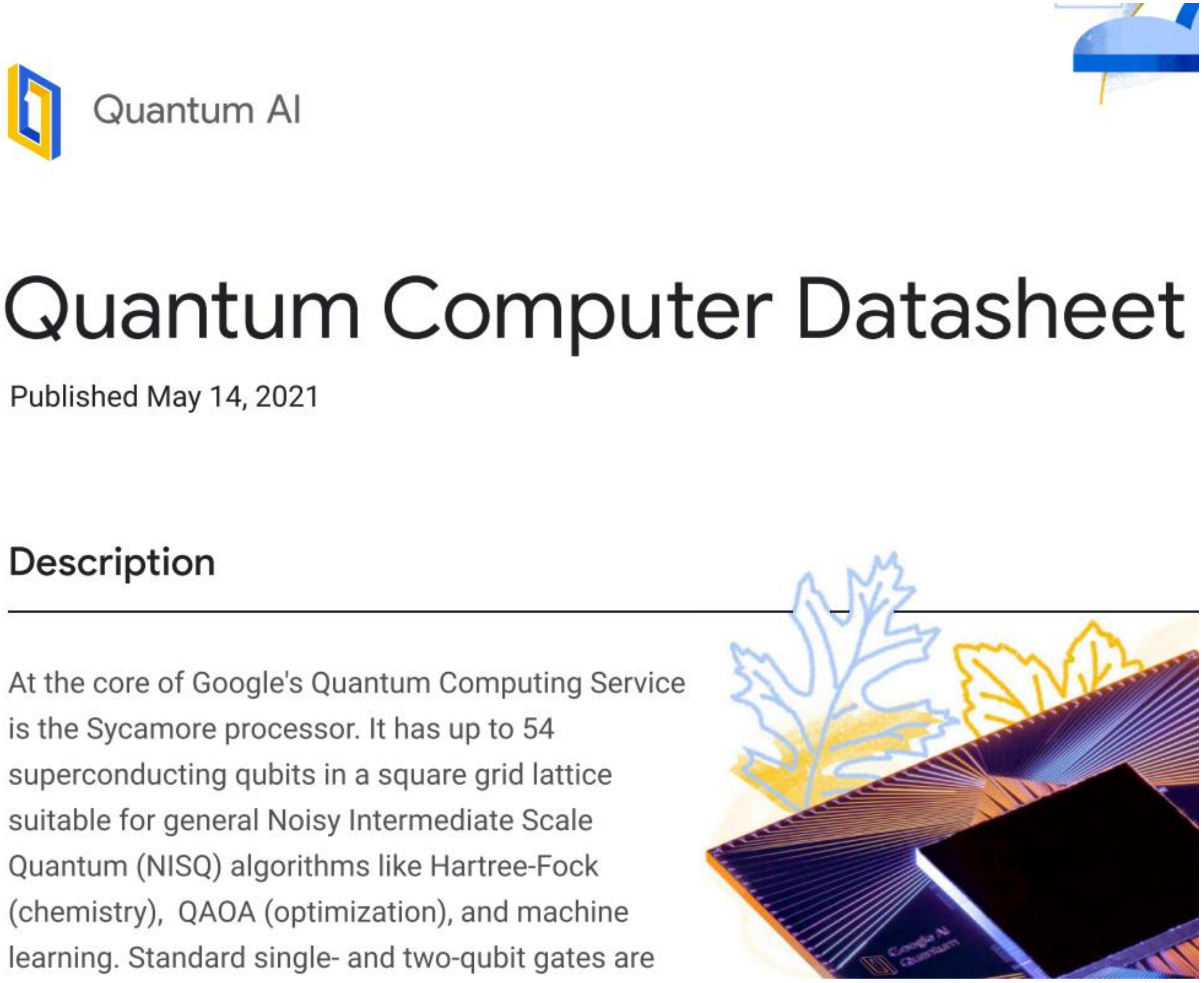
Description

At the core of Google's Quantum Computing Service is the Sycamore processor. It has up to 54 superconducting qubits in a square grid lattice suitable for general Noisy Intermediate Scale Quantum (NISQ) algorithms like Hartree-Fock (chemistry), QAOA (optimization), and machine learning. Standard single- and two-qubit gates are



Building a Quantum Computer

- On May 14, 2021, Google made public the technical characteristics of its latest quantum computer, Sycamore
- Sycamore is a fairly important machine in the history of quantum computers: it was the first computer to achieve an early form of quantum supremacy (October 2019)
- Warning: this does not mean that Sycamore is more powerful than any supercomputer in the world


Quantum AI

Quantum Computer Datasheet

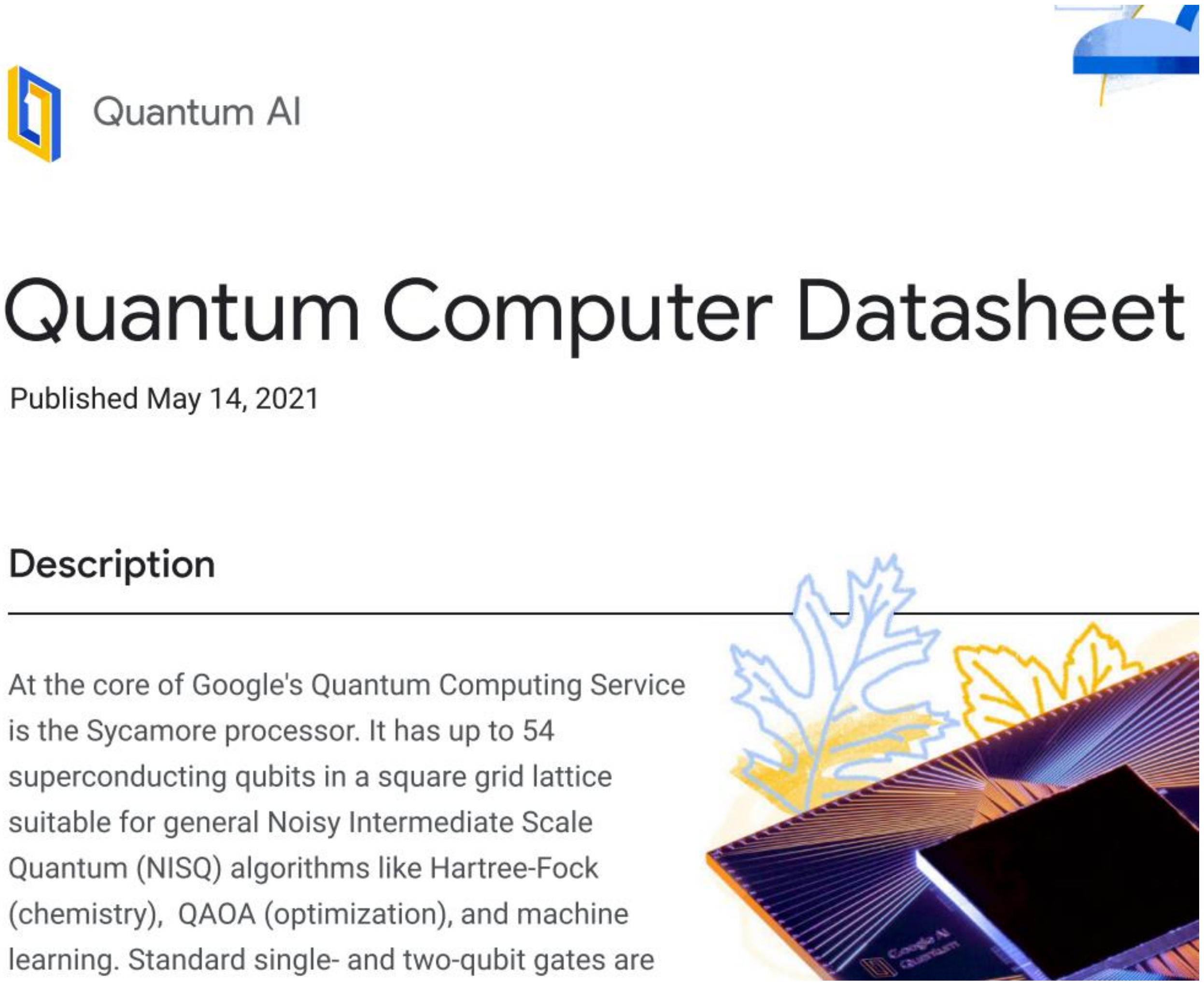
Published May 14, 2021

Description

At the core of Google's Quantum Computing Service is the Sycamore processor. It has up to 54 superconducting qubits in a square grid lattice suitable for general Noisy Intermediate Scale Quantum (NISQ) algorithms like Hartree-Fock (chemistry), QAOA (optimization), and machine learning. Standard single- and two-qubit gates are

Building a Quantum Computer

- On May 14, 2021, Google made public the technical characteristics of its latest quantum computer, Sycamore
- Sycamore is a fairly important machine in the history of quantum computers: it was the first computer to achieve an early form of quantum supremacy (October 2019)
- Warning: this does not mean that Sycamore is more powerful than any supercomputer in the world
- The quantum supremacy achieved by the Google device is related to the implementation of a particular sampling algorithm: in the implementation of this particular algorithm, Sycamore also beat Summit


Quantum AI

Quantum Computer Datasheet

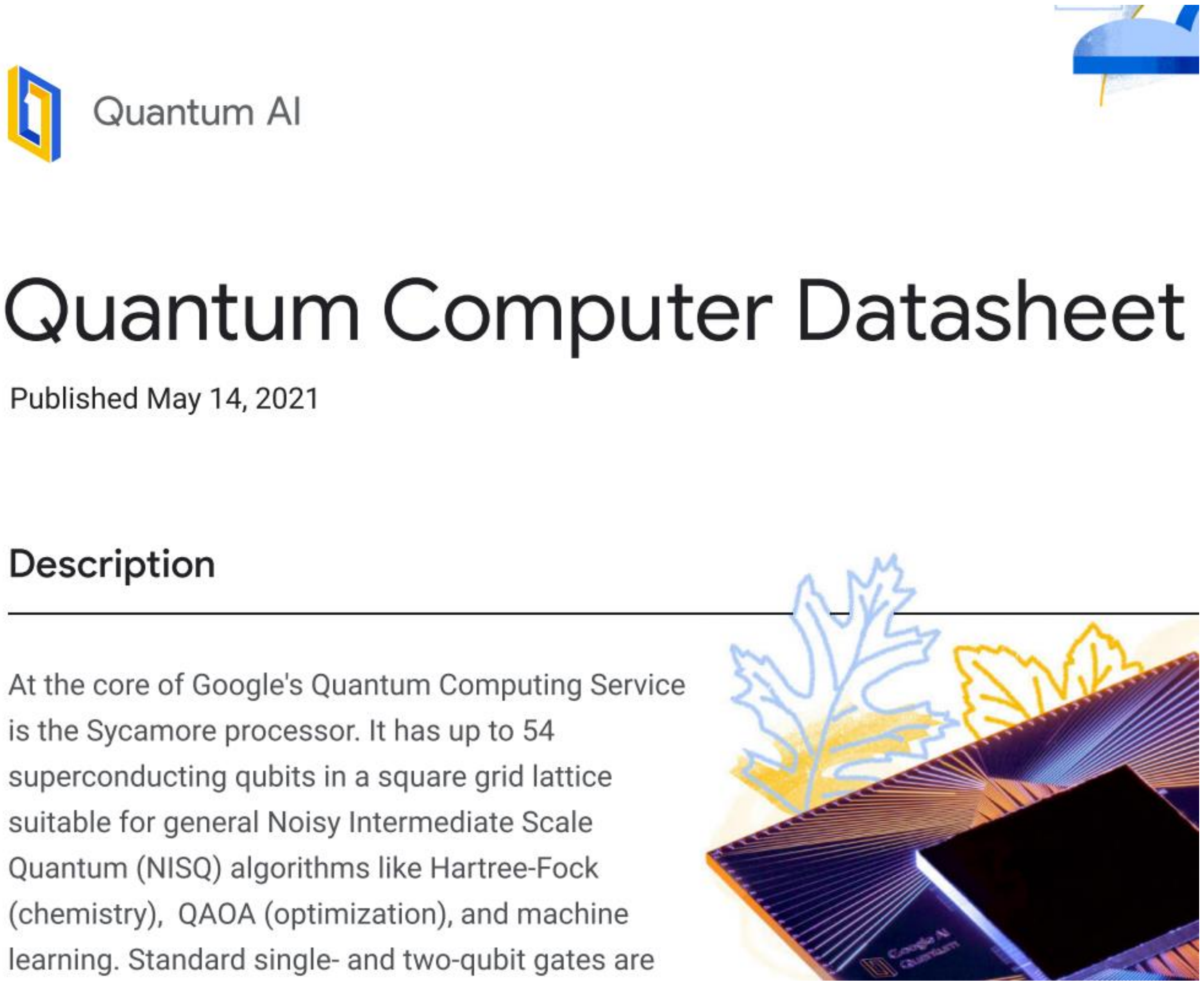
Published May 14, 2021

Description

At the core of Google's Quantum Computing Service is the Sycamore processor. It has up to 54 superconducting qubits in a square grid lattice suitable for general Noisy Intermediate Scale Quantum (NISQ) algorithms like Hartree-Fock (chemistry), QAOA (optimization), and machine learning. Standard single- and two-qubit gates are

Building a Quantum Computer

- On May 14, 2021, Google made public the technical characteristics of its latest quantum computer, Sycamore
- Sycamore is a fairly important machine in the history of quantum computers: it was the first computer to achieve an early form of quantum supremacy (October 2019)
- Warning: this does not mean that Sycamore is more powerful than any supercomputer in the world
- The quantum supremacy achieved by the Google device is related to the implementation of a particular sampling algorithm: in the implementation of this particular algorithm, Sycamore also beat Summit
- Let's take a look at the power of Sycamore


Quantum AI

Quantum Computer Datasheet

Published May 14, 2021

Description

At the core of Google's Quantum Computing Service is the Sycamore processor. It has up to 54 superconducting qubits in a square grid lattice suitable for general Noisy Intermediate Scale Quantum (NISQ) algorithms like Hartree-Fock (chemistry), QAOA (optimization), and machine learning. Standard single- and two-qubit gates are

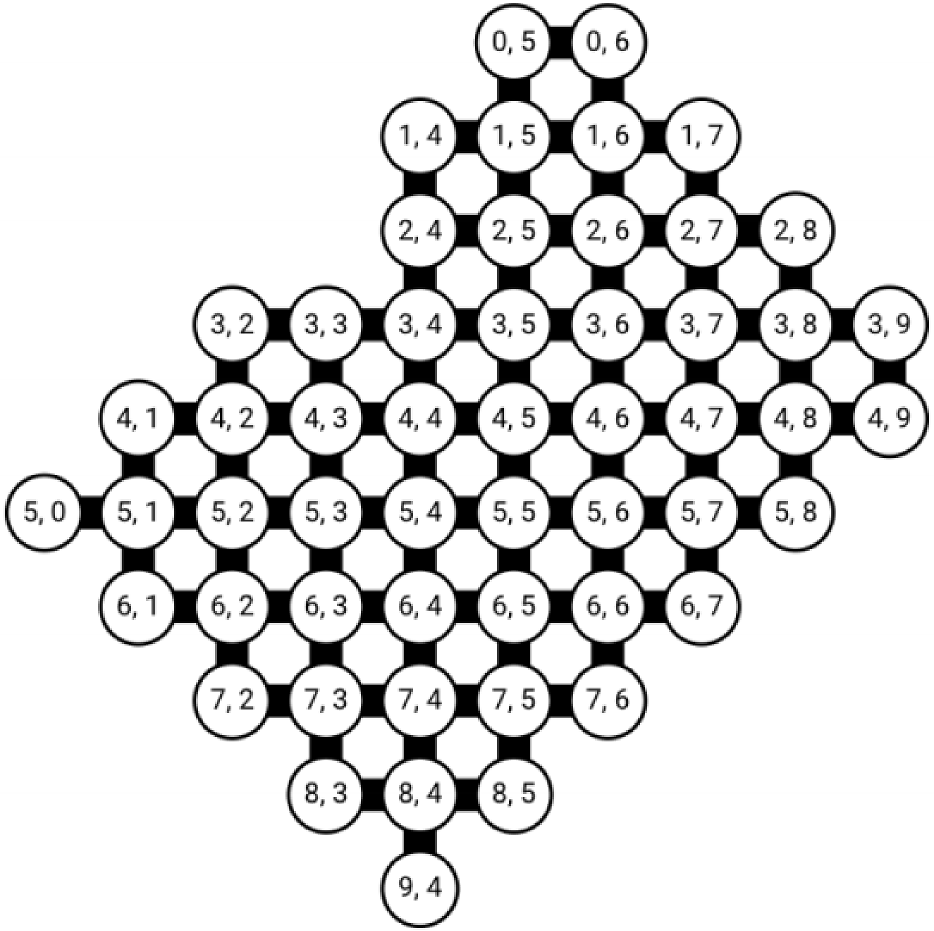
Weber Quantum Computer

Qubit Grid

processor_id	Weber
Family	Sycamore
Supported two-qubit gates	√iSWAP, Sycamore
Number of qubits in the grid	53

Qubit Layout

Example grid for Weber. Use QCS Console for up-to-date layout.



Building a Quantum Computer

Qubit Operations

Type	Gate	Duration[1]	Matrix
Single Qubit Gates	Phased XZ	25 ns	$\begin{bmatrix} \cos(\pi x/2) & -i\sin(\pi x/2)e^{i\pi a} \\ -i\sin(\pi x/2)e^{i\pi(a+z)} & \cos(\pi x/2)e^{i\pi z} \end{bmatrix}$
	Virtual Z	0 ns	$\begin{bmatrix} 1 & 0 \\ 0 & e^{(i\pi t)} \end{bmatrix}$
	Physical Z	20 ns	$\begin{bmatrix} 1 & 0 \\ 0 & e^{(i\pi t)} \end{bmatrix}$
Two Qubit Gates	Sycamore	12 ns	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & e^{(-i\pi/6)} \end{bmatrix}$
	√iSWAP	32 ns	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm 1/\sqrt{2} & \pm i/\sqrt{2} & 0 \\ 0 & \pm i/\sqrt{2} & \pm 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
	CZ	In development	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Performance

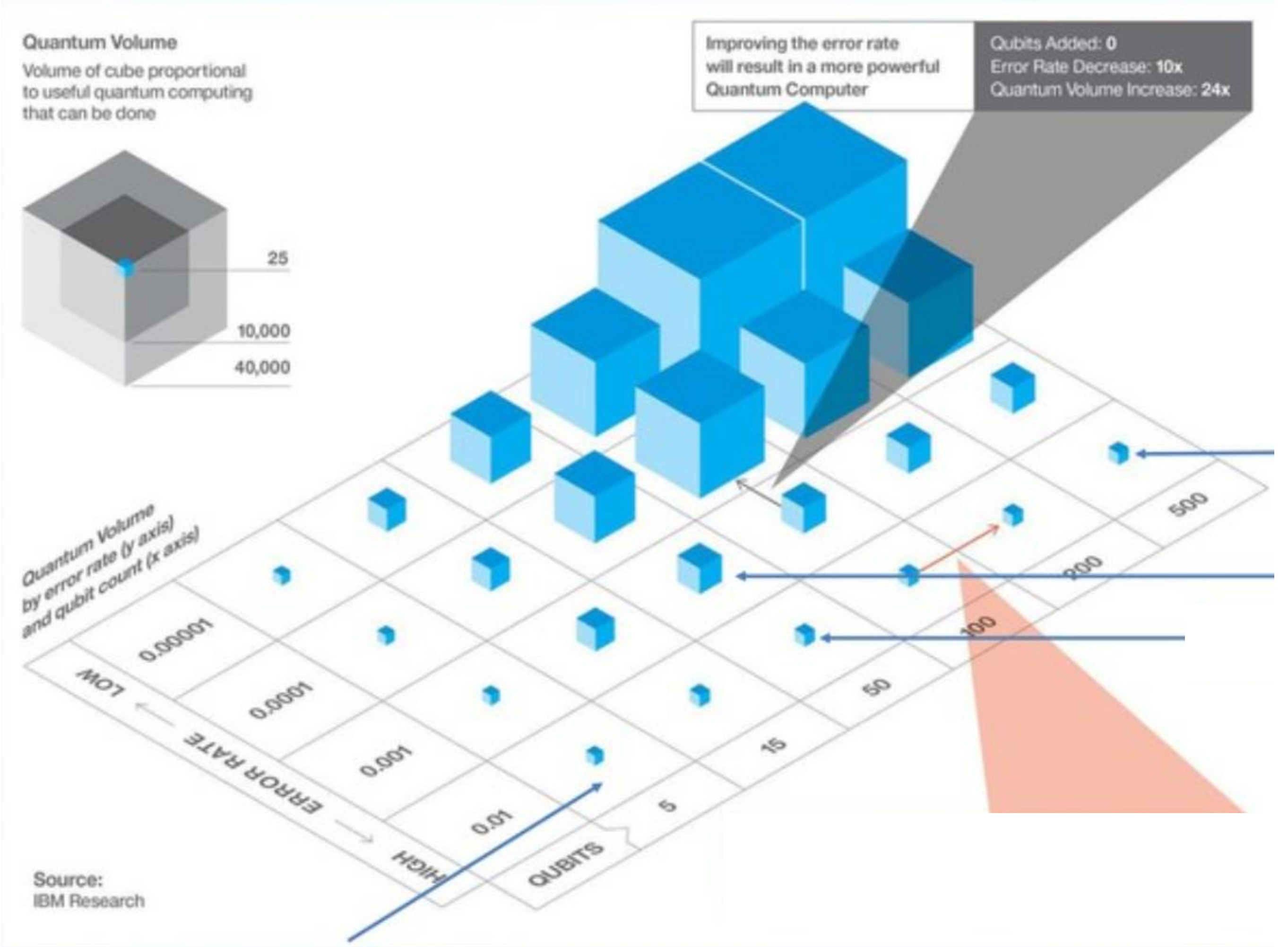
Metric	Symbol	Condition	Low[1]	Typ[2]	High[3]	Units	Description
Single-qubit gate error rate	e_1	Isolated	0.1	0.1	0.2	% error per gate	Randomized benchmarking
Two-qubit gate error rate (√iSWAP)	e_2 (√iSWAP)	Isolated	0.7	0.9	1.9	% error per gate	Cross-entropy benchmarking (XEB)
		Parallel	0.8	1.4	3.3	% error per gate	Cross-entropy benchmarking

Building a Quantum Computer

Readout error |0⟩	e_{r0}	Isolated	0.5	1.1	2.6	% error	Confusion matrix: prepare |0⟩ and observe |1⟩; includes state prep error
		Simultaneous	1	2	3	% error	Confusion matrix: prepare |0⟩ and observe |1⟩; includes state prep error
Readout error |1⟩	e_{r1}	Isolated	3	5	9	% error	Confusion matrix: prepare |1⟩ and observe |0⟩; includes state prep error
		Simultaneous	3	7	9	% error	Confusion matrix: prepare |1⟩ and observe |0⟩; includes state prep error
Relaxation	T_1	Isolated	11	15	21	μs	Direct measurement of |1⟩ population relaxation

Quantum Volume

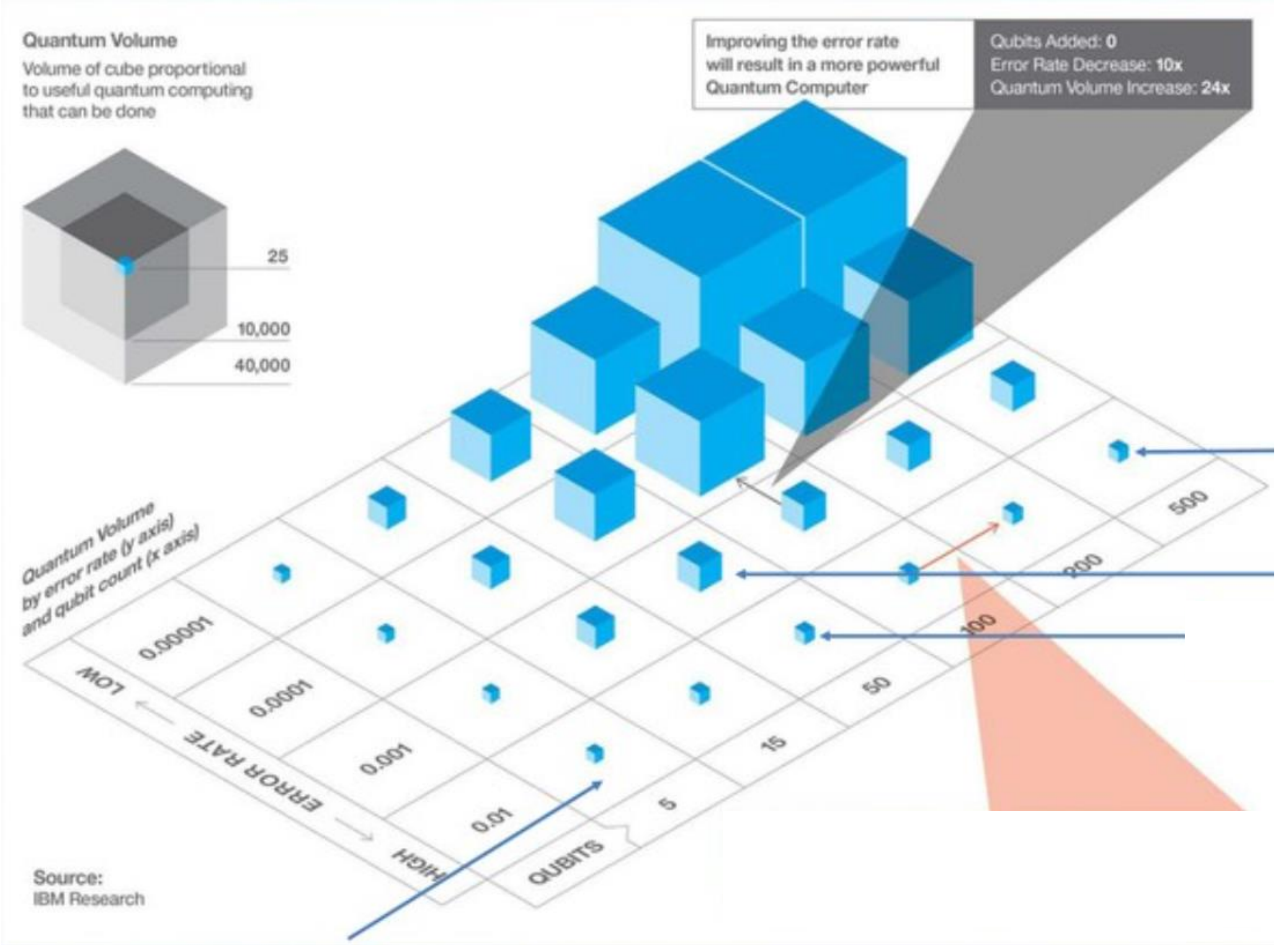
- Quantum Volume is a metric that can be used to express the power of a quantum computer.



CINECA QUANTUM COMPUTING LAB

Quantum Volume

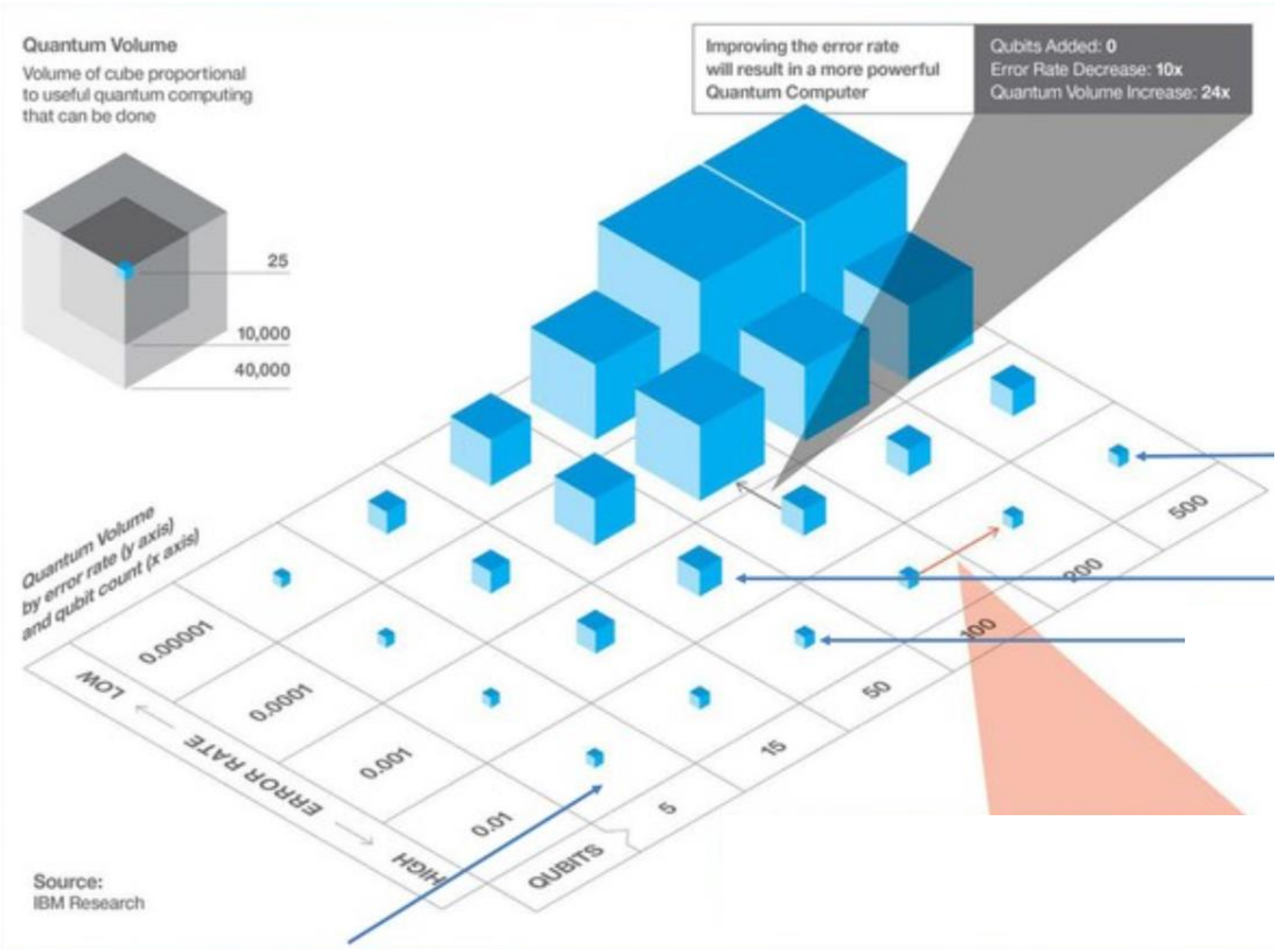
- Quantum Volume is a metric that can be used to express the power of a quantum computer.
- Developed by IBM research, try to enclose all the key characteristics of a quantum computer in a single value



CINECA QUANTUM COMPUTING LAB

Quantum Volume

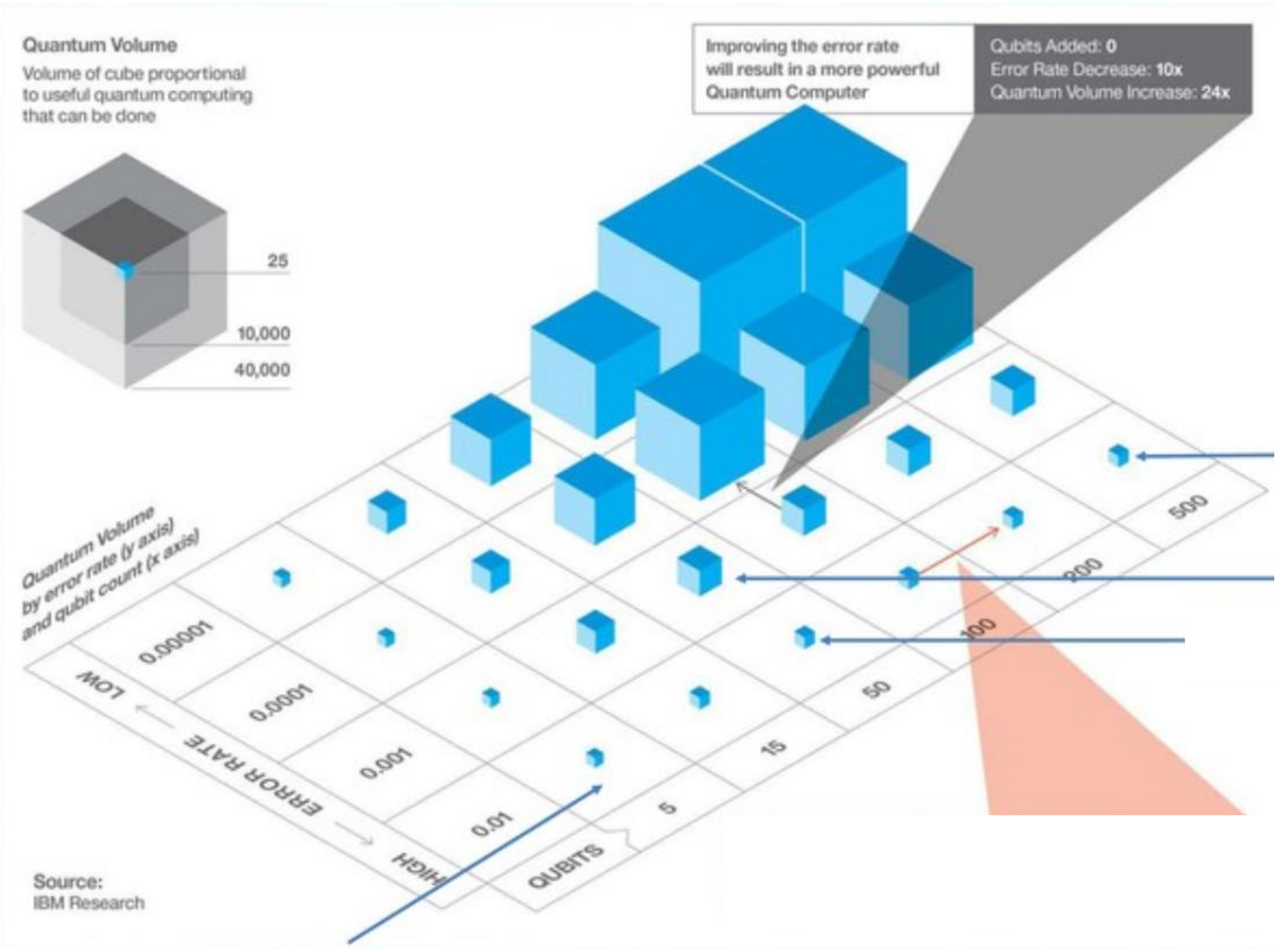
- Quantum Volume is a metric that can be used to express the power of a quantum computer.
- Developed by IBM research, try to enclose all the key characteristics of a quantum computer in a single value
- In particular, it takes into account connectivity, number of qubits and error rates.



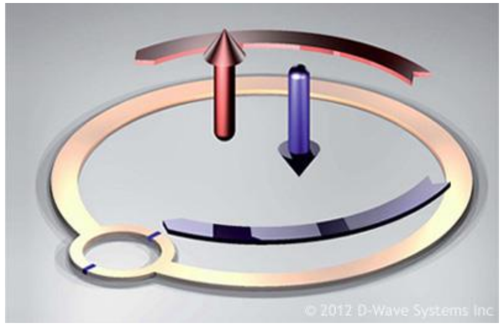
Quantum Volume

- Quantum Volume is a metric that can be used to express the power of a quantum computer.
- Developed by IBM research, try to enclose all the key characteristics of a quantum computer in a single value
- In particular, it takes into account connectivity, number of qubits and error rates.
- It is not intuitive to calculate

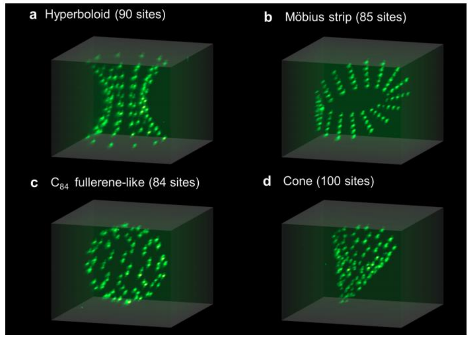
$$\log_2 V_Q = \underset{n \leq N}{\arg\max} \{\min [n, d(n)]\}$$



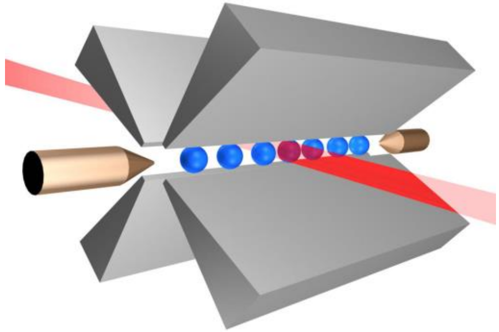
Making a Qubit



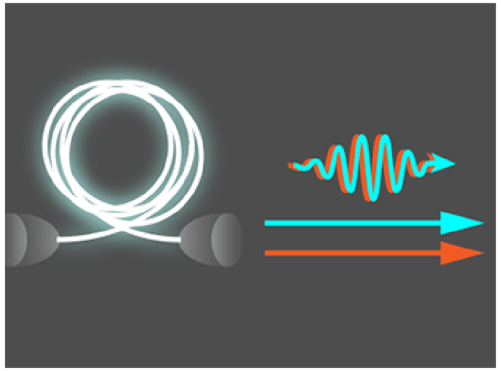
Superconducting



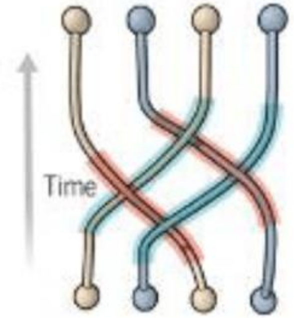
Neutral Atoms



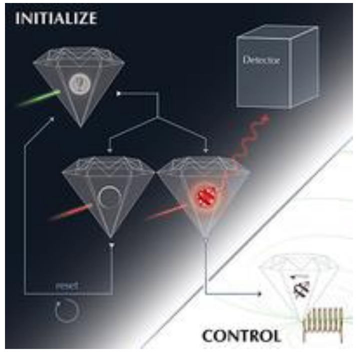
Trapped-Ions



Photonic



Topological



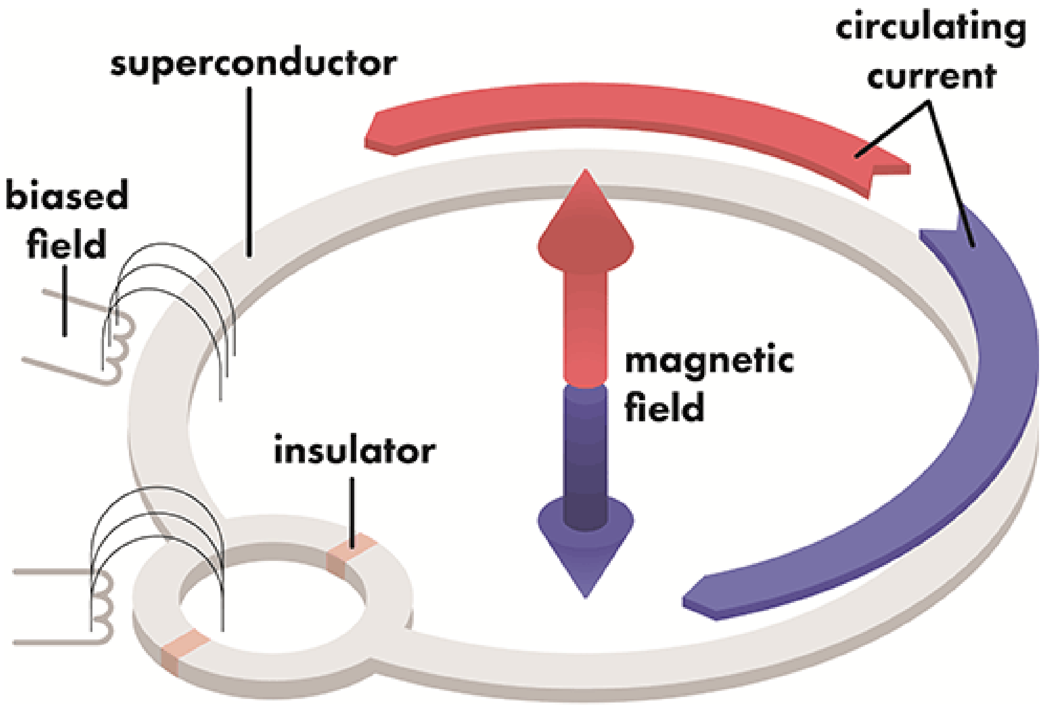
Diamond Center

Superconducting Qubits Technology

- The first technology we are going to analyze is also the most widespread

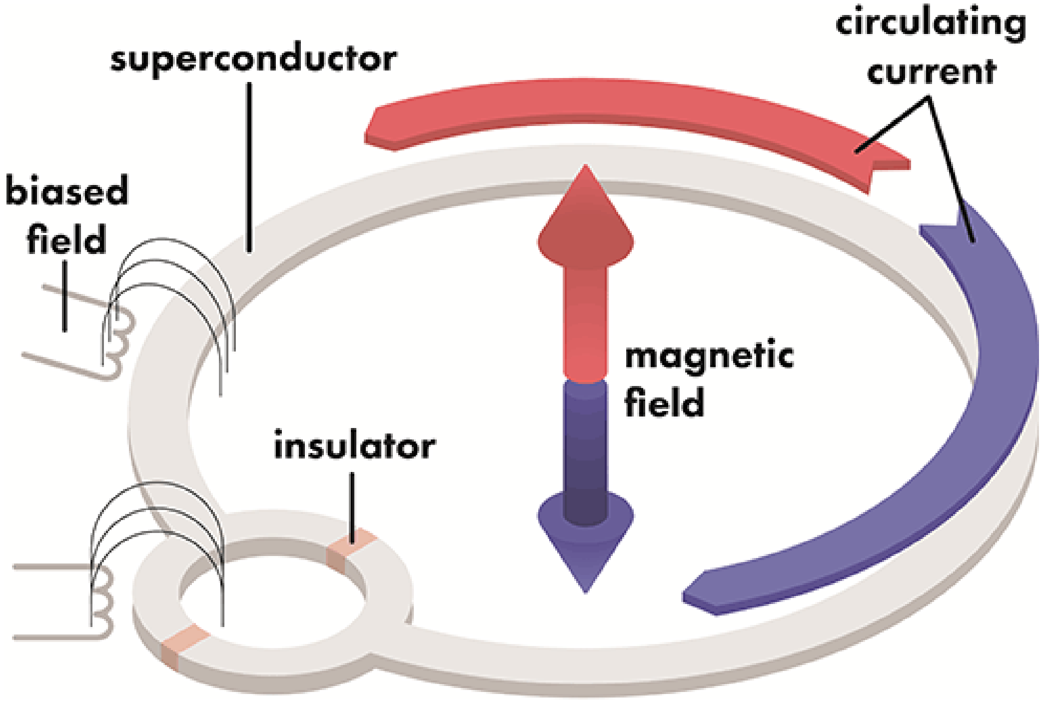
Superconducting Qubits Technology

- The first technology we are going to analyze is also the most widespread
- Superconducting qubits are rings of superconducting material crossed by electric current



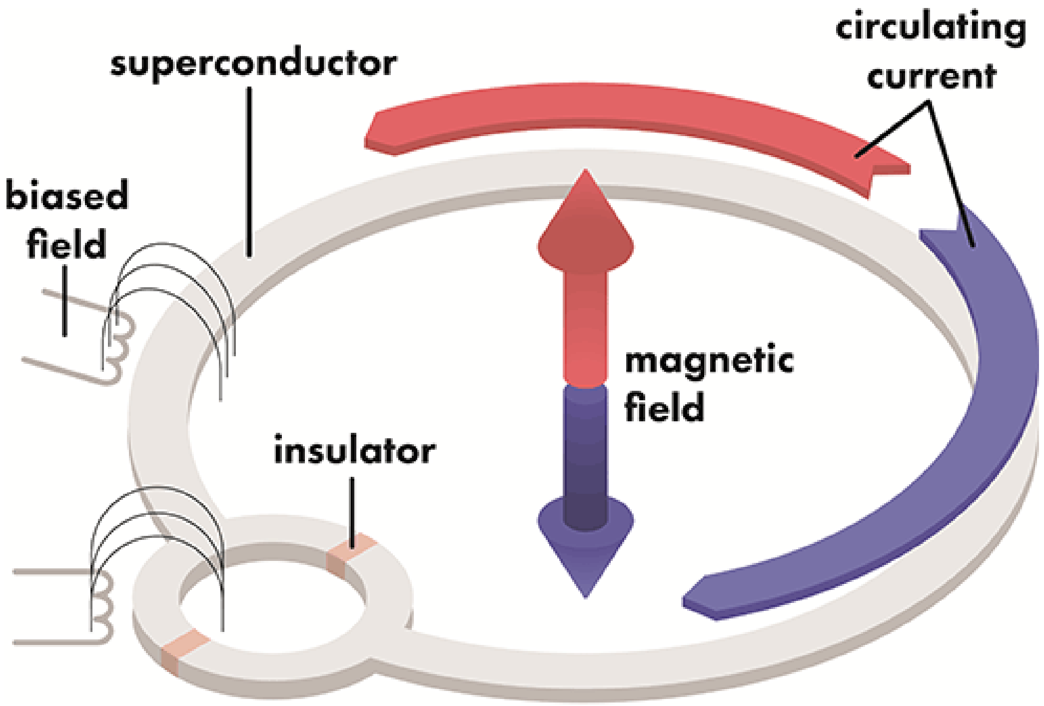
Superconducting Qubits Technology

- The first technology we are going to analyze is also the most widespread
- Superconducting qubits are rings of superconducting material crossed by electric current
- States 0 and 1 are identified with the flow of the electric current



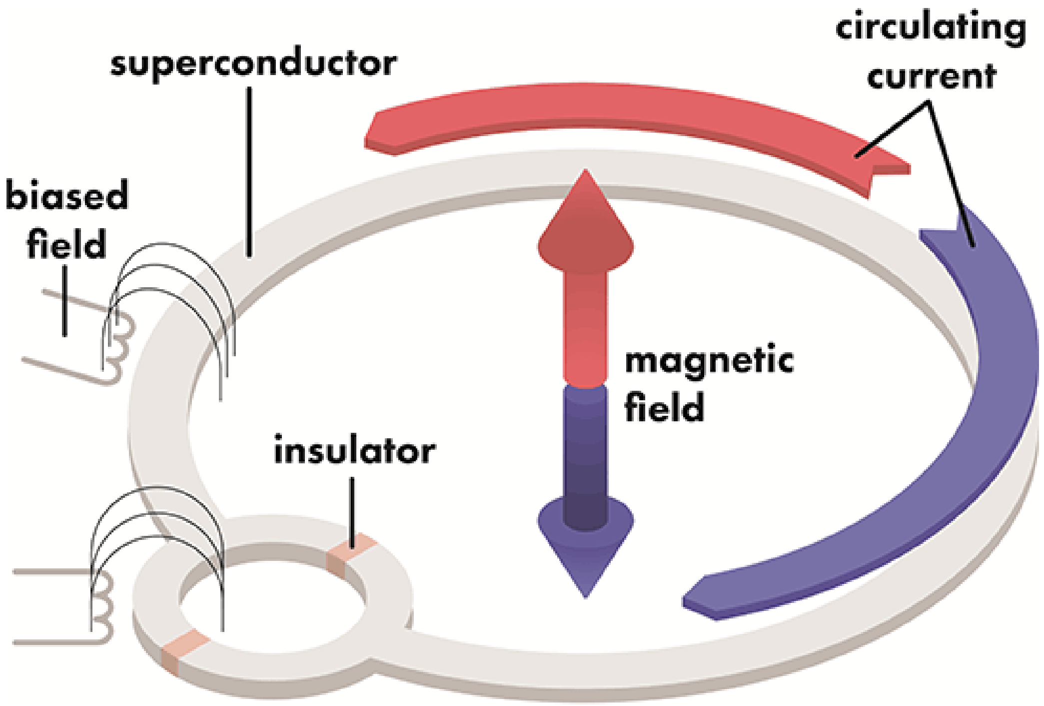
Superconducting Qubits Technology

- The first technology we are going to analyze is also the most widespread
- Superconducting qubits are rings of superconducting material crossed by electric current
- States 0 and 1 are identified with the flow of the electric current
- The material with which they are made, Niobium, is a superconducting material which, if cooled to temperatures close to absolute 0 (Kelvin), become able to be crossed by electric current in both directions at the same time.



Superconducting Qubits Technology

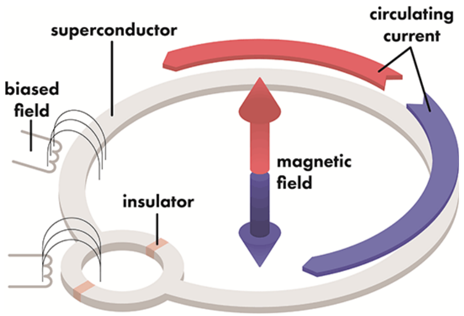
- The first technology we are going to analyze is also the most widespread
- Superconducting qubits are rings of superconducting material crossed by electric current
- States 0 and 1 are identified with the flow of the electric current
- The material with which they are made, Niobium, is a superconducting material which, if cooled to temperatures close to absolute 0 (Kelvin), become able to be crossed by electric current in both directions at the same time.
- In this way, the superconducting qubit is able to reproduce superposition effects

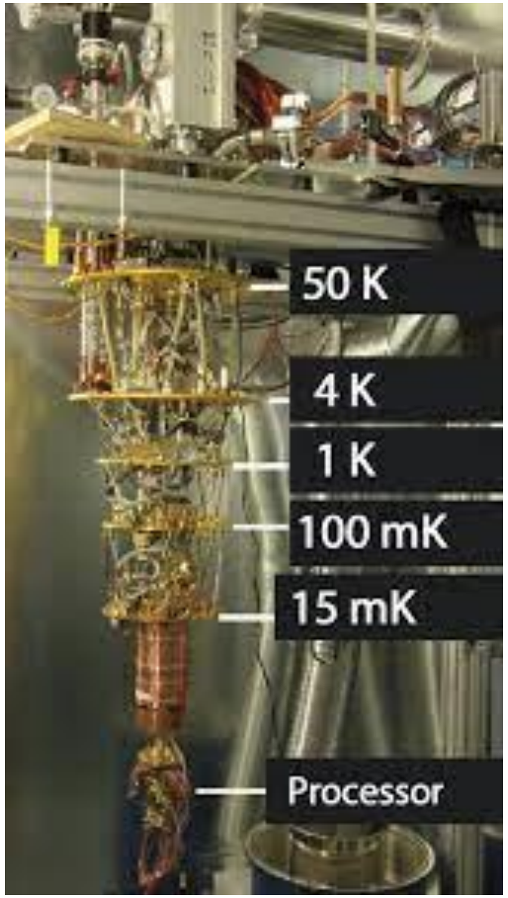


Superconducting Qubits Technology

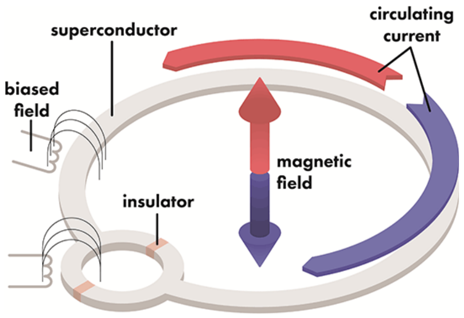
IBM Q

- IBM Quantum Division: American Big Tech, already owner of the most powerful supercomputer in the world (Summit), IBM was one of the first large companies to seriously embark on the development of quantum computers
- In 2016 they created *ibmquantumexperience*, a website where anybody can use their smallest quantum computers, made available for free to the public.
- To date, all models with 5 and 16 qubits can be used for free
- Their largest computer is 65 qubits
- They have a very aggressive roadmap: they predict 1000 qubits in 2023

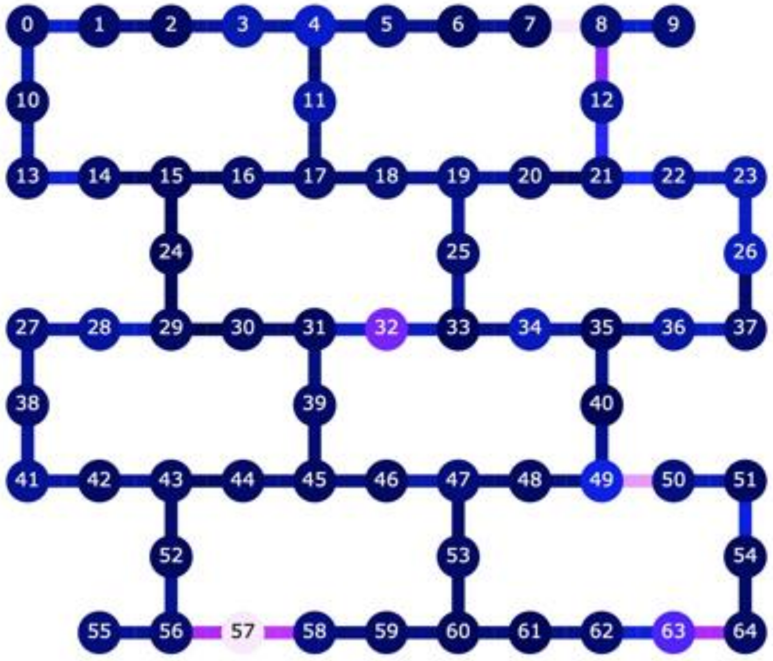


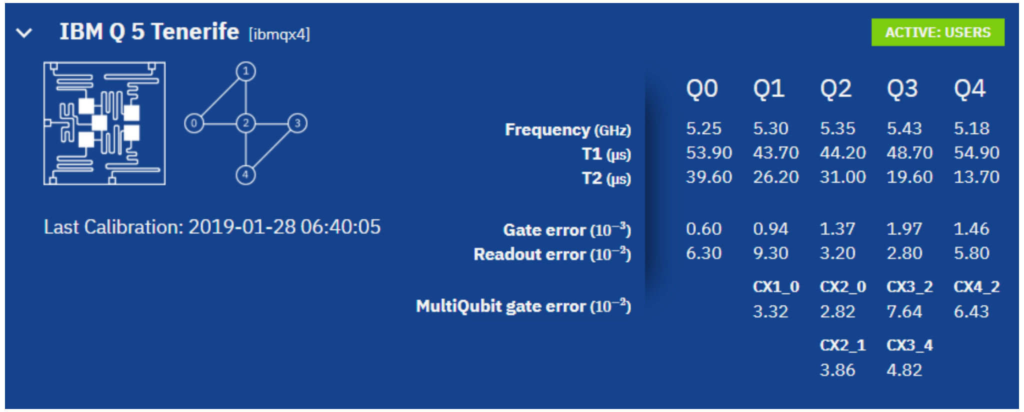


Superconducting Qubits Technology

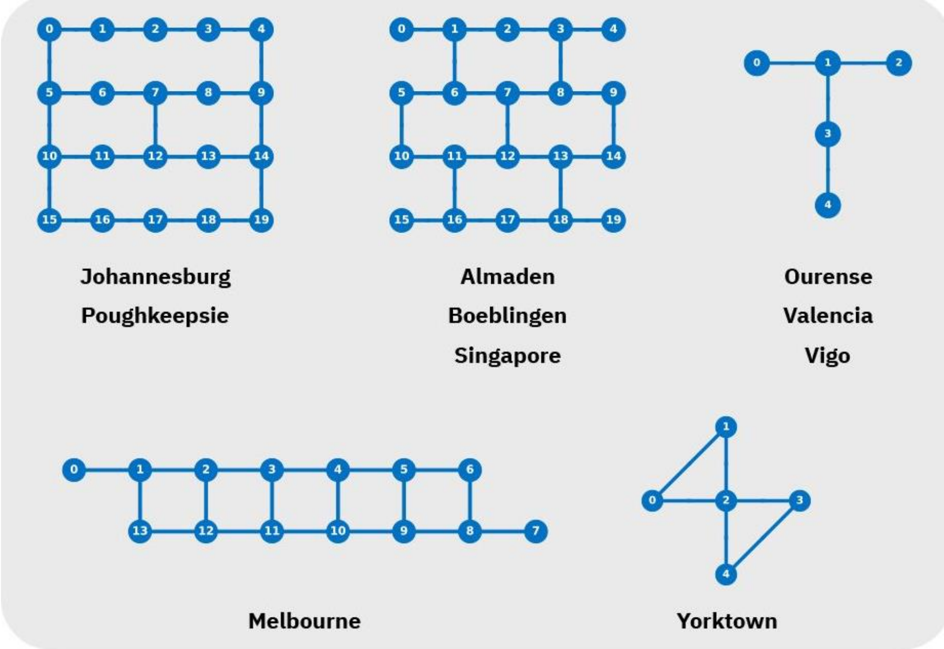


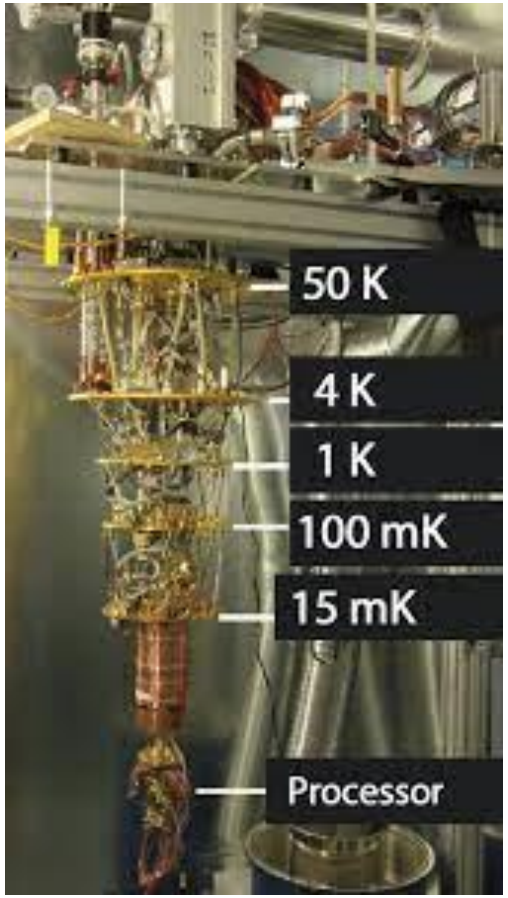
IBM Q



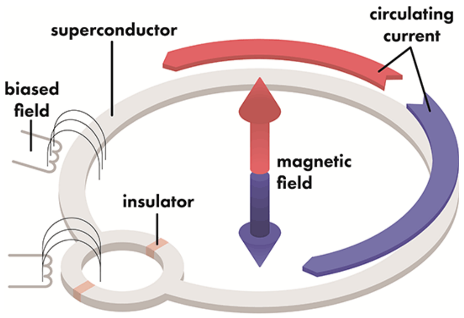


	Q0	Q1	Q2	Q3	Q4
Frequency (GHz)	5.25	5.30	5.35	5.43	5.18
T1 (µs)	53.90	43.70	44.20	48.70	54.90
T2 (µs)	39.60	26.20	31.00	19.60	13.70
Gate error (10^{-3})	0.60	0.94	1.37	1.97	1.46
Readout error (10^{-2})	6.30	9.30	3.20	2.80	5.80
		CX1_0	CX2_0	CX3_2	CX4_2
MultiQubit gate error (10^{-2})		3.32	2.82	7.64	6.43
			CX2_1	CX3_4	
			3.86	4.82	





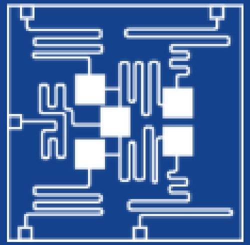
Superconducting Qubits Technology

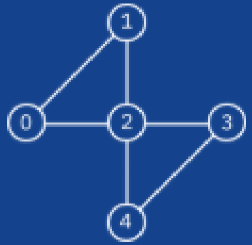


IBM Q

IBM Q 5 Tenerife [ibmqx4]

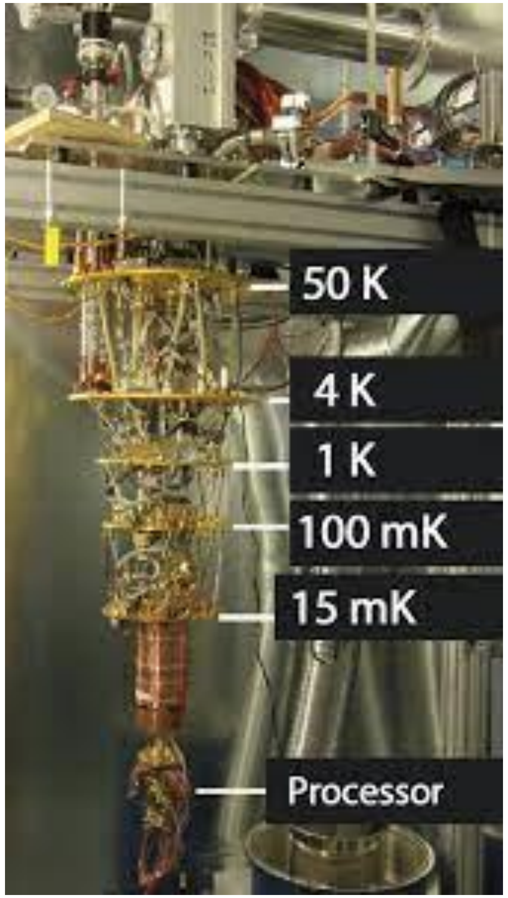
ACTIVE: USERS



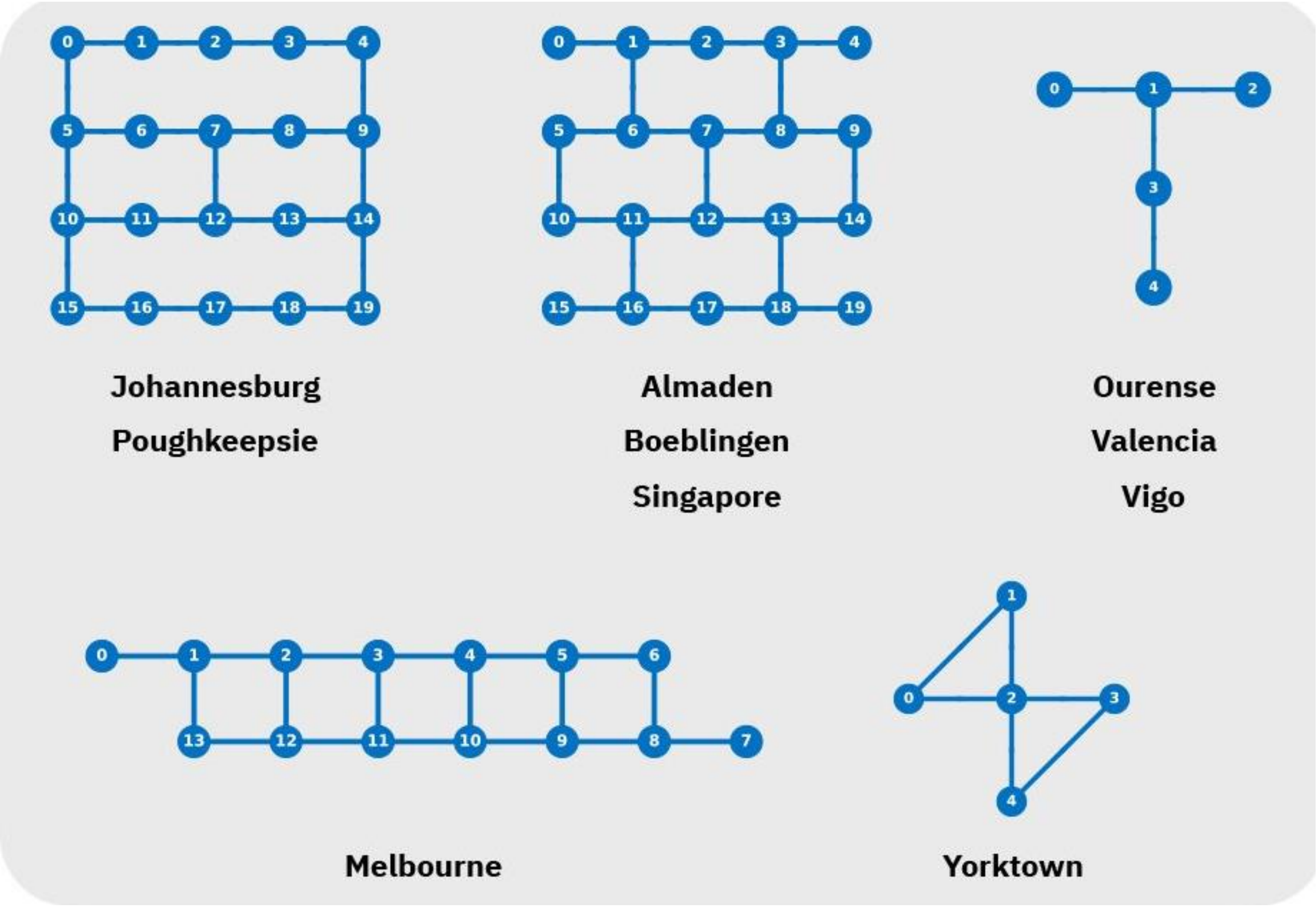


Last Calibration: 2019-01-28 06:40:05

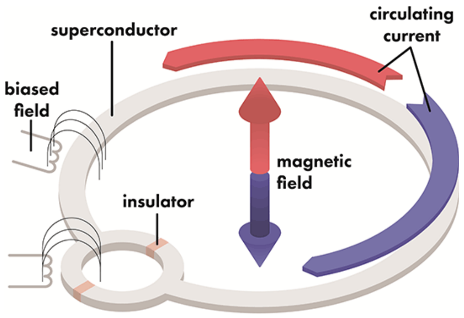
	Q0	Q1	Q2	Q3	Q4
Frequency (GHz)	5.25	5.30	5.35	5.43	5.18
T1 (µs)	53.90	43.70	44.20	48.70	54.90
T2 (µs)	39.60	26.20	31.00	19.60	13.70
Gate error (10^{-3})	0.60	0.94	1.37	1.97	1.46
Readout error (10^{-2})	6.30	9.30	3.20	2.80	5.80
		CX1_0	CX2_0	CX3_2	CX4_2
MultiQubit gate error (10^{-2})		3.32	2.82	7.64	6.43
			CX2_1	CX3_4	
			3.86	4.82	

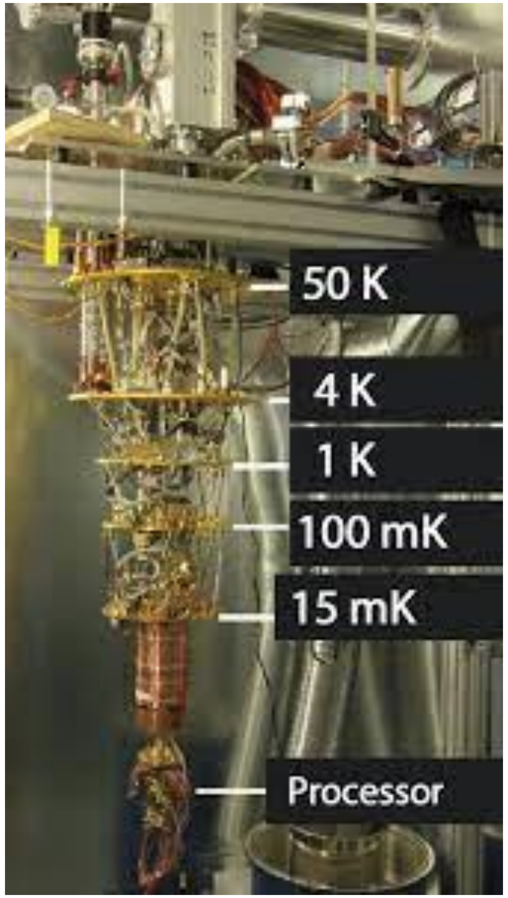


Superconducting Qubits Technology

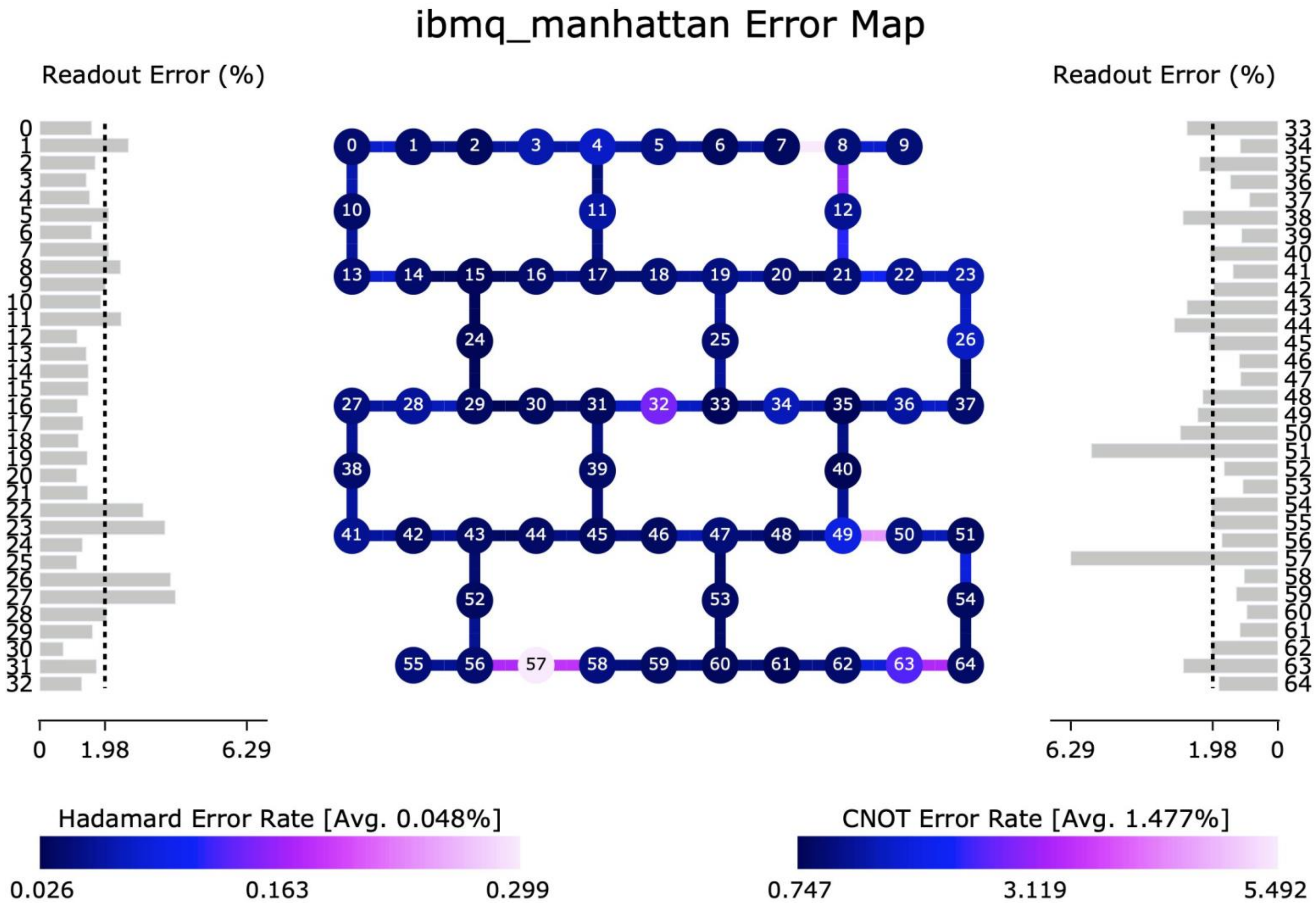


IBM **Q**

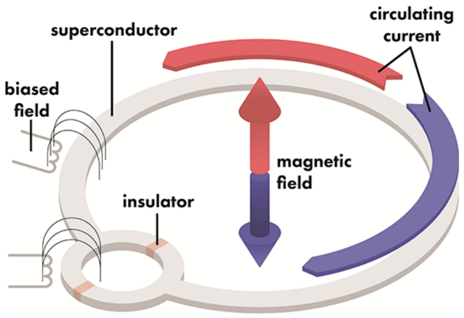


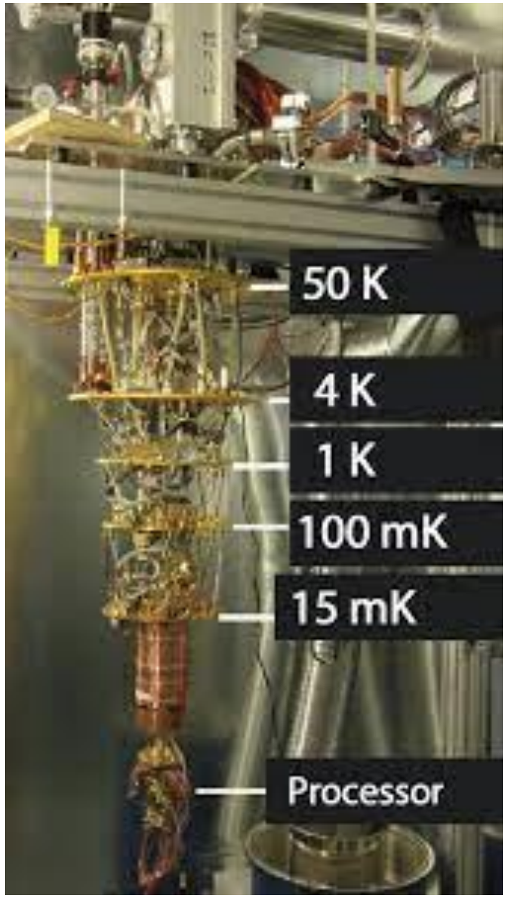


Superconducting Qubits Technology

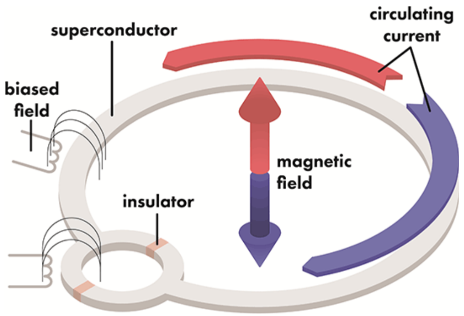


IBM Q

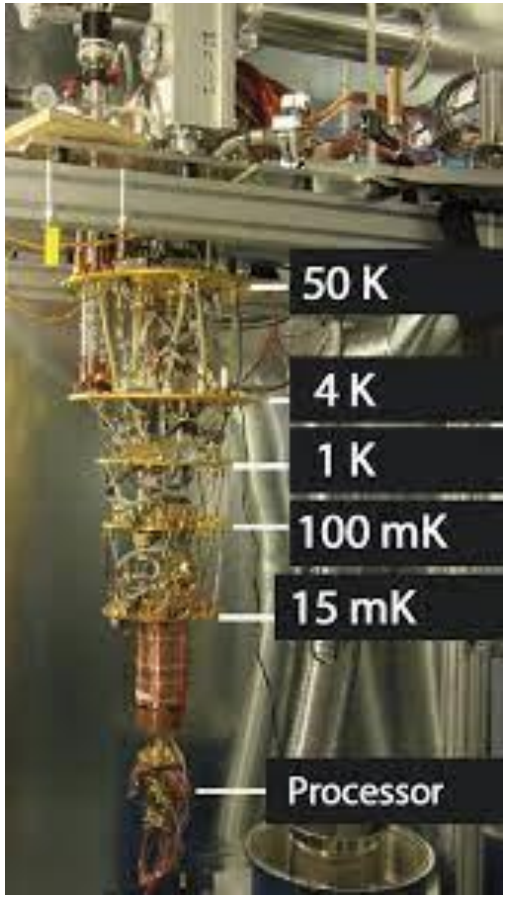




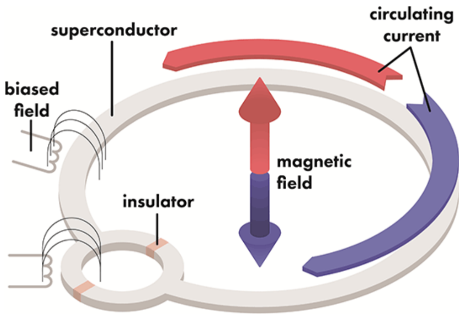
Superconducting Qubits Technology



- American startup born in 2013 in California
- They also manufacture their own general purpose quantum computers using superconducting qubits
- Most powerful machine: Aspen-9 (32 qubits)
- They have announced that they are close to building a model with 128 qubits

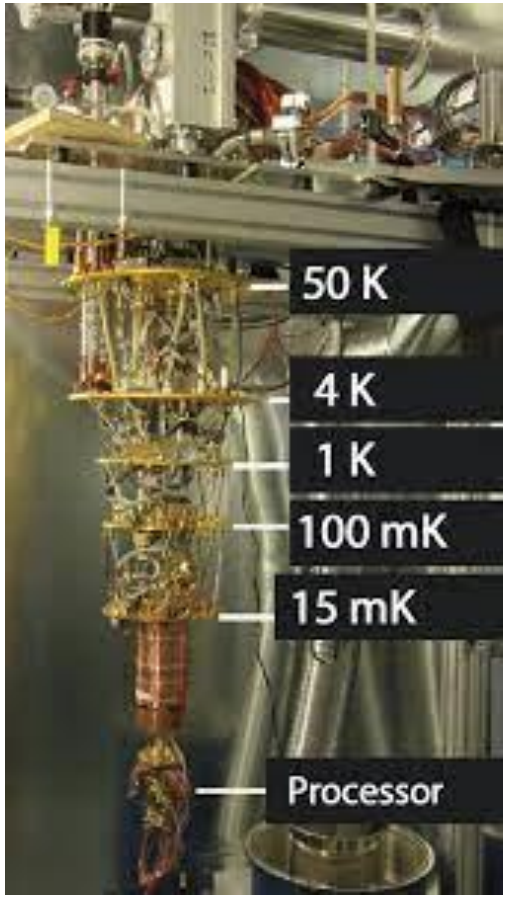


Superconducting Qubits Technology



Rigetti right now

Aspen-9		Median Time Duration (μs)		Median Fidelity (per op.)	
Deployed	07.02.21	T1 Lifetime	27	Single-qubit gates	99.8%
Qubits	31	T2 Lifetime	19	Two-qubit gates (CZ)	95.8%
				Two-qubit gates (XY)	95.4%

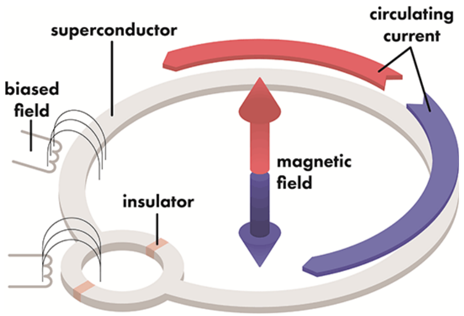


rigetti

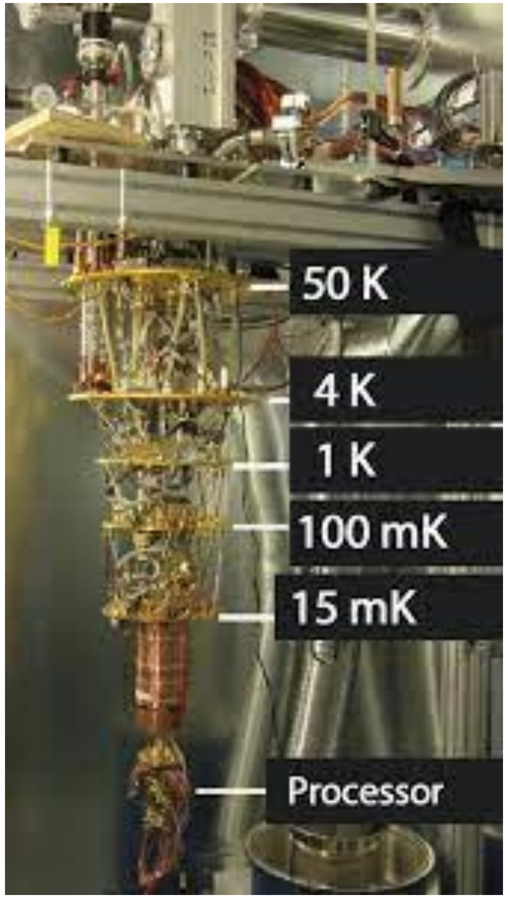
CINECA QUANTUM COMPUTING LAB

Superconducting Qubits Technology

D:Wave
The Quantum Computing Company™

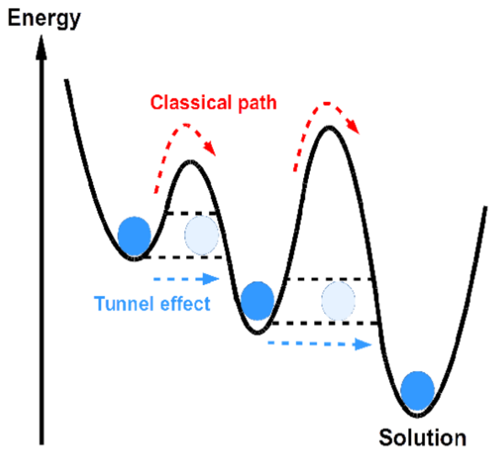


- D-Wave is our latest example of a quantum computer company with superconducting qubits
- Among all those we have seen so far, it is undoubtedly the most particular: it does not produce general purpose quantum computers, but special purpose
- What does it mean? In practice it means that it is not possible to use their computers to implement any quantum algorithm
- Their computers are made with the sole purpose of solving optimization problems by implementing a particular algorithm called Quantum Annealing
- With this choice, they have shown that it is possible to make great chipsets already: their latest model has more than 5600 qubits!

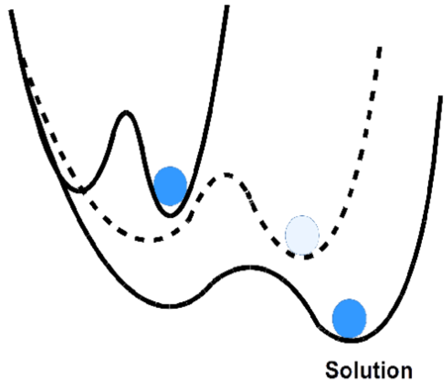


Superconducting Qubits Technology

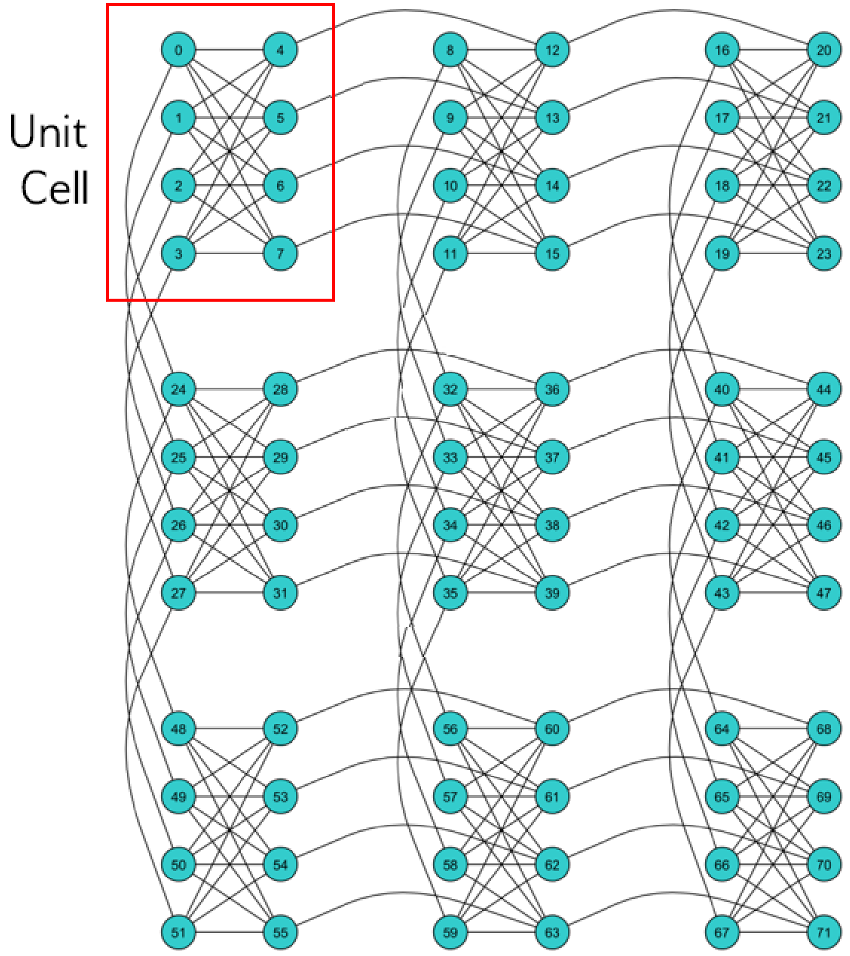
D:Wave
The Quantum Computing Company™

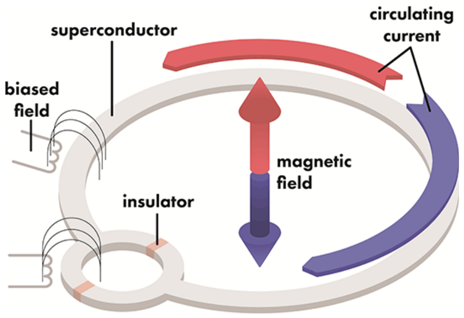


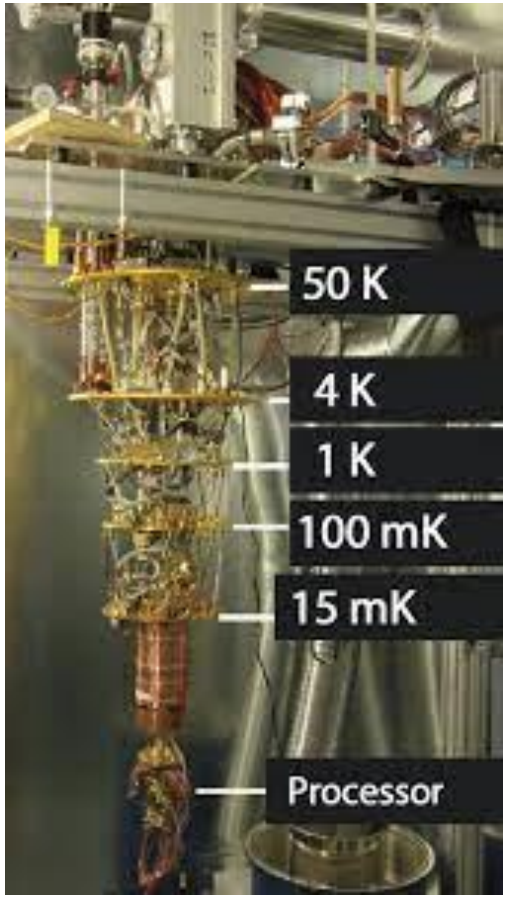
Quantum Tunnelling



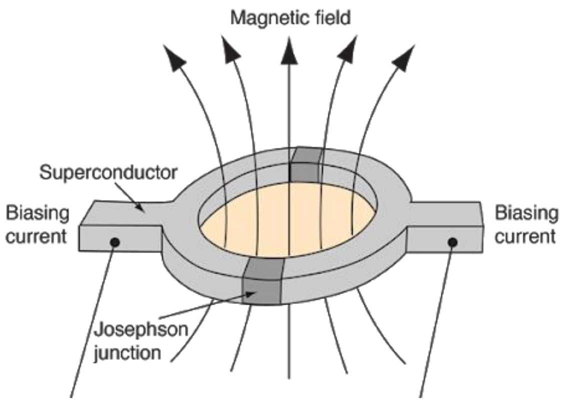
Adiabatic evolution







Superconducting Qubits Technology



SUPERCONDUCTING

PRO

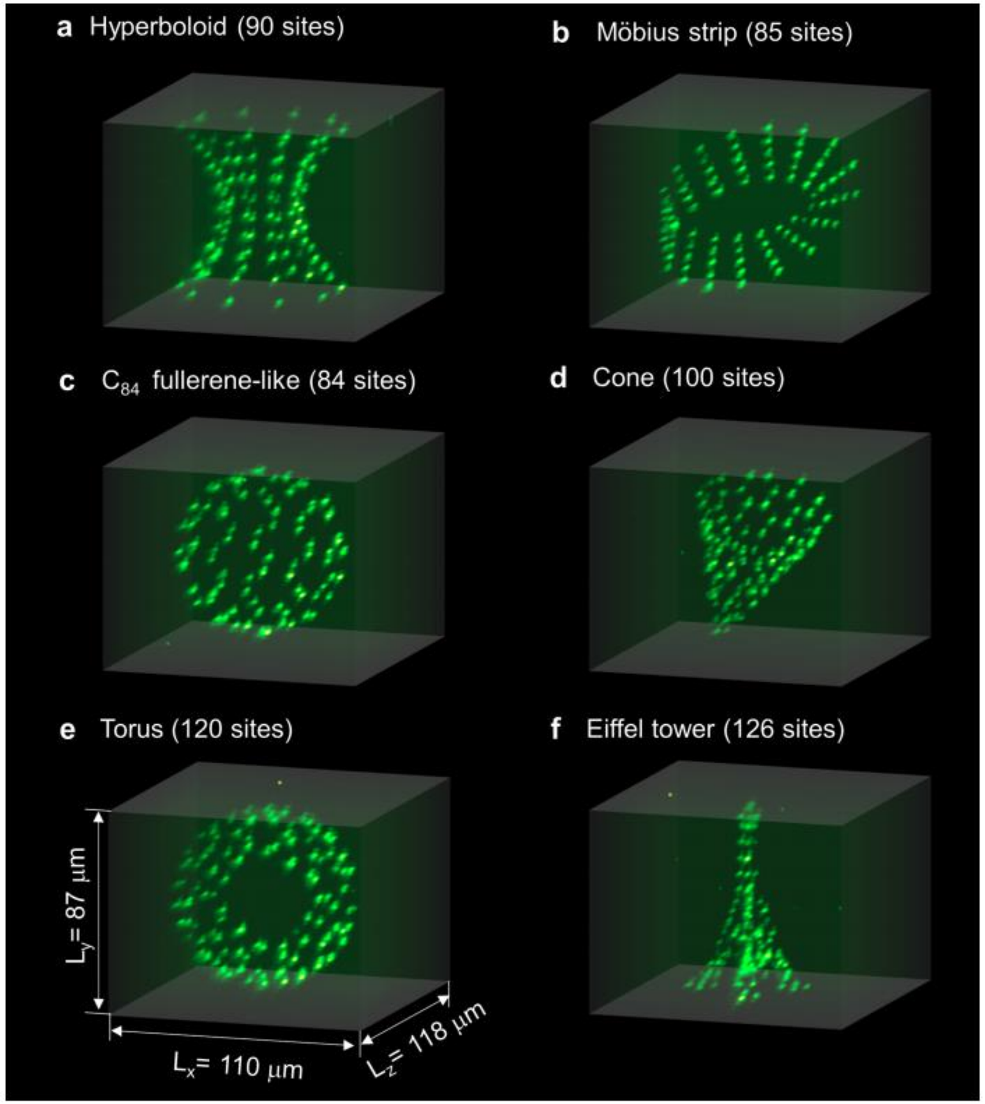
- Macroscopic
- Very large experience
- Fast operations

CONS

- Very low coherence time
- Low fidelity
- Need to be cooled down

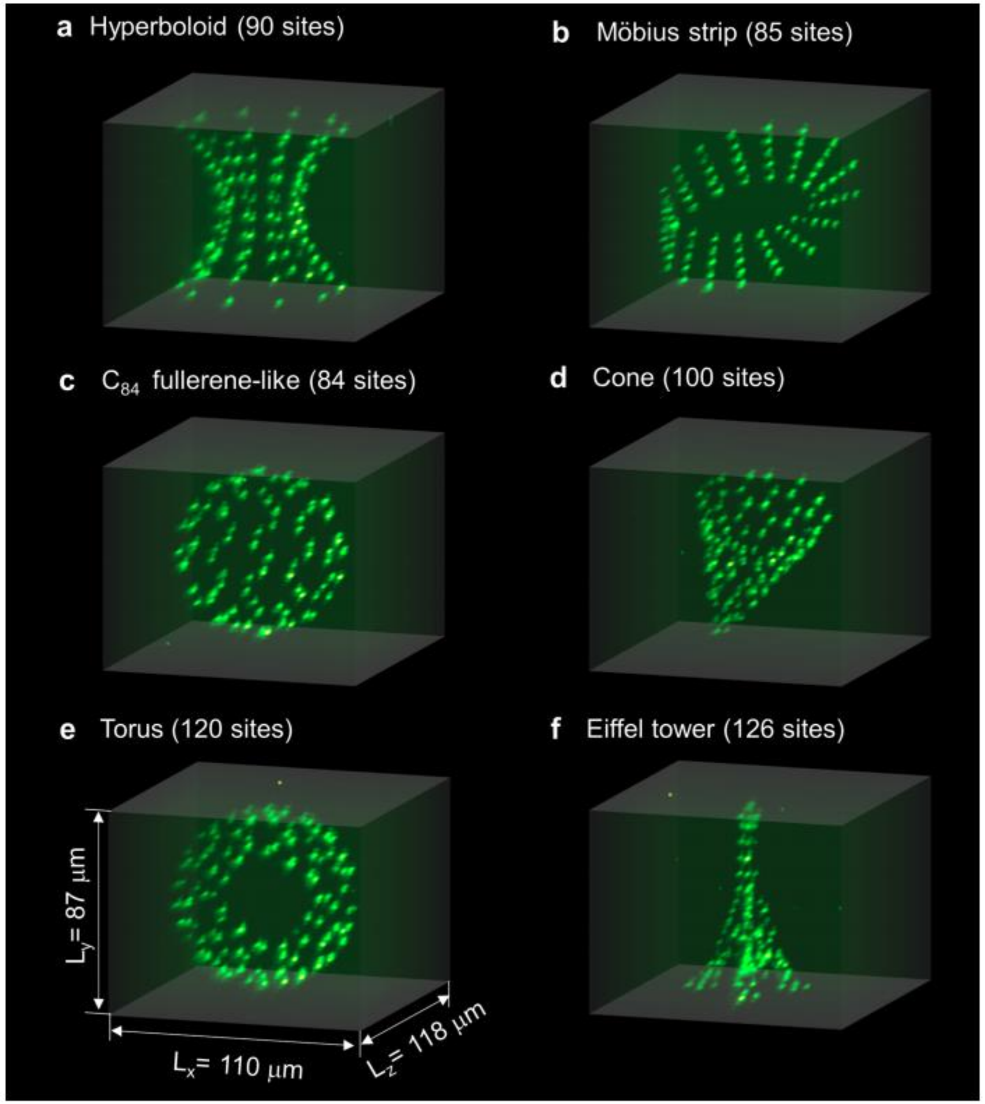
Neutral Atoms Qubits Technology

- The second qubits technology that we are going to analyze is the Neutral Atoms technology



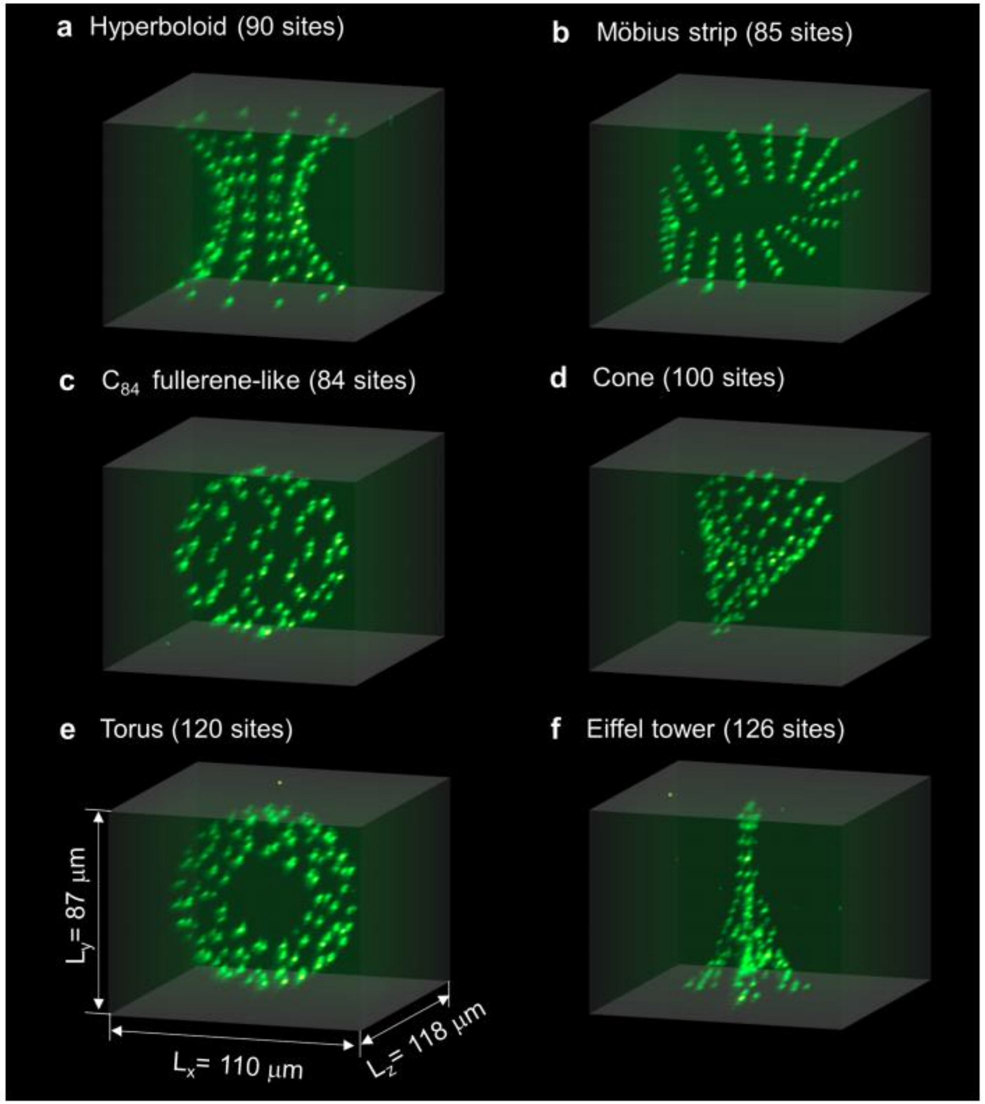
Neutral Atoms Qubits Technology

- The second qubits technology that we are going to analyze is the Neutral Atoms technology
- To date, the only technology that can be considered mature is Superconducting technology: all the others that we will see are in a more or less experimental phase.



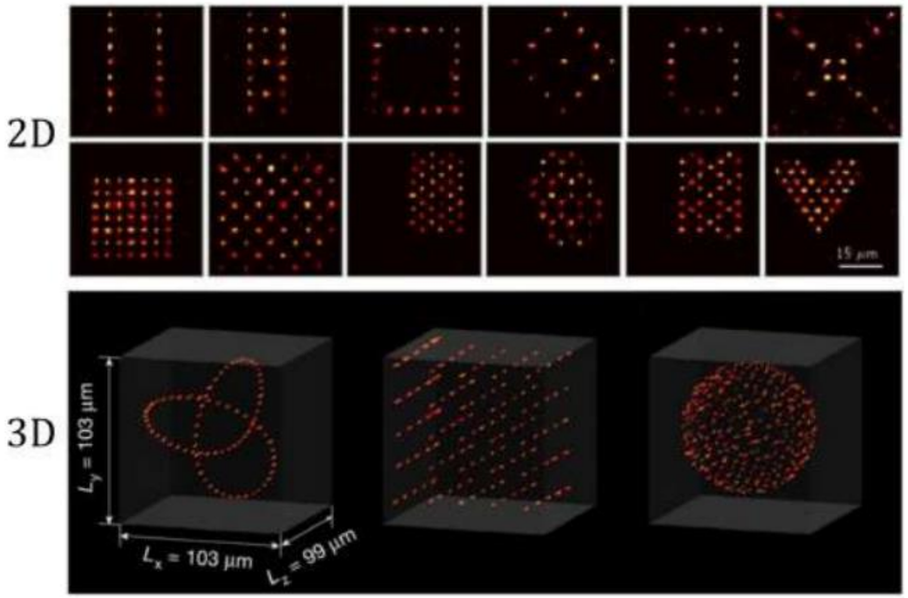
Neutral Atoms Qubits Technology

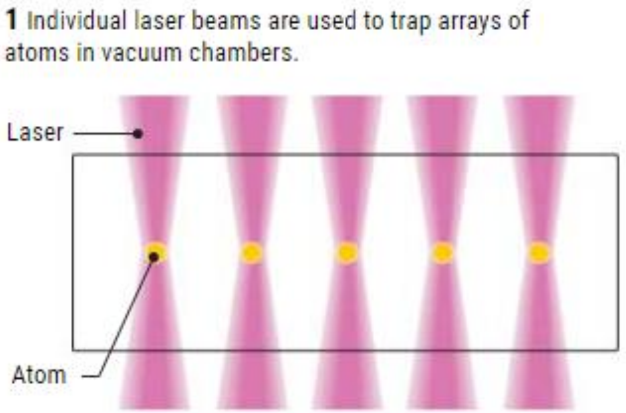
- The second qubits technology that we are going to analyze is the Neutral Atoms technology
- To date, the only technology that can be considered mature is Superconducting technology: all the others that we will see are in a more or less experimental phase.
- In the case of neutral atoms, however, we are close to perfecting QC capable of competing with superconducting ones

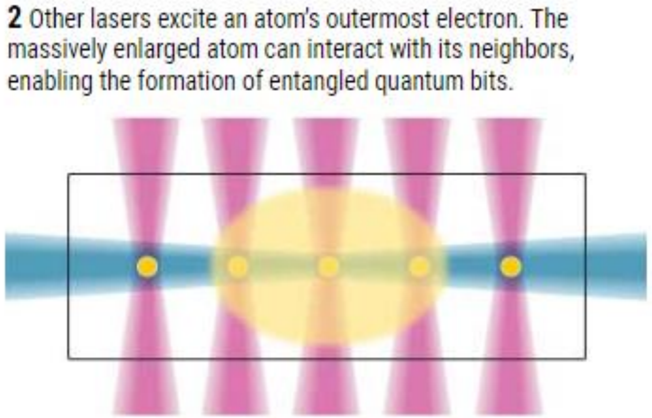


Neutral Atoms Qubits Technology

- The second qubits technology that we are going to analyze is the Neutral Atoms technology
- To date, the only technology that can be considered mature is Superconducting technology: all the others that we will see are in a more or less experimental phase.
- In the case of neutral atoms, however, we are close to perfecting QC capable of competing with superconducting ones
- Qubits Neutral Atoms are Rubidium atoms, which can be manipulated through the use of special lasers

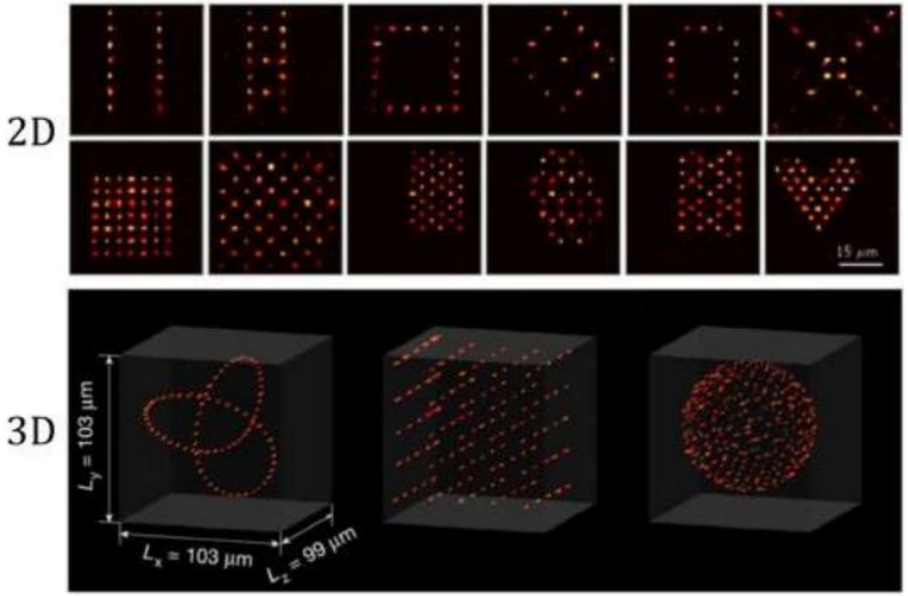


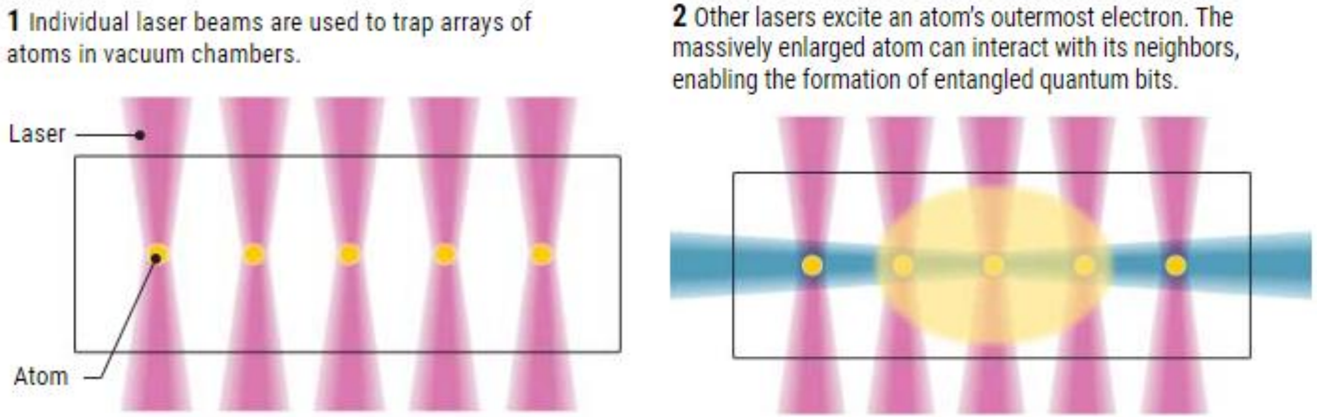




Neutral Atoms Qubits Technology

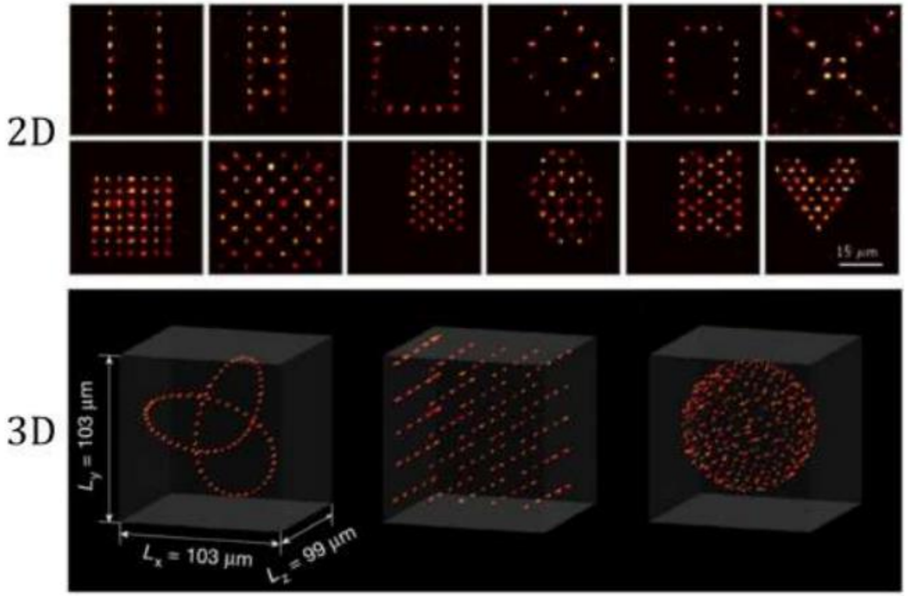
- The second qubits technology that we are going to analyze is the Neutral Atoms technology
- To date, the only technology that can be considered mature is Superconducting technology: all the others that we will see are in a more or less experimental phase.
- In the case of neutral atoms, however, we are close to perfecting QC capable of competing with superconducting ones
- Qubits Neutral Atoms are Rubidium atoms, which can be manipulated through the use of special lasers
- Manipulation occurs both in space and between ground state and excited state

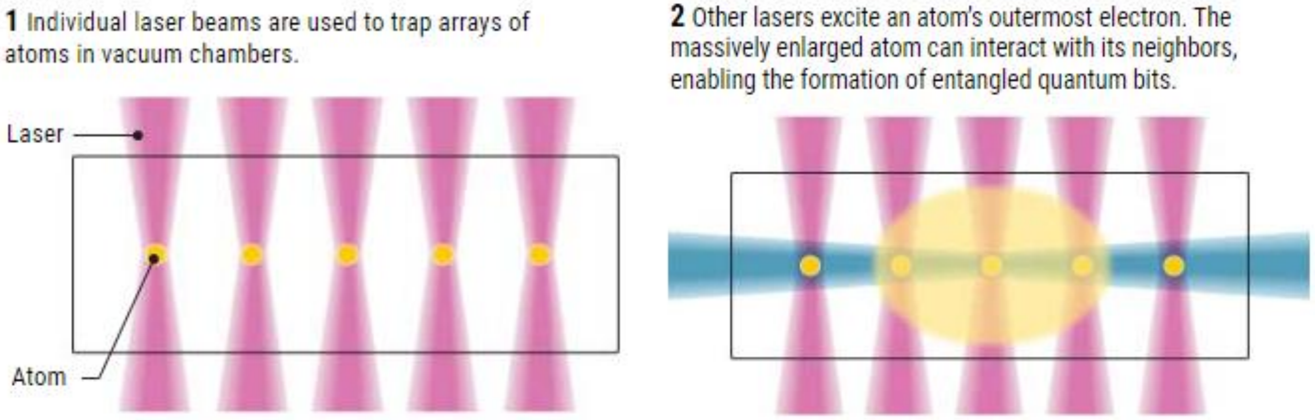




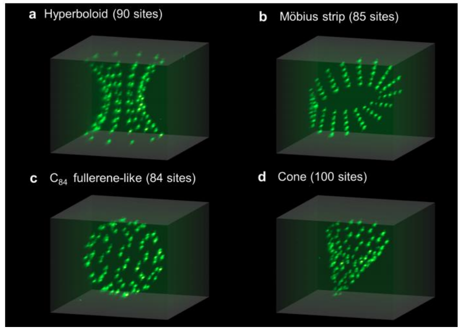
Neutral Atoms Qubits Technology

- The second qubits technology that we are going to analyze is the Neutral Atoms technology
- To date, the only technology that can be considered mature is Superconducting technology: all the others that we will see are in a more or less experimental phase.
- In the case of neutral atoms, however, we are close to perfecting QC capable of competing with superconducting ones
- Qubits Neutral Atoms are Rubidium atoms, which can be manipulated through the use of special lasers
- Manipulation occurs both in space and between ground state and excited state
- Connectivity is established by the Rydberg radius



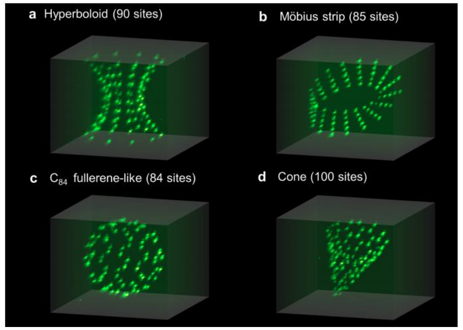


Neutral Atoms Qubits Technology

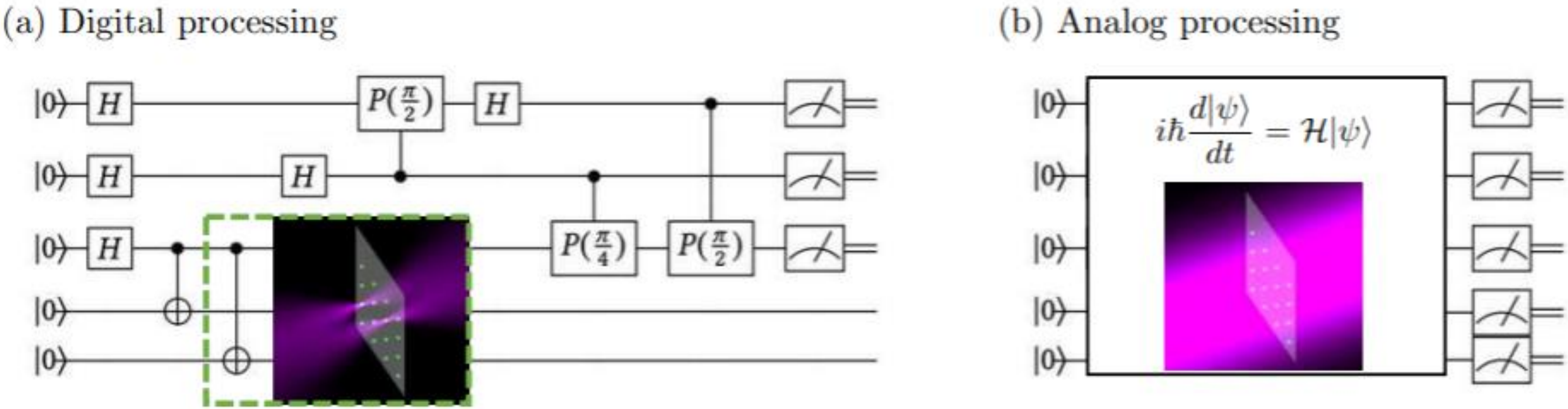


- Pasqal is a French startup born in Paris in 2019
- Spin-off of the Institut d'Optique Graduate School, its team boasts among the greatest experts in quantum computing using neutral atoms
- The company was born after demonstrating the effectiveness of their qubits with a working laboratory prototype
- In this first phase, Pasqal's quantum machine is a machine capable of performing analog calculations
- In other words, it is not yet possible to program it as a real QC, but it can be used to simulate the evolution of particular Hamiltonians
- They plan to have a computer with 100 qubits by September this year and to increase the number of qubits to a thousand by 2023.

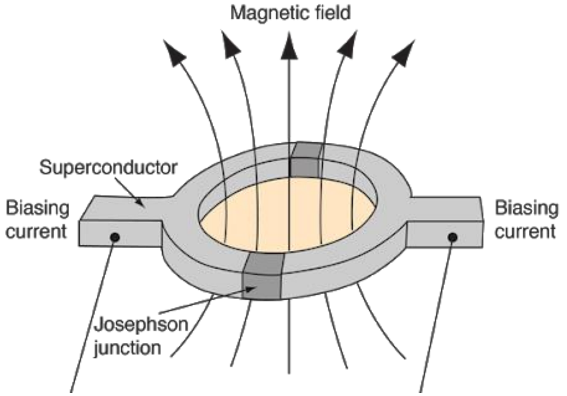
Neutral Atoms Qubits Technology

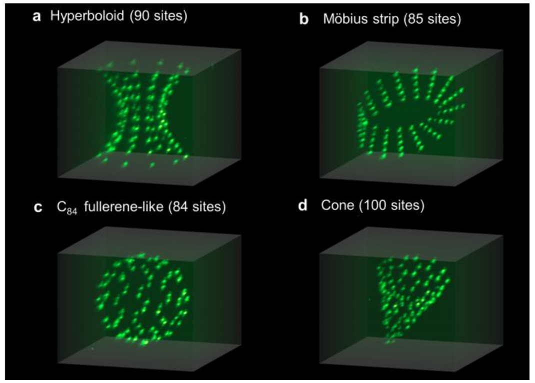


PASQAL



Building a Quantum Computer





SUPERCONDUCTING

PRO

- Macroscopic
- Very large experience
- Fast operations

CONS

- Very low coherence time
- Low fidelity
- Need to be cooled down

NEUTRAL ATOMS

PRO

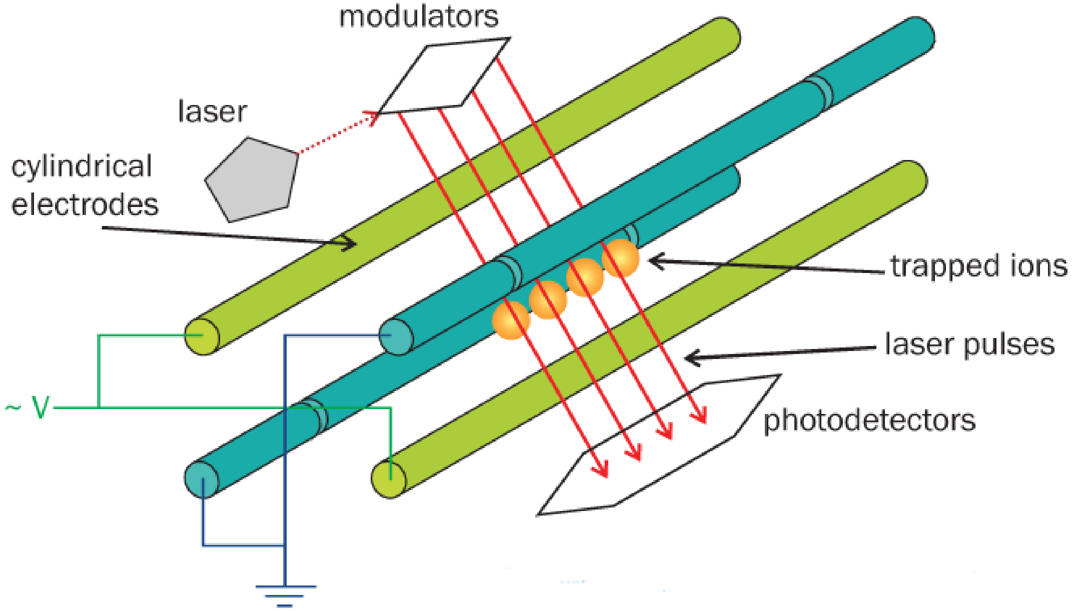
- Good coherence
- interchangeable topology
- Room temperature

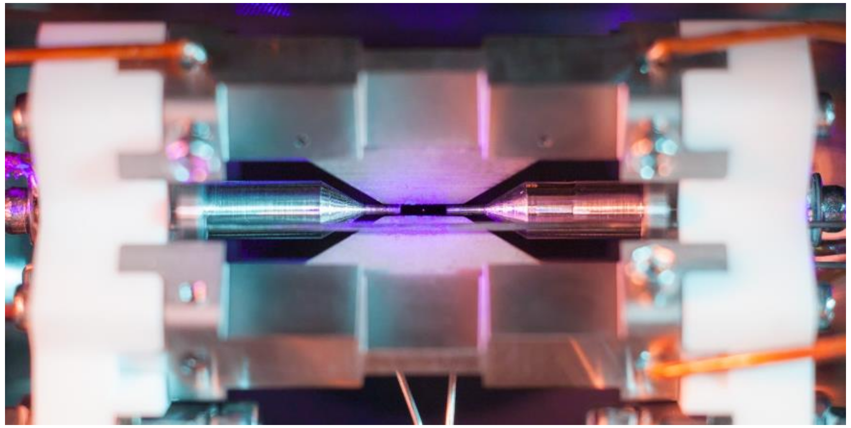
CONS

- Experimental technology

Trapped Ions Qubits Technology

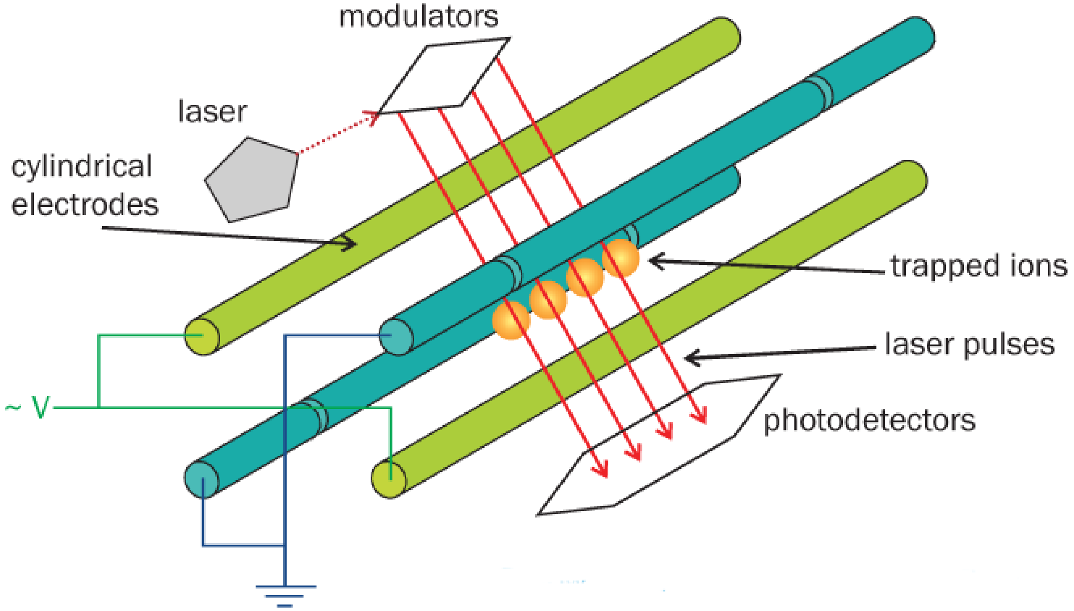
- Trapped-ions is the third and qubit technology that we see in detail today

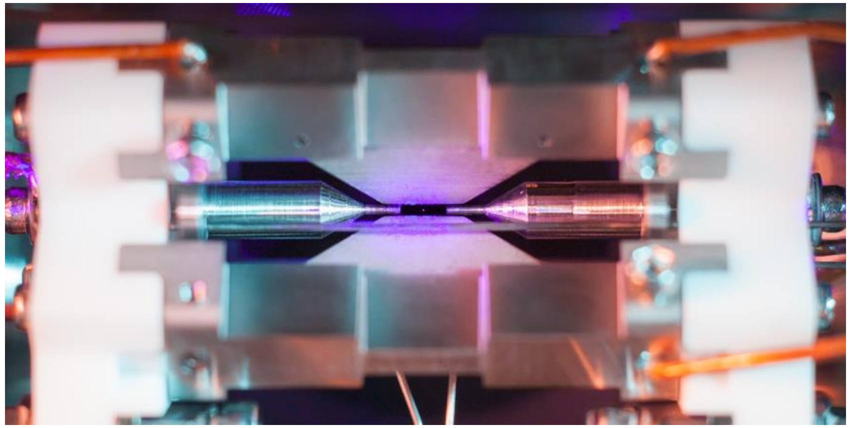




Trapped Ions Qubits Technology

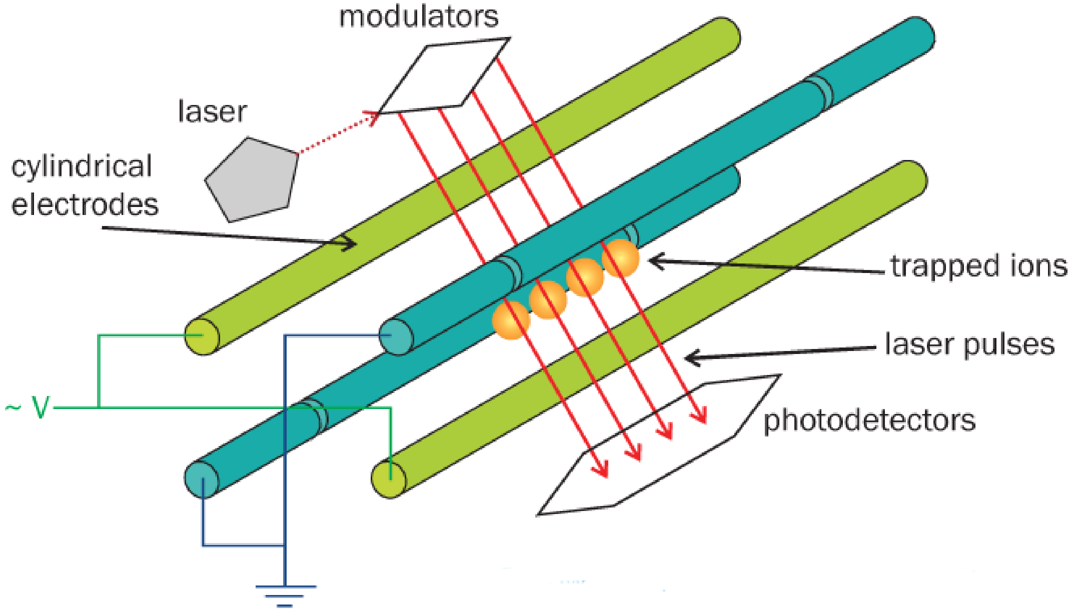
- Trapped-ions is the third and qubit technology that we see in detail today
- In this case, trapped ions are used as qubits

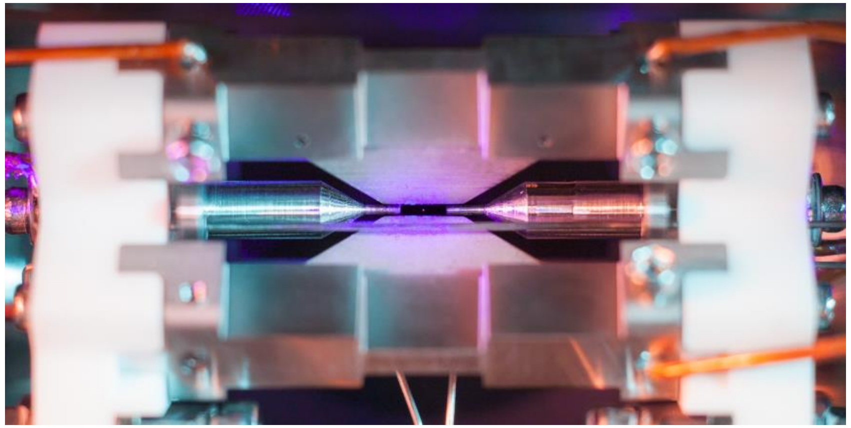




Trapped Ions Qubits Technology

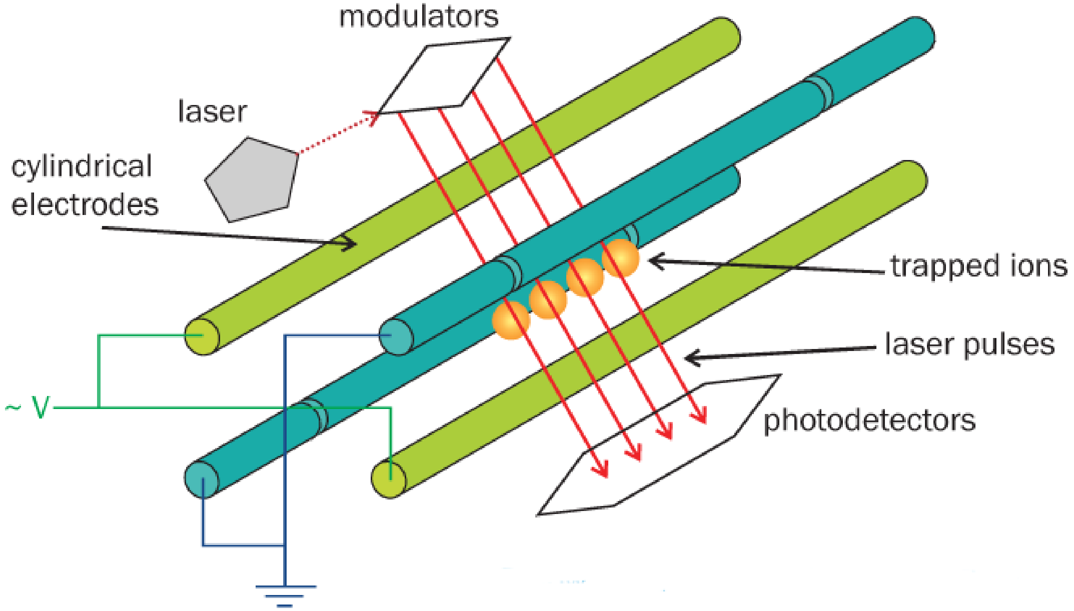
- Trapped-ions is the third and qubit technology that we see in detail today
- In this case, trapped ions are used as qubits
- Through a mechanism like the one in the figure it is possible to trap an ion which can be manipulated using laser pulses

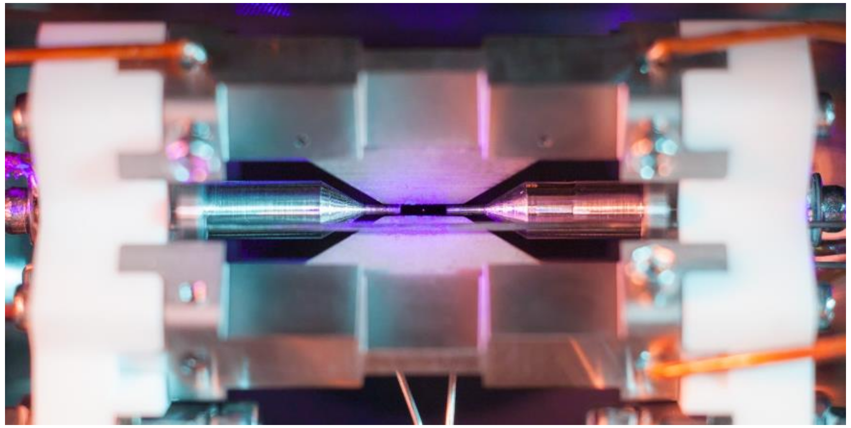




Trapped Ions Qubits Technology

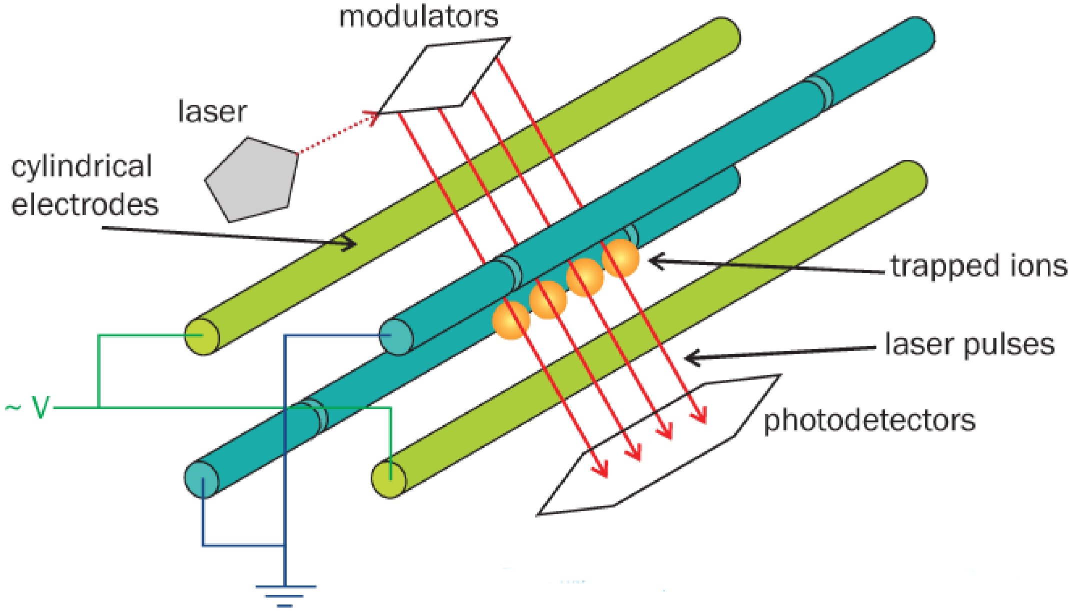
- Trapped-ions is the third and qubit technology that we see in detail today
- In this case, trapped ions are used as qubits
- Through a mechanism like the one in the figure it is possible to trap an ion which can be manipulated using laser pulses
- This is also a very interesting technology, albeit not so mature

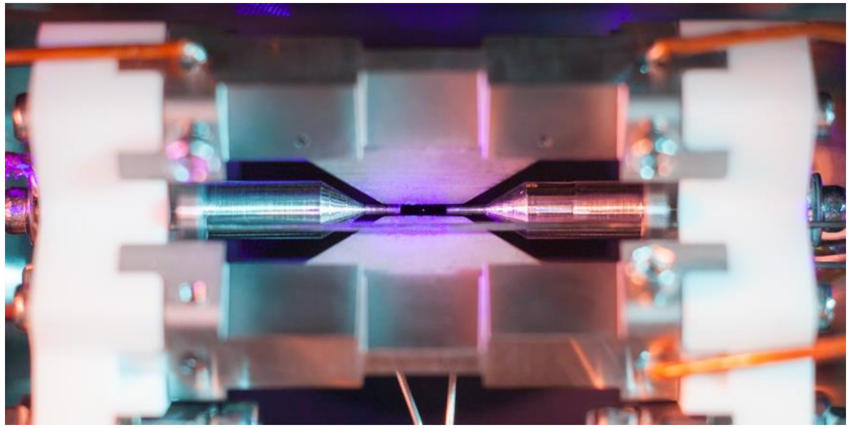




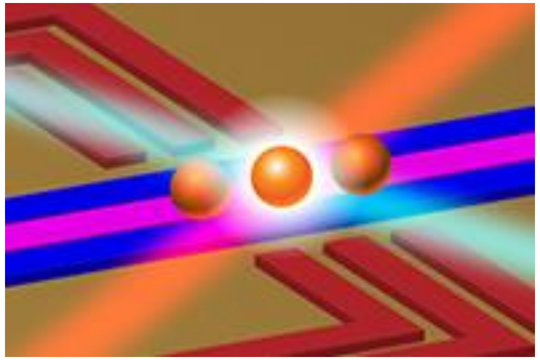
Trapped Ions Qubits Technology

- Trapped-ions is the third and qubit technology that we see in detail today
- In this case, trapped ions are used as qubits
- Through a mechanism like the one in the figure it is possible to trap an ion which can be manipulated using laser pulses
- This is also a very interesting technology, albeit not so mature
- A great strength of this technology is the connectivity of the qubits: thanks to the Coulomb forces, every chipset made with trapped-ion atoms will always naturally be completely connected.



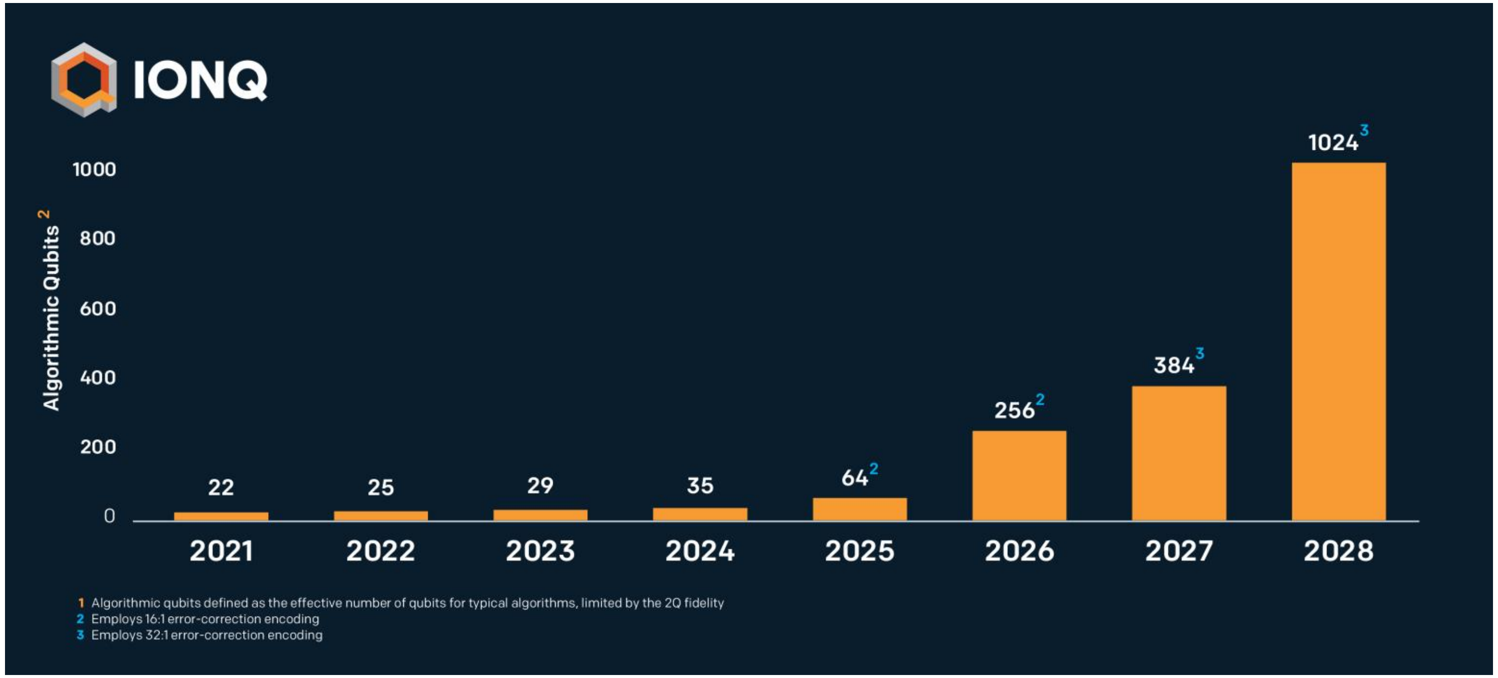


Trapped Ions Qubits Technology

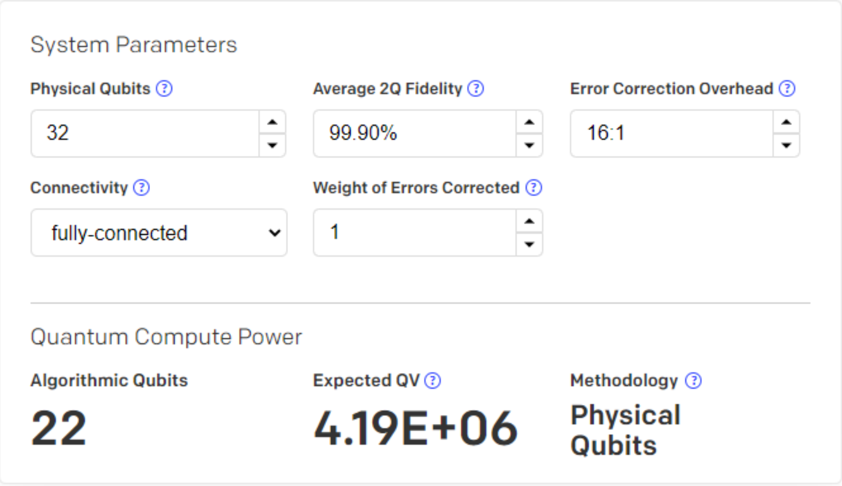


- IonQ is a quantum computing hardware and software company based in College Park, Maryland. They are developing a general-purpose trapped ion quantum computer and software to generate, optimize, and execute quantum circuits
- It was founded in 2015
- They are considered the most advanced startup in their field
- Their biggest machine is a fully connected quantum computer with 32 qubits
- They consider it the most powerful quantum computer in the world, but to date no work has yet come out confirming this.

Trapped Ions Qubits Technology



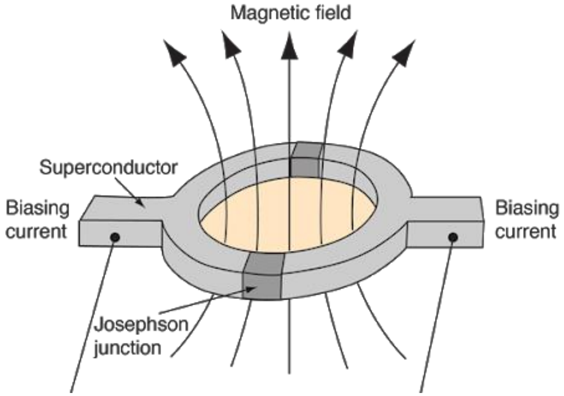
Algorithmic Qubit Calculator



11 qubits → **32 qubits**

Hence, we introduce Algorithmic Qubits (AQ), which is defined as the largest number of effectively perfect qubits you can deploy for a typical quantum program[1]. It's a similar idea to Quantum Volume, but takes error-correction into account and has a clear, direct relationship to qubit count. In the absence of error-correction encoding, AQ = $\log_2(QV)$, or inversely, QV = 2^{AQ}. AQ represents the number of "useful" encoded qubits in a particular quantum computer and is a simple proxy for the ability to execute real quantum algorithms for a given input size.

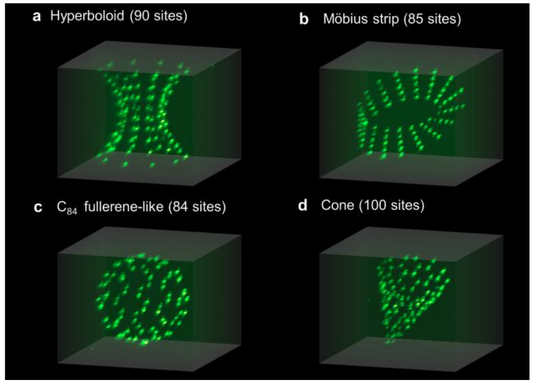
Building a Quantum Computer



SUPERCONDUCTING

PRO
- Macroscopic
- Very large experience
- Fast operations

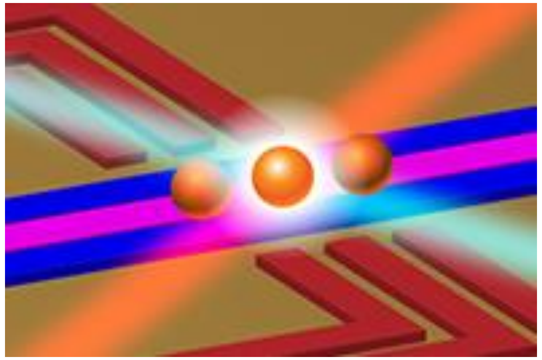
CONS
- Very low coherence time
- Low fidelity
- Need to be cooled down



NEUTRAL ATOMS

PRO
- Good coherence
- interchangeable topology
- Room temperature

CONS
- Experimental technology

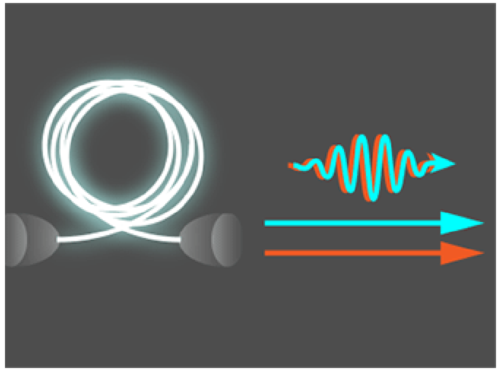


TRAPPED IONS

PRO
- Very long coherence
- Promising fidelity
- Room temperature

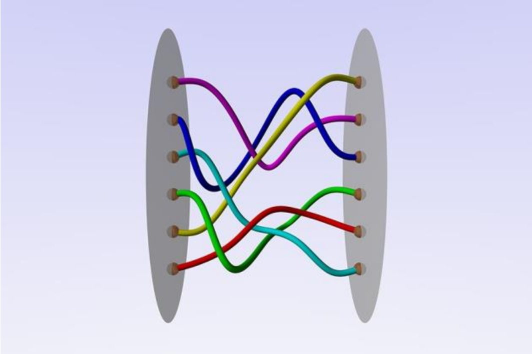
CONS
- Slow operations
- Many lasers needed

Building a Quantum Computer



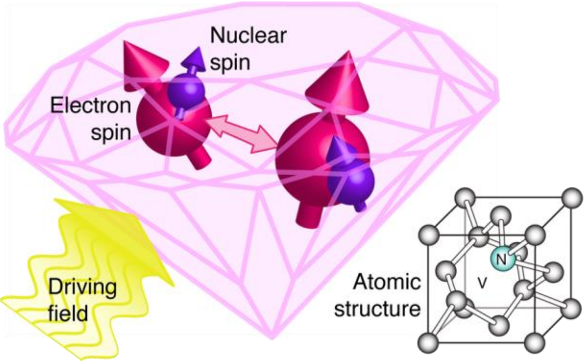
PHOTONIC
PRO
- Good scalability
- Room temperature

CONS
- Very experimental technology
- Connectivity not clear



TOPOLOGICAL
PRO
- Almost perfect on paper

CONS
- Existence not confirmed



DIAMOND VACANCIES
PRO
- Good coherence time
- Room temperature

CONS
- Not so precise
- Difficult to entangle

Emulate a Quantum Computer

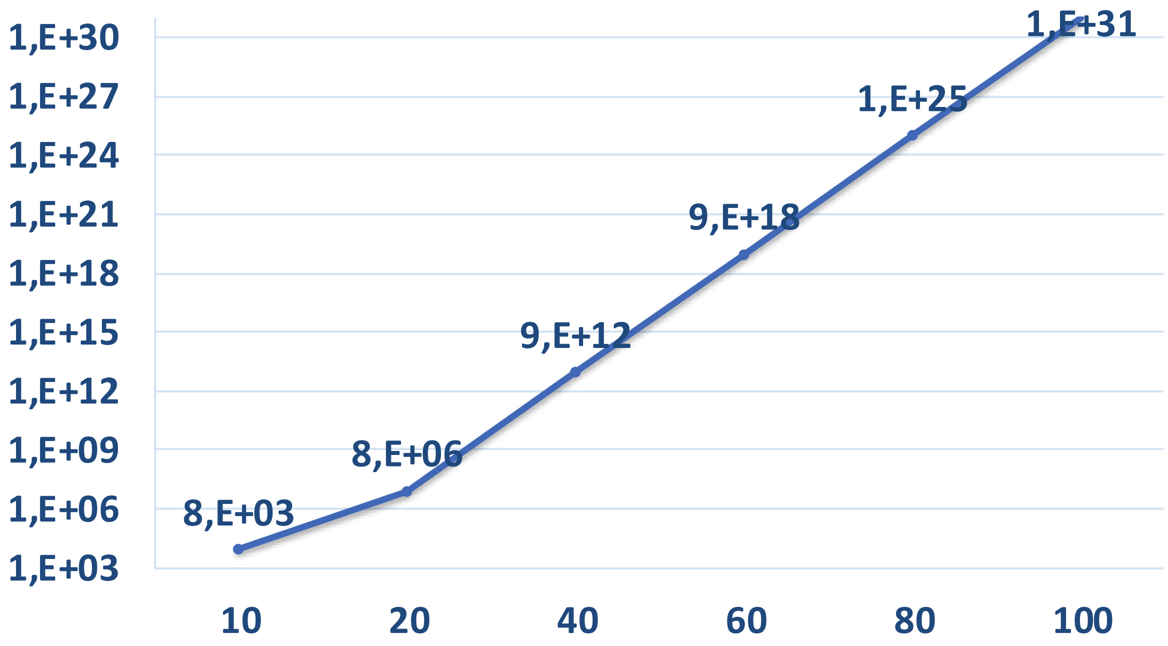
- Emulating a quantum computer on a classic computer is possible, but it requires a considerable amount of resources, especially if you want to simulate a fair amount of qubits.

- Emulating a qubit requires a lot of RAM:

  8 (4 + 4) bytes (for the coefficient) X 2 bits (the states)

  In general, emulating N qubits requires

  8×2^N bytes



- Furthermore, the emulation of a quantum circuit requires computational time for the application of quantum gates. Natural phenomena in real quantum computers, in classical computers they are calculations to be made, often very expensive

kilo-	k or K **	10^3
mega-	M	10^6
giga-	G	10^9
tera-	T	10^{12}

peta-	P	10^{15}
exa-	E	10^{18} *
zetta-	Z	10^{21} *
yotta-	Y	10^{24} *

Emulate a Quantum Computer

MARCONI - 100

Nodes: 980

Processors: 2x16 cores IBM POWER9 AC922 at 3.1 GHz

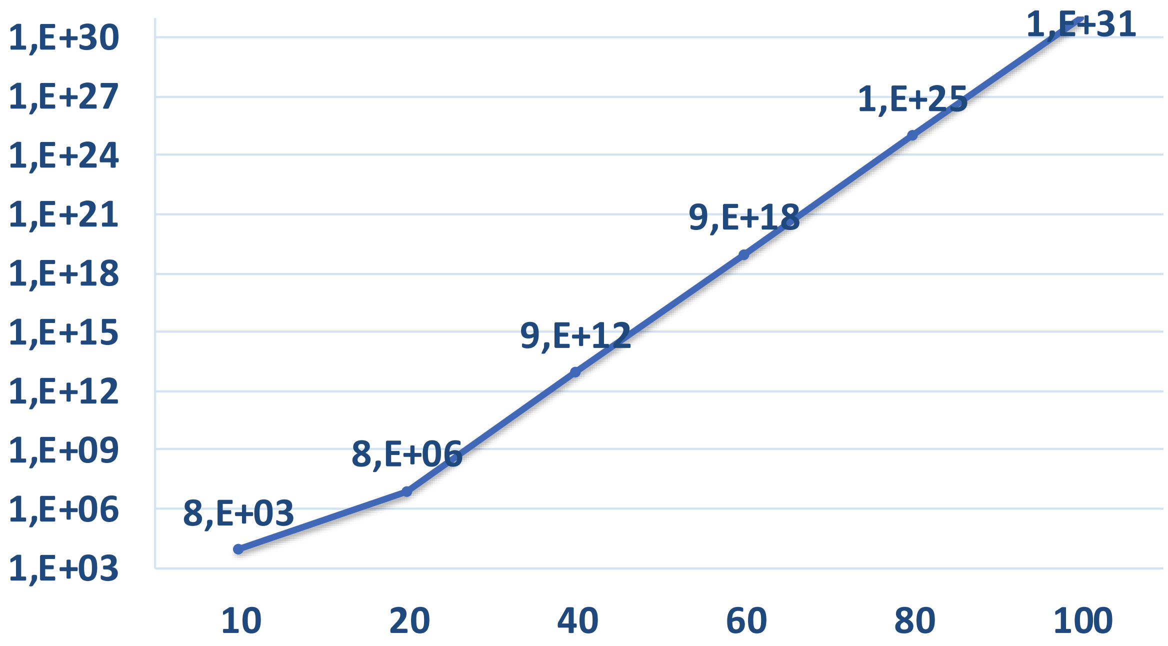
Accelerators: 4 x NVIDIA Volta V100 GPUs, Nvlink 2.0, 16GB

Cores: 32 cores/node

RAM: 256 GB/node

Peak Performance: ~32 PFlop/s

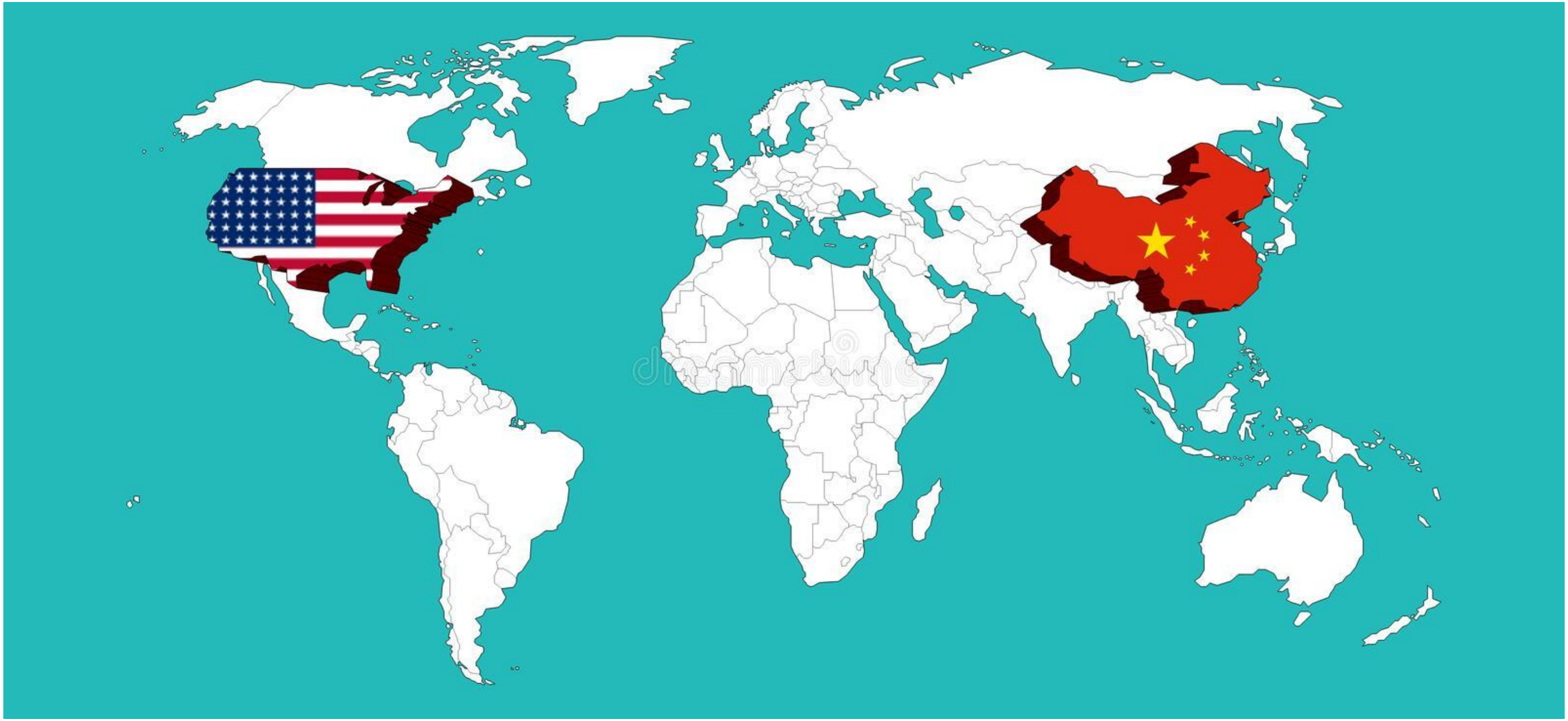
256 x 980 = 250880 GB = 2,5E+14



kilo-	k or K **	10^3	peta-	P	10^{15}
mega-	M	10^6	exa-	E	10^{18} *
giga-	G	10^9	zetta-	Z	10^{21} *
tera-	T	10^{12}	yotta-	Y	10^{24} *

CINECA QUANTUM COMPUTING LAB

Quantum Computing In The World



CINECA QUANTUM COMPUTING LAB

Quantum Computing In The World

- 2018: National Quantum Initiative Act
- 1.2 Billion $ over 10 years + 625 million $ over 5 years for five research centres + 340 million $ from the private sector
- Big Tech Companies: Google, IBM, Intel, Microsoft (<100 qubits)
  - Google: ***Quantum Supremacy* with superconducting qubits** (October 2019).
  - IBM: very aggressive roadmap, more than 1000 qubits in 2023
  - Microsoft: no QC yet, developed Azure Quantum Network. Searching for the Holy Grail of the Qubits: The topological realization
- Very Promising Start-up:
  - Rigetti Computing: QC producer, superconducting qubits technology. Announced 128 qubits model
  - IonQ: Trapped-Ion quantum computers
  - Honeywell: Trapped-Ion quantum computers
  - Xanadu: Photonic quantum computers
  - D-WAVE (Canadian start-up, to be precise): manufacturers of *Quantum Annealers*, special purpose quantum computers able to solve optimization problems. More than 5000 qubits

Quantum Computing In The World

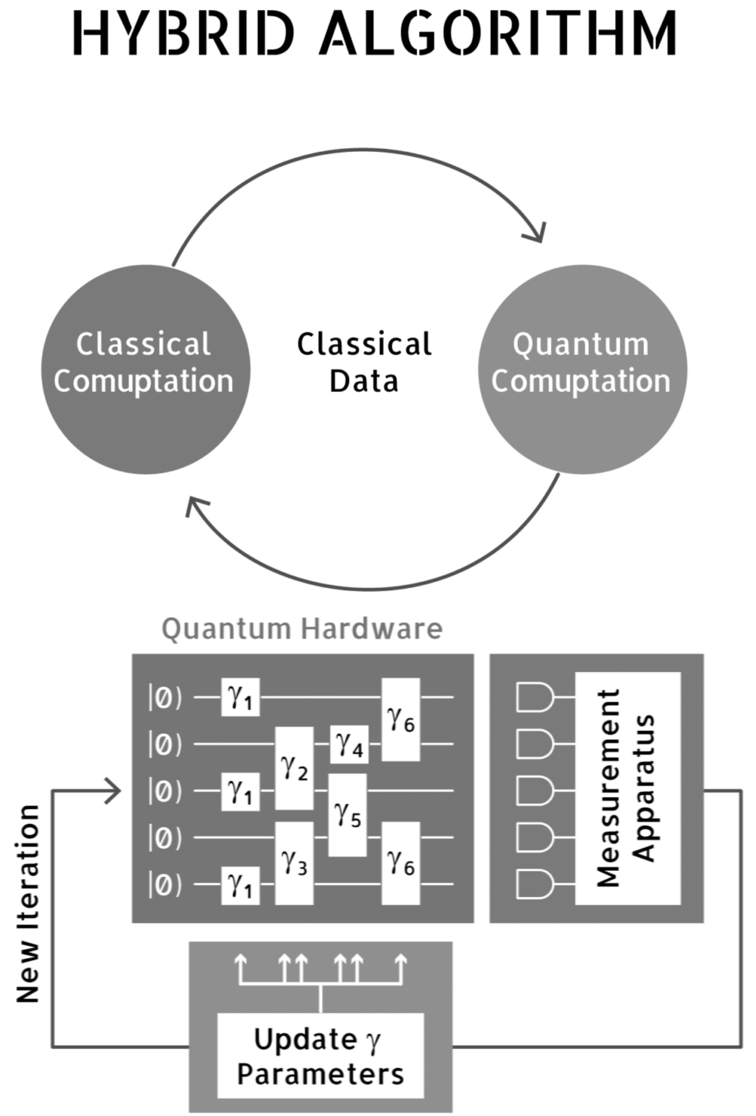
- China had made quantum technology a key priority in its thirteenth five-year plan (2016-2020)
- *Micius: Quantum Experiments at Space Scale:* carrying out a range of ground-to-space experiments in quantum communication
- 2017: $10 billion for national laboratory for quantum information sciences
- December 2020: China claims to have achieved **quantum supremacy with *Jiuzhang*, a photonic quantum computer** capable of detecting up to 76 photons (USTC: University of Science and Technology of China)
- Big Tech Companies: Alibaba, Baidu
    - In 2015, Alibaba set up its own quantum computing laboratory in order to produce a **prototype** 50 to 100 **qubit quantum** computer for general use by 2030
    - Baidu, the company behind the eponymous search engine, announced the creation of a quantum computing institute in March 2018

Quantum Computing In The World

Company	Type	Technology	Now	Next Goal
Intel	Gate	Superconducting	49	TBD
Google	Gate	Superconducting	53 / 72	TBD
IBM	Gate	Superconducting	64	127
Rigetti	Gate	Superconducting	32	128
USTC (China)	Gate	Superconducting	10	20
USTC (China)	Gate	Photonic	76	TBD
IonQ	Gate	Ion Trap	32	79
IQOQI/Univ. Ulm/Univ. Innsbruck	Gate	Ion Trap	20	TBD
NSF STAQ Project	Gate	Ion Trap	N/A	≥64
Intel	Gate	Spin	26	TBD
Silicon Quantum Computing	Gate	Spin	N/A	10
CEA-Leti/INAC/Institut Néel	Gate	Spin	N/A	100
Pasqal	Gate	Neutral Atoms	100	1000
D-Wave	Annealing	Superconducting	5000+	TBD

Far from Universal QC, entering the NISQ era

- Although the results obtained so far **do not allow us to observe significant applications of the most famous quantum algorithms**, this does not mean that the computers produced so far are completely useless.
- Some **examples of quantum supremacy** have been achieved
- Before reaching **universal quantum supremacy**, we will arrive at **local quantum supremacy**
- By **combining** the power of **quantum computers and current supercomputers**, we can achieve great benefits
- Quantum computers that exist today are about to reach the power needed to run hybrid algorithms (**NISQ Computers**)



And what about Europe?

- May 2016: over 3400 figures from research and corporate world signed the *Quantum Manifesto*
  - The Manifesto, addressed to the European Commission, said in essence: *we have the opportunity to compete for a new kind of technological independence, let's take it.*
- October 2018: The European Commission launched the **Quantum Flagship programme**: 1.3 billions of Euro to support 10 year of quantum technologies research and development.
- The **European High Performance Computing Joint Undertaking (EuroHPC JU)** is a joint initiative between the EU, European countries and private partners to **develop a World Class Supercomputing Ecosystem in Europe**.
- The **European Processor Initiative (EPI)** is a project whose aim is to **design and implement a roadmap for a new family of low-power European processors** for extreme scale computing, high-performance Big-Data and a range of emerging applications.

Quantum Manifesto
A New Era of Technology
May 2016

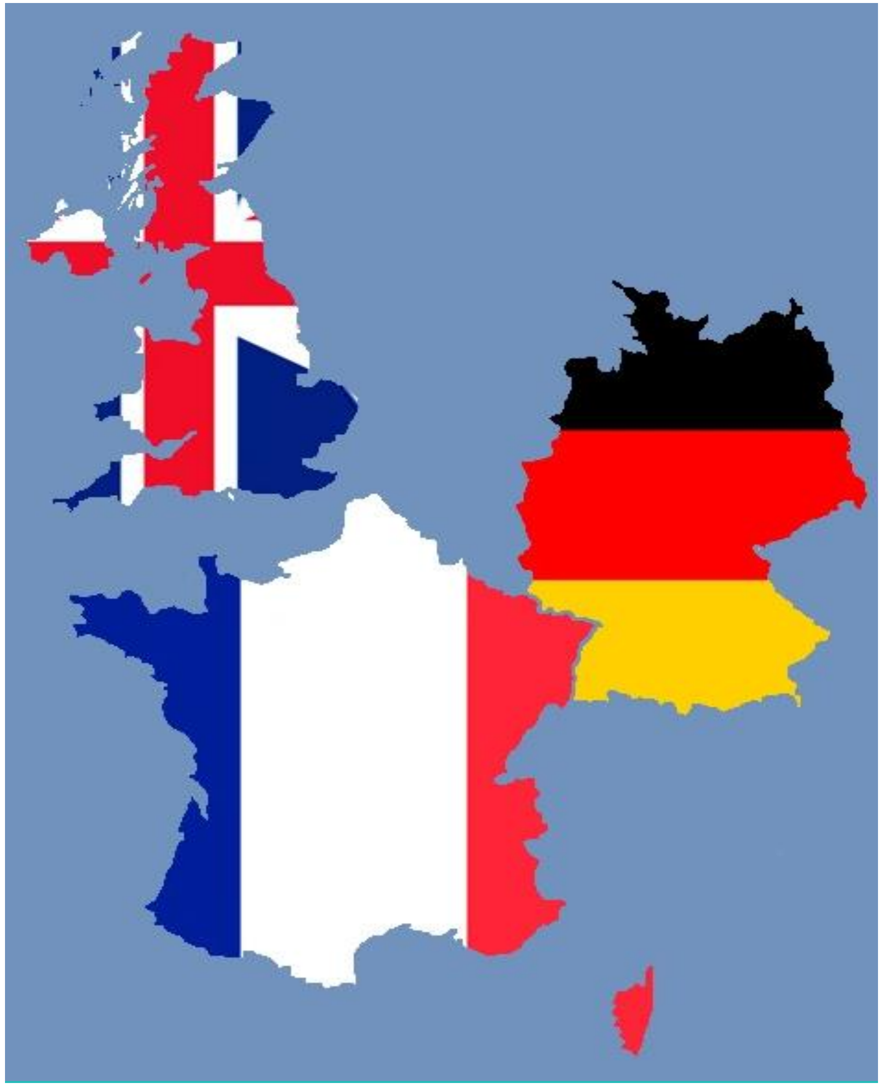
QUANTUM FLAGSHIP

EuroHPC Joint Undertaking

European Processor Initiative epi

European Union countries quantum initiatives

- 2013: UK (Still in Europe!) became the **first country in Europe to announce its own quantum strategy**, investing €370 million over five years
    - 2018: realization of a National Quantum Computing centre with the **aim to build a Quantum Computer**
    - 2019: additional investment of £153 million in quantum computing
- 2018: Germany announced a framework programme with the aim to **develop quantum technologies**
    - The framework programme was funded with over €650 million
    - May 2021: Germany will spend about **2 billion euros** to support the development of its first quantum computer and related technologies in the next four years
- 2019: the French government instructs a task force to implement a **national strategy for quantum technologies**
    - January 2021: Emmanuel Macron announced the **National Quantum Plan**
    - The Quantum Plan provides for actions in support of research (especially for quantum computers, sensors, and communications), industry, and academic and professional training. It is financed with €1.8 billion



Quantum Computing in Italy

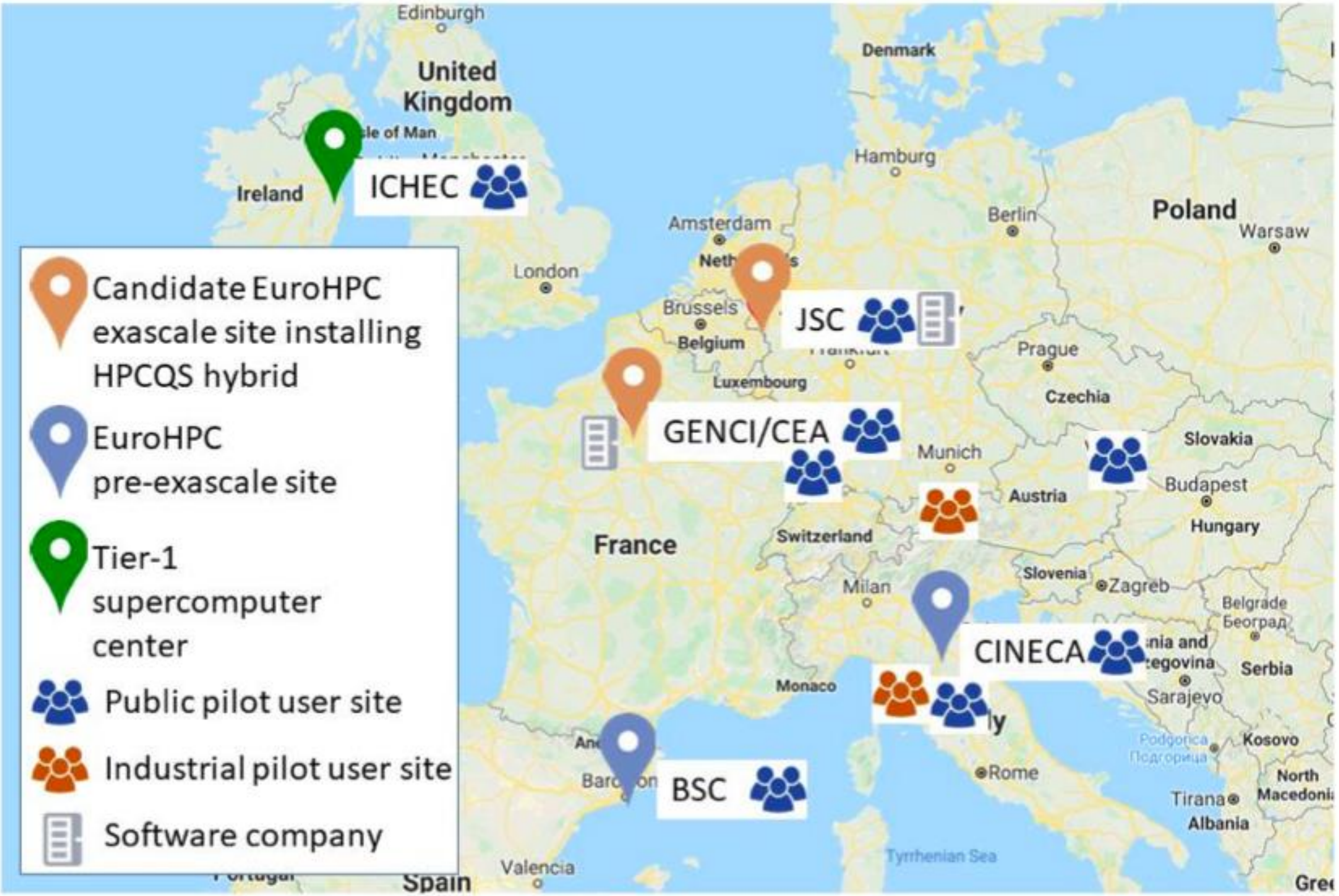
- Quantum Computing is also becoming a reality in Italy.
- With the Italian Recovery Fund (*PNRR*) the Italian government has provided a large amount of money to be donated to **research on strategic issues**. Quantum Computing has been defined as such.
- For technology transfer, the government aims to use €1.3 billion to create 20 regional innovation hubs for research and development, jointly funded by the public and private sectors and modelled after the Fraunhofer institutes for applied research in Germany.
- Another €1.6 billion will be used to launch seven new centres on key technologies: artificial intelligence, **quantum computing**, agricultural technology, energy, hydrogen, technologies for finance and pharmaceutical research

HPC and Quantum Computing: HPCQS Consortium

FZJ (Coordinator)	CNRS
ParTec (LTP*[1])	Sorbonne (LTP*)
CEA	SUPELEC (LTP*)
GENCI	INRIA
ATOS	Pasqal
CNR	CINECA
NUIG-ICHEC	BSC
	FLS
UIBK	Parity QC
EURICE	Fraunhofer IAF

- The HPCQS consortium was born with the idea of **combining HPC and QC hardware and software**.
- For the **realization of Quantum Computers**, the French company **PASQAL** was chosen, which produces quantum computers based on Neutral Atoms technology
- During the 4 years of the project, the most efficient way to **connect Pasqal computers to EuroHPC supercomputers** will be studied.
- In fact, **latency** between computing systems **is a significant problem**
- The ultimate goal of the project is the **creation of an interconnected network of quantum computers throughout Europe**, able to communicate with each other and through the support of EUROHPC supercomputers.

HPC and Quantum Computing: HPCQS Consortium



EuroHPC Joint Undertaking

European Processor Initiative

PASQAL

FZJ (Coordinator)	CNRS
ParTec (LTP*[1])	Sorbonne (LTP*)
CEA	SUPELEC (LTP*)
GENCI	INRIA
ATOS	Pasqal
CNR	CINECA
NUIG-ICHEC	BSC
	FLS
UIBK	Parity QC
EURICE	Fraunhofer IAF

HPC and QC in Italy: role of CINECA

- **CINECA**, the Italian supercomputing center and inter-university consortium, **started working in the field of quantum computing 3 years ago**, in 2018.
- Today **CINECA**, together with the most important European computing centers, **is part of the HPCQS consortium**
- From March 2021 it distributes **DWAVE quantum computing hours and General Purpose Quantum Computing emulators** free of charge to Italian universities through the **ISCRA project**.
- At the beginning of 2021 the **CINECA Quantum Computing Lab** was inaugurated (www.quantumcomputinglab.cineca.it)

HPC and QC in Italy: role of CINECA

Also in March 2021 a **collaboration with Pasqal** began. This collaboration, which runs parallel to the HPCQS project, was born with the **aim of testing the hardware status of the Pasqal machine**.

- In addition to scientific research, CINECA is also **developing a series of "HPC-ready" emulators based on different systems** (including *tensor network* emulators of not perfect quantum systems, able to make the most of the available computational resources)
- On this front collaborations are proceeding with **Pasqal and UniPD**
- Not only software and algorithm development: organization of **annual conferences** (**HPCQC**, this year in its **fourth edition**) and **schools** (**Introduction to Quantum Computing**, first edition at the end of **June 2021**)