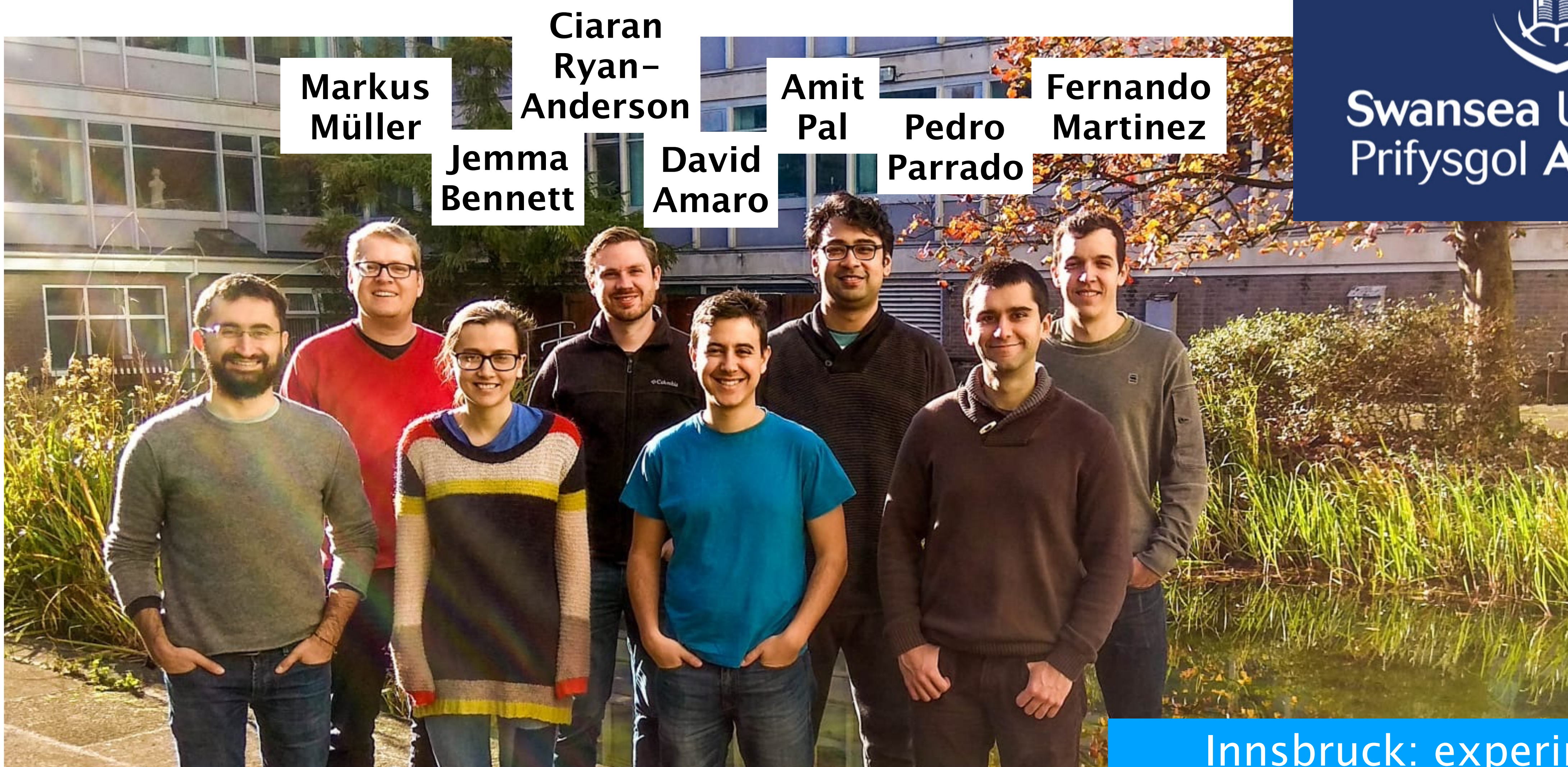
# Fighting Qubit Loss in Topological Quantum Memories

Davide Vodola
Department of Physics and Astronomy
Bologna University

Bologna, 19/12/2019

Quantum info and simulation lab

Where this work started...




Swansea University
Prifysgol Abertawe

Innsbruck: experiment

Roman Stricker

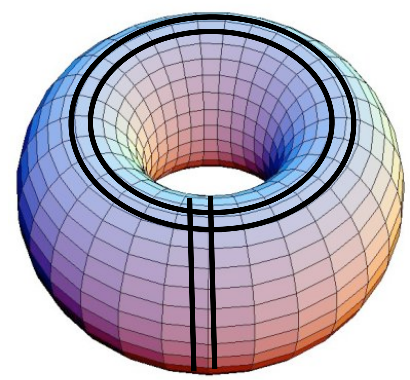
Philipp Schindler

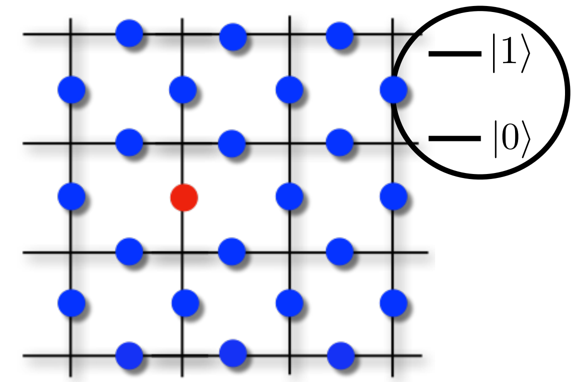
Thomas Monz

Rainer Blatt

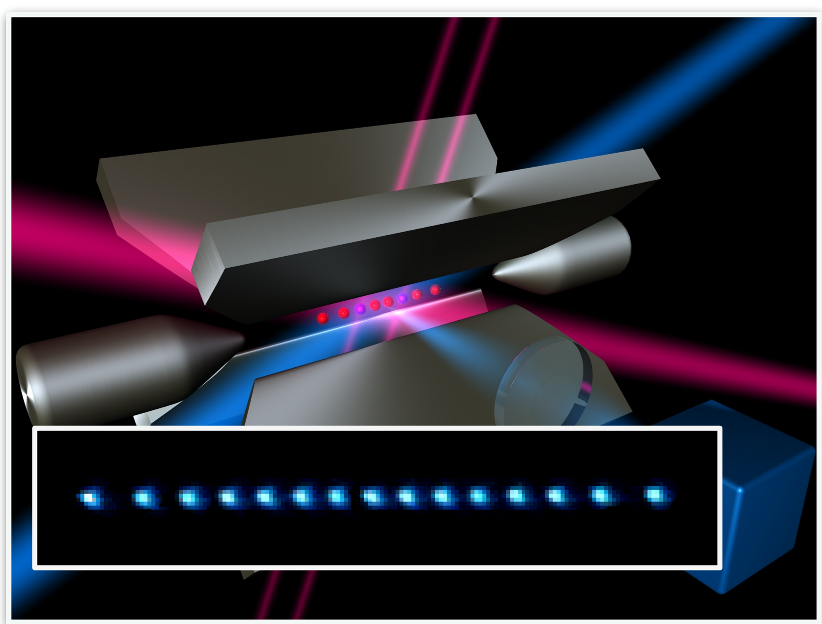
# Outline of the talk

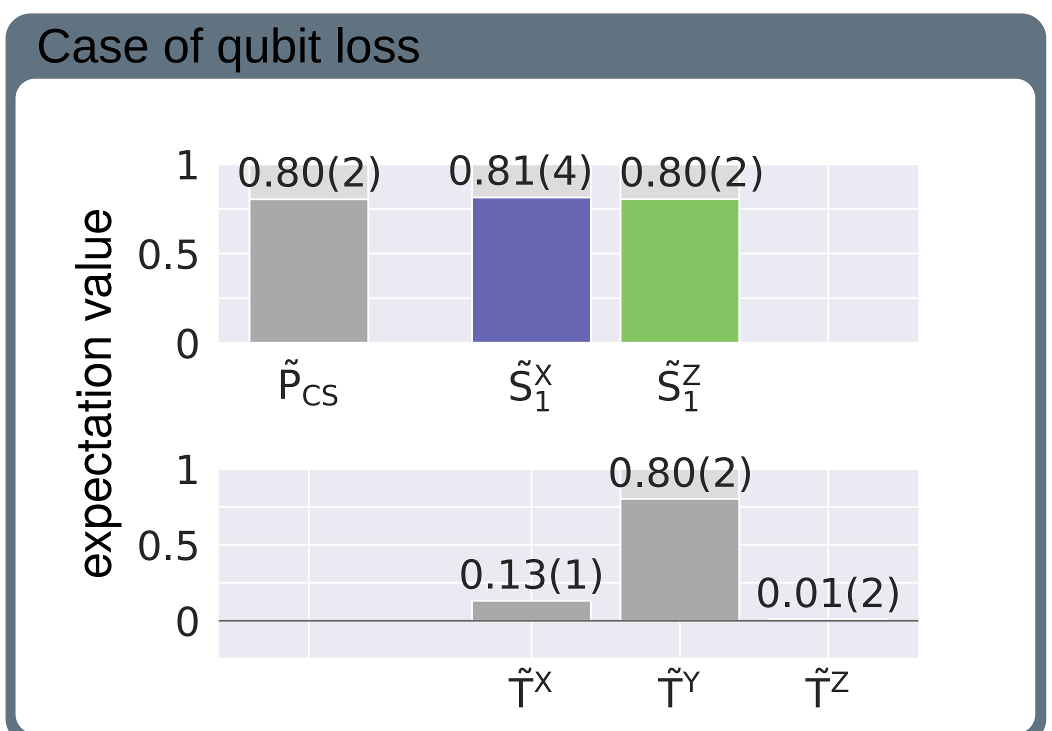
## 1 - Brief introduction to topological quantum memories: Kitaev's Toric Code






## 2 - Qubit Loss Error Correction: Theory and Experiment



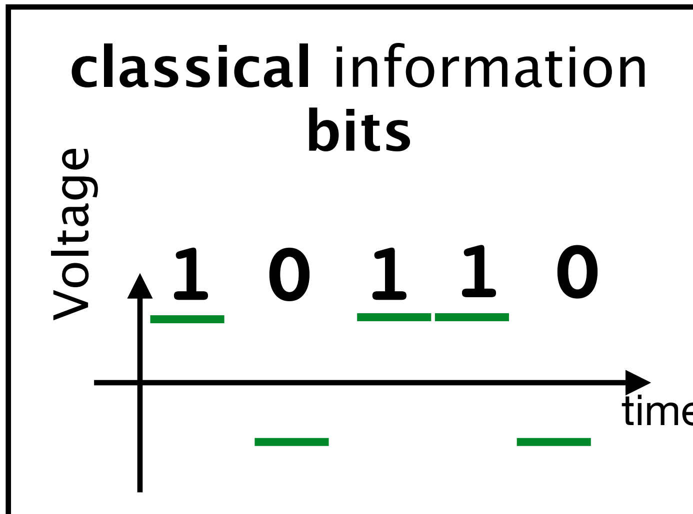


# A quantum computer is ...

a computer ...
which works based on the laws of
**quantum physics**

<u>Central ingredients:</u>

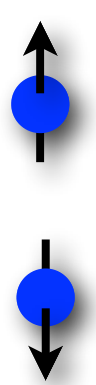
- quantum **superposition** principle
- quantum mechanical **entanglement**




Basic unit in **quantum** information:
two-level system = quantum bit (**qubit**)

— $|1\rangle$

— $|0\rangle$

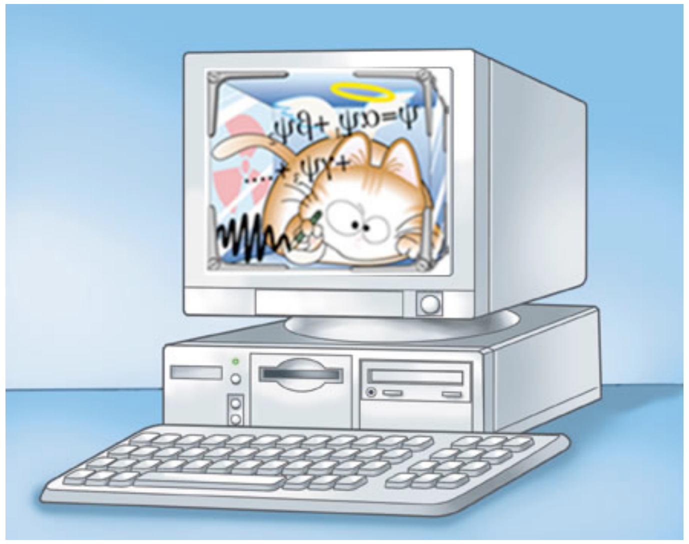


$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$$
$$c_0, c_1 \in \mathbb{C}$$

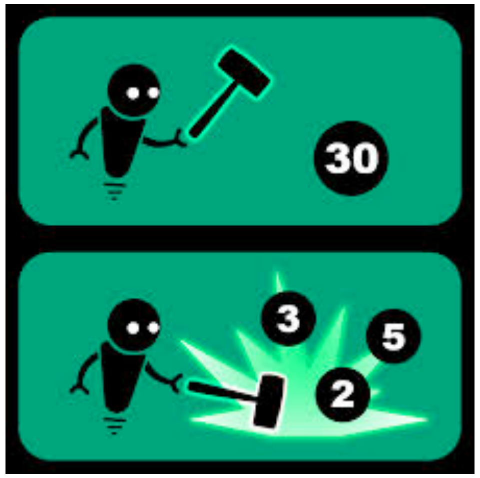
Where?

- spins of electrons
- two energy levels of atoms/ions
- polarization states of photons
- Josephson junctions

# Why should we build a large-scale and fault-tolerant quantum computer?

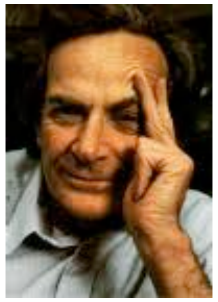


prime factoring
(Shor's alg.)




data base search
(Grover's alg.)

...

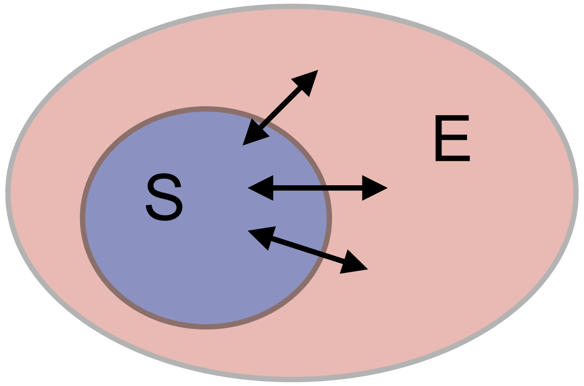


many–body
quantum
systems

universal quantum
simulation

# Main obstacle towards quantum computers: decoherence & errors

Coupling to the environment causes decoherence




Examples

**1. Magnetic field fluctuations**

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$ quantum state

dephasing

$$\rho = |\alpha_0|^2|0\rangle\langle 0| + |\alpha_1|^2|1\rangle\langle 1|$$ classical state

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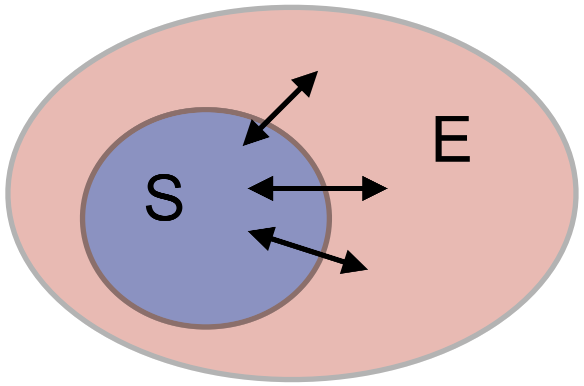


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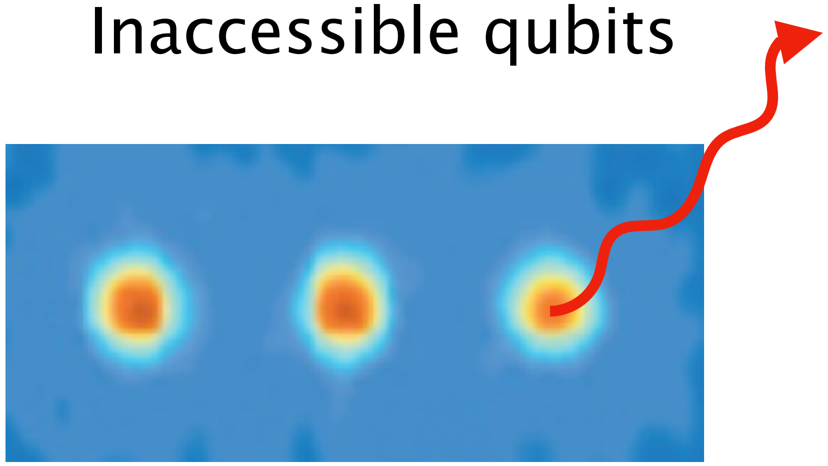
**2. Losses**

This talk

Inaccessible qubits



# Need for error correction: naive approach ...

- Classical world:
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...011010...

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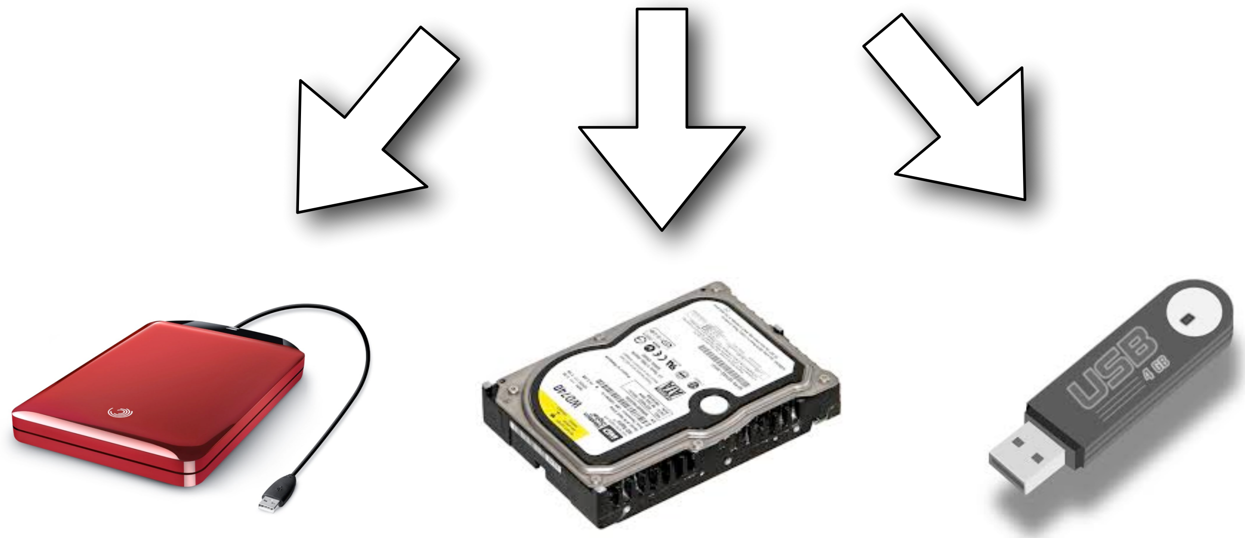

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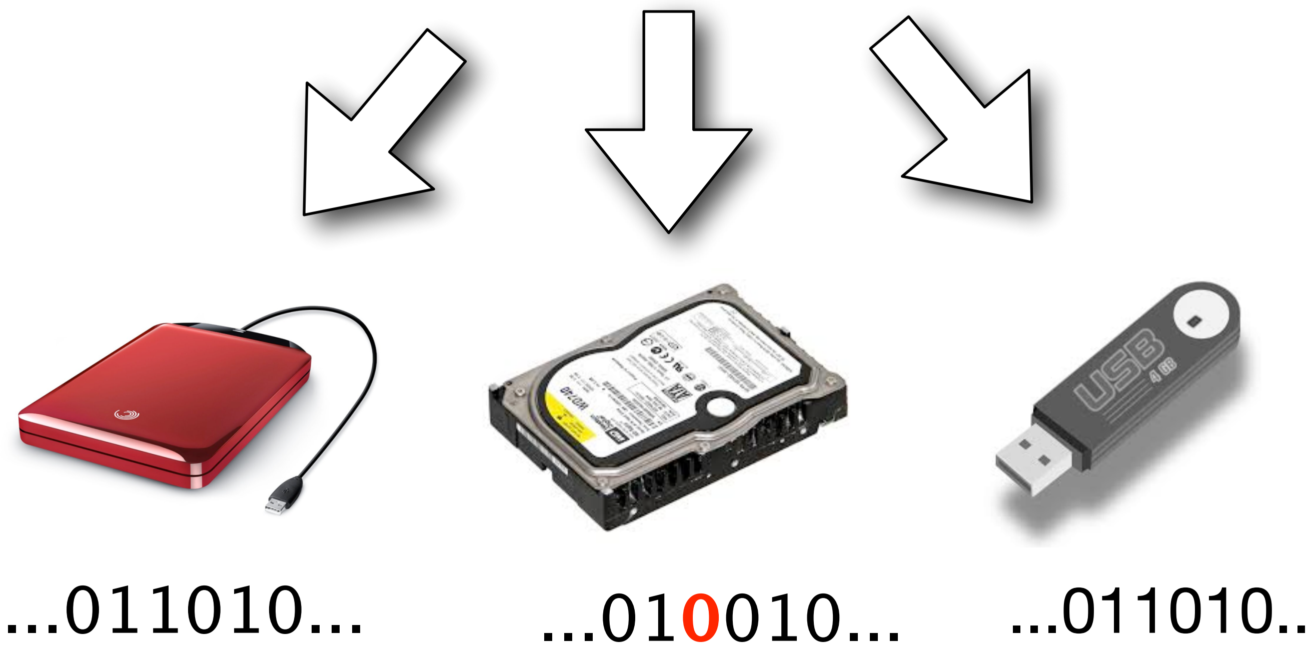

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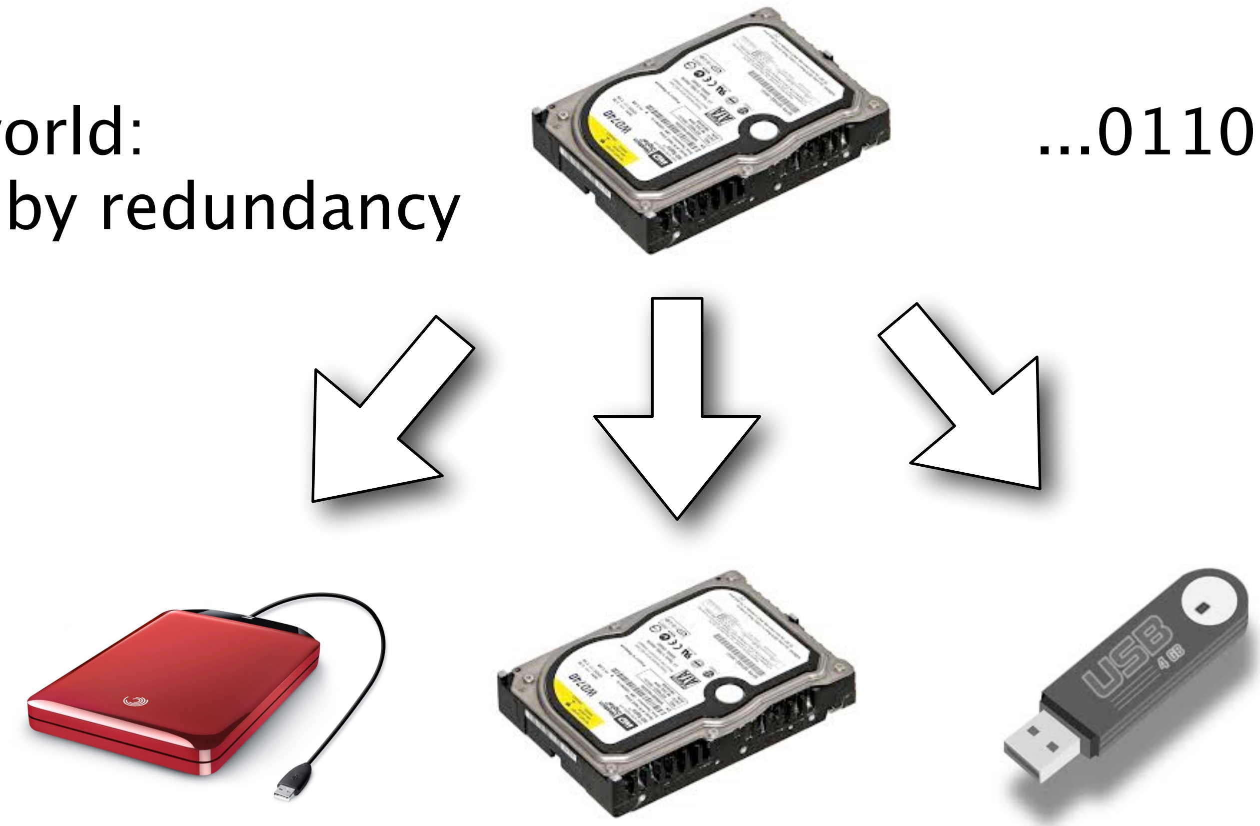


...011010...

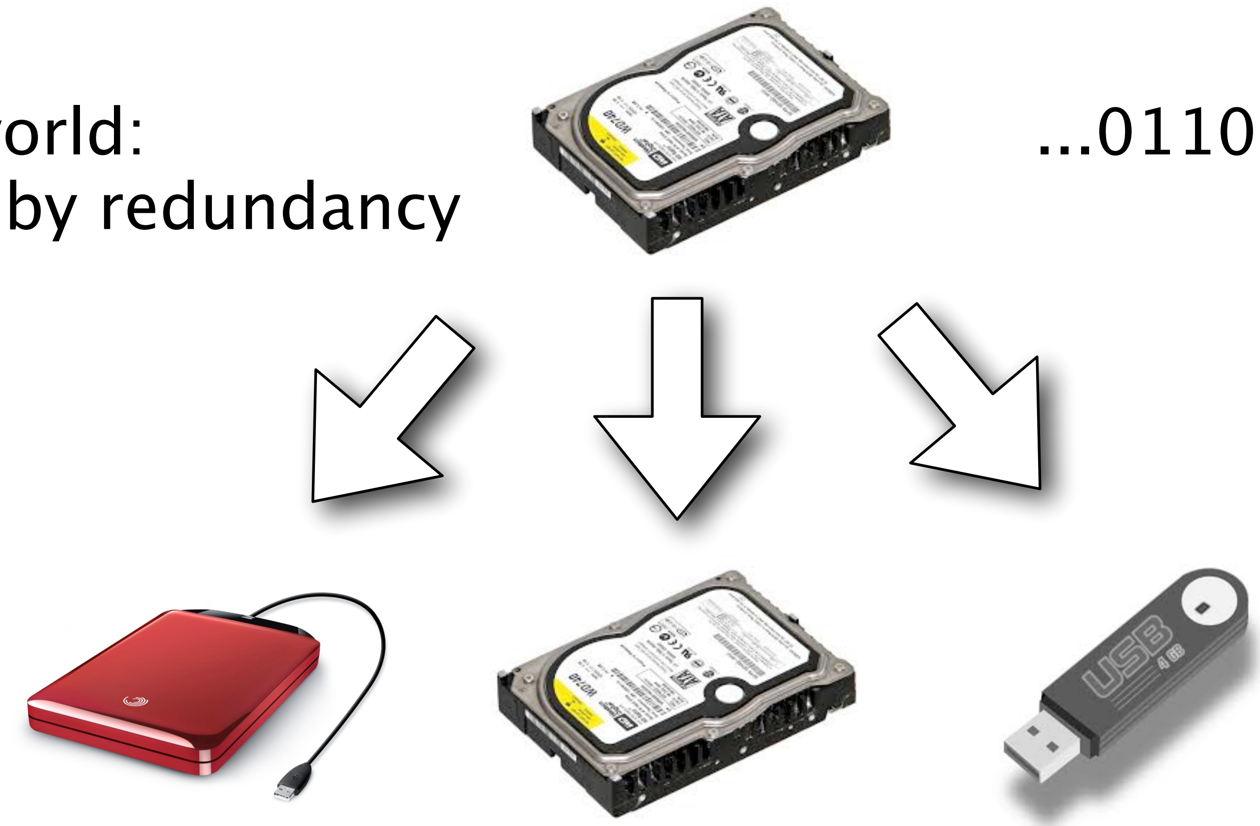
...011010...    ...010010...    ...011010...

recover: ...011010...

# Need for error correction: naive approach ... fails

- Classical world:
  protection by redundancy

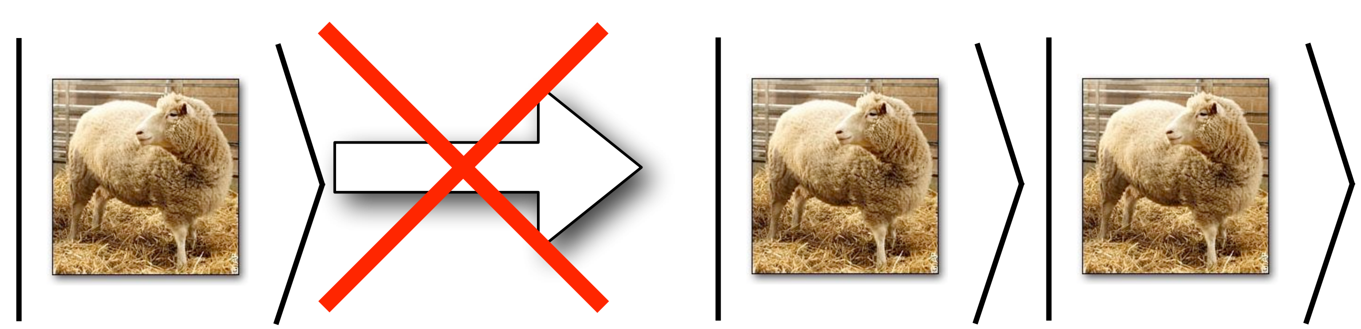
...011010...

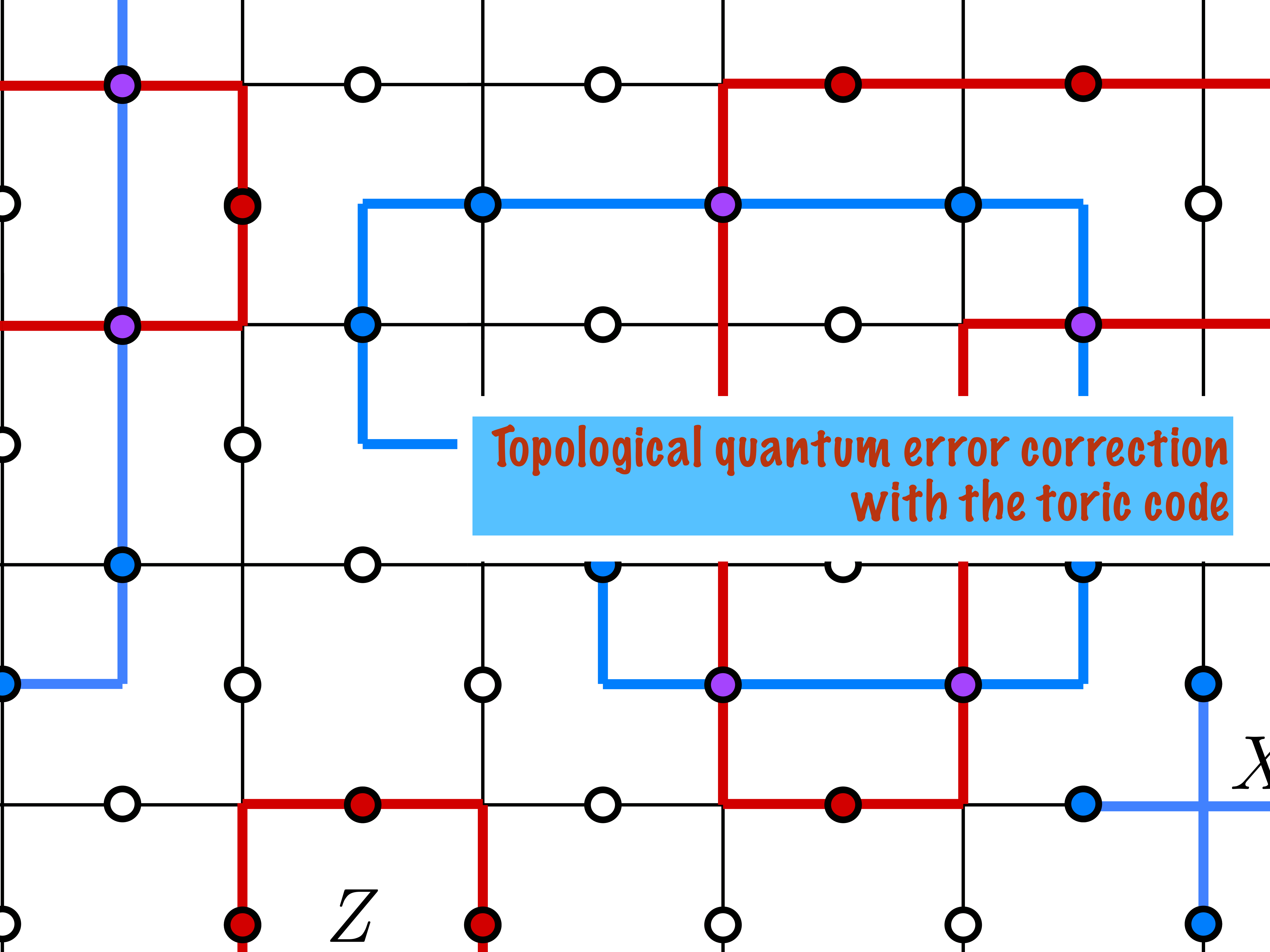



...011010... ...010010... ...011010... recover: ...011010...

- Quantum world: not possible in this way
  **no-cloning theorem for quantum states**

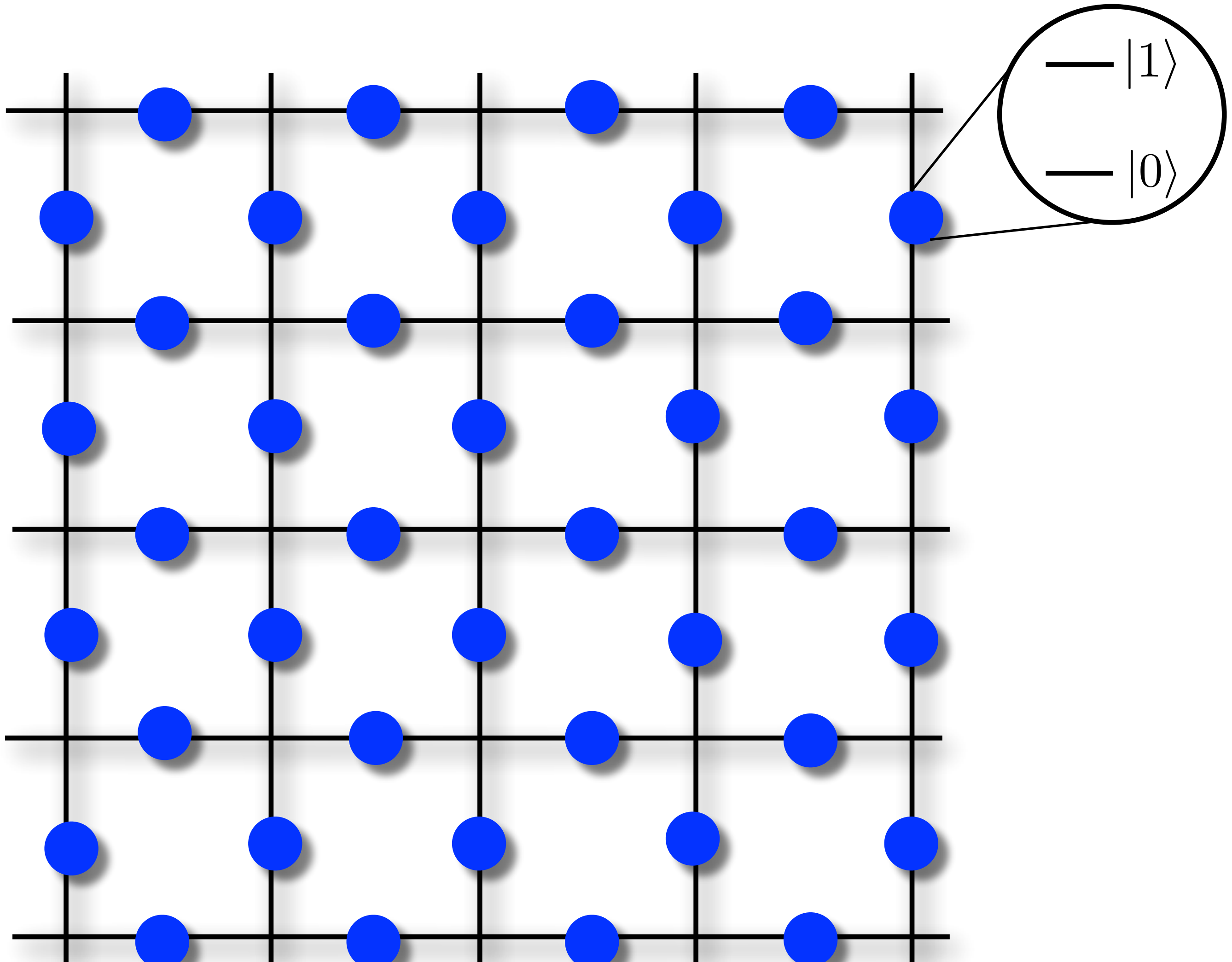
Quantum states can't be copied!



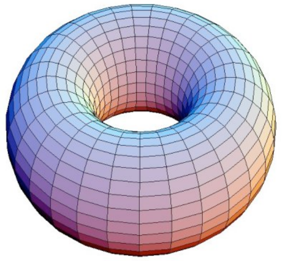

Topological quantum error correction
with the toric code
Z
X

# Kitaev's toric code

A. Yu. Kitaev, *Annals of Physics*, **303** (2003)




- Qubits on the links / bonds of a 2D square lattice



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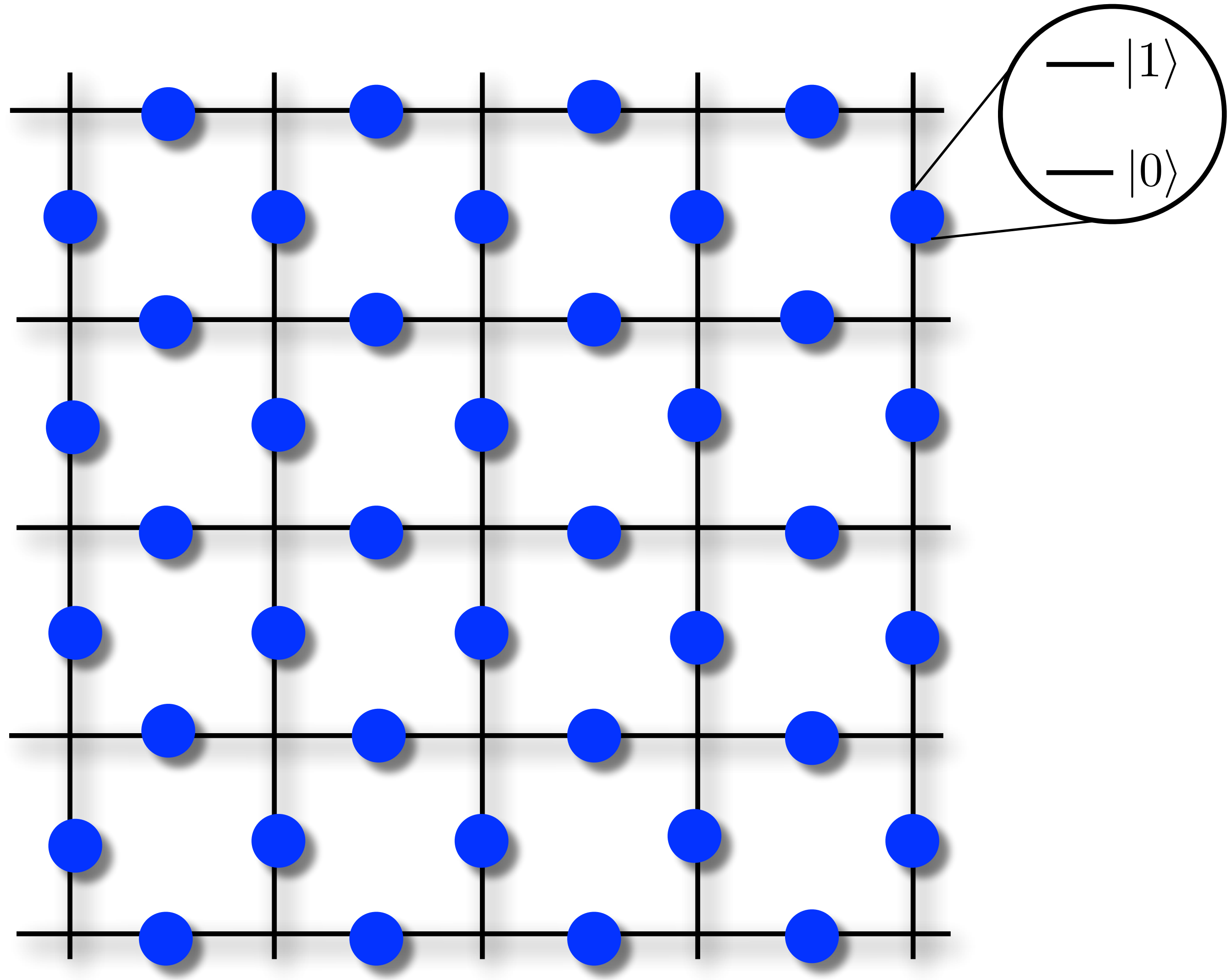


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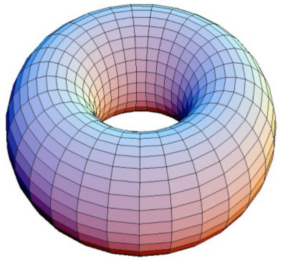

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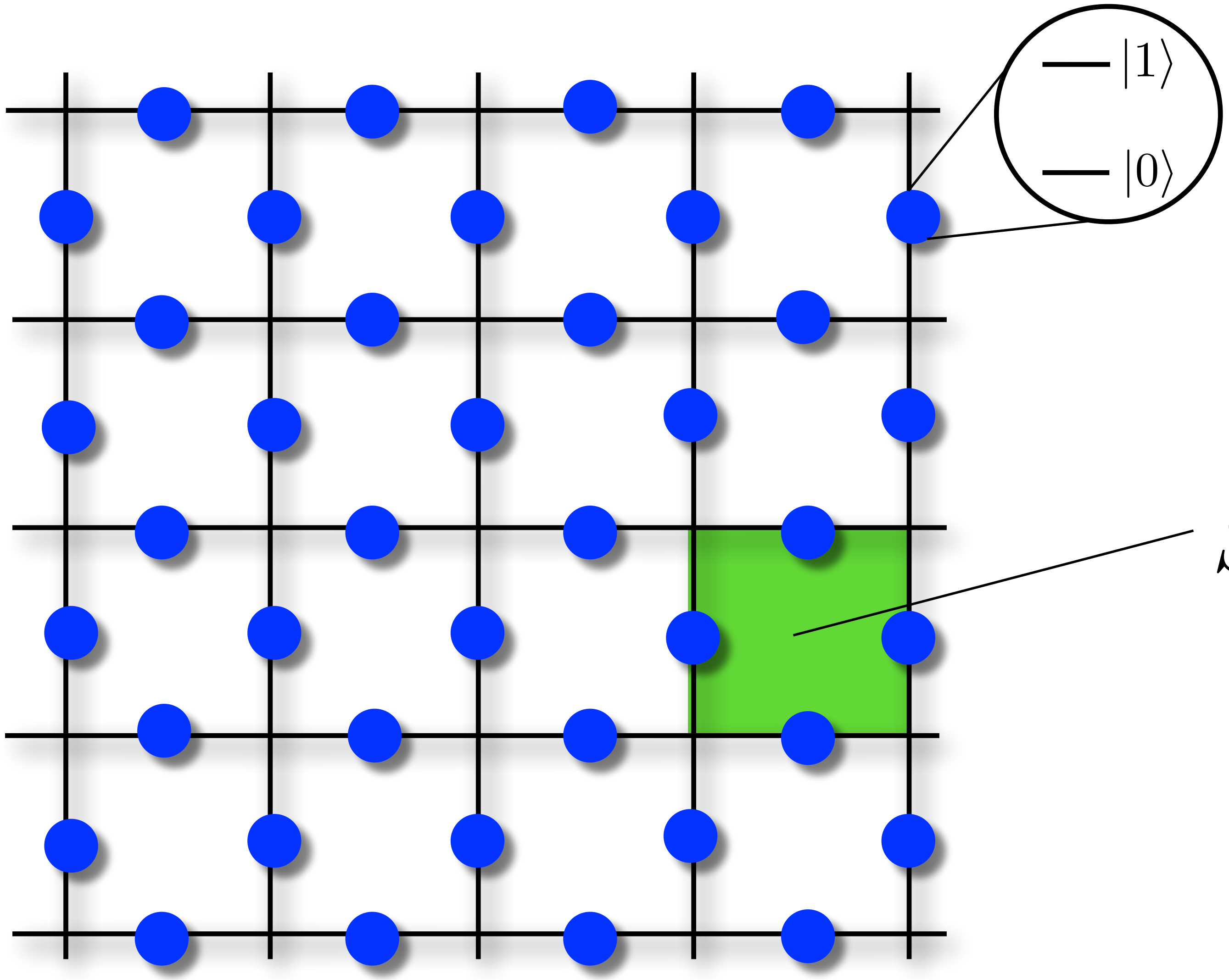


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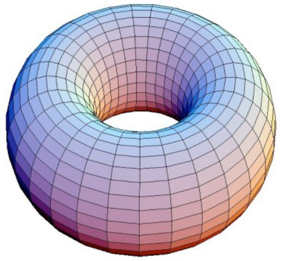

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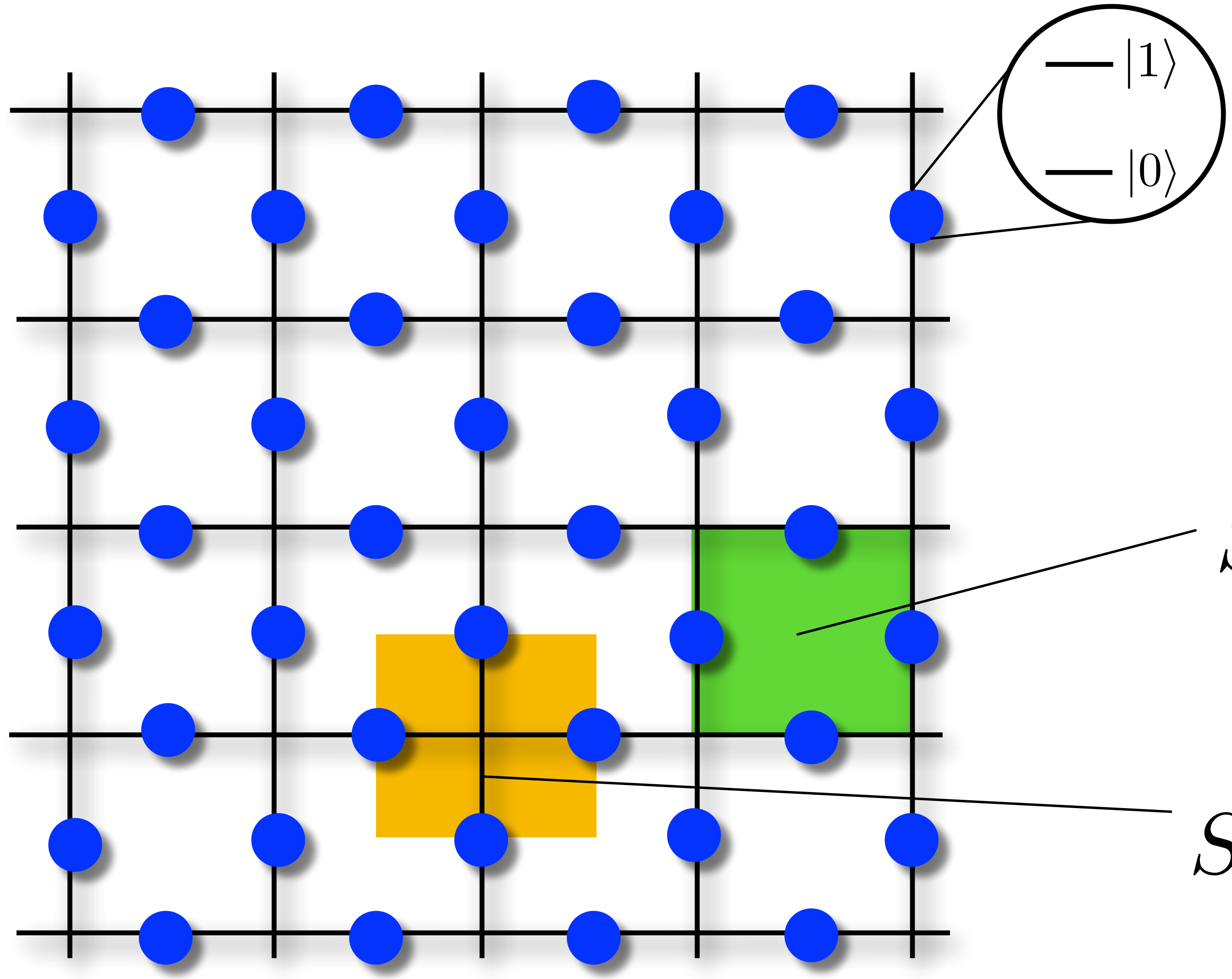
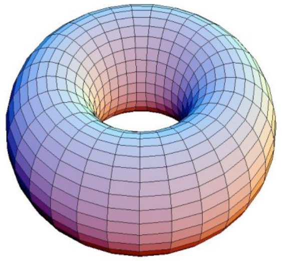



- Qubits (blue dots) on the links / bonds of a 2D square lattice



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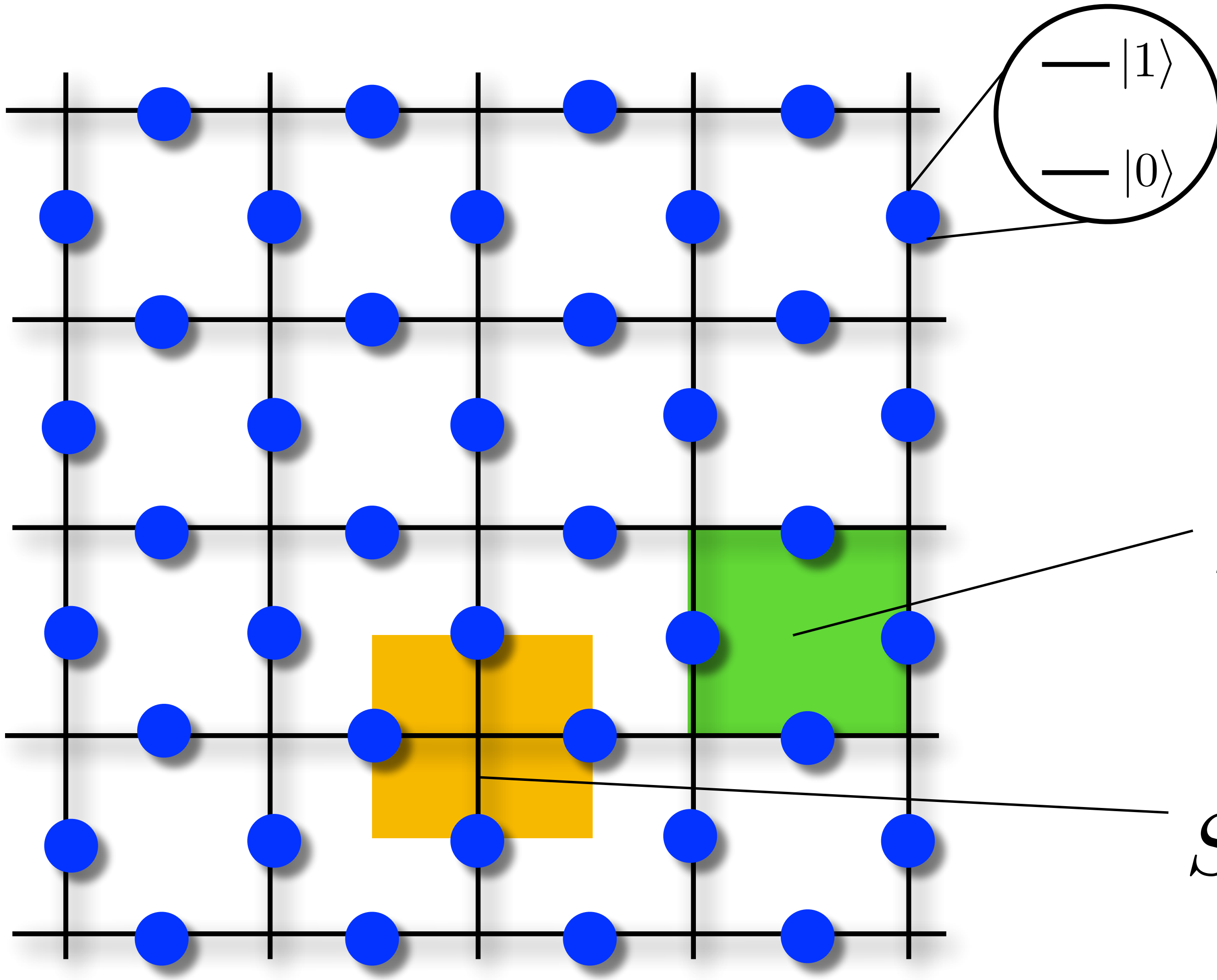


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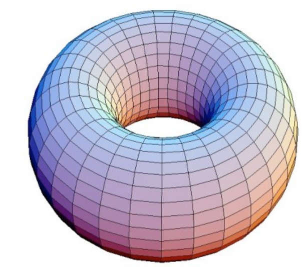

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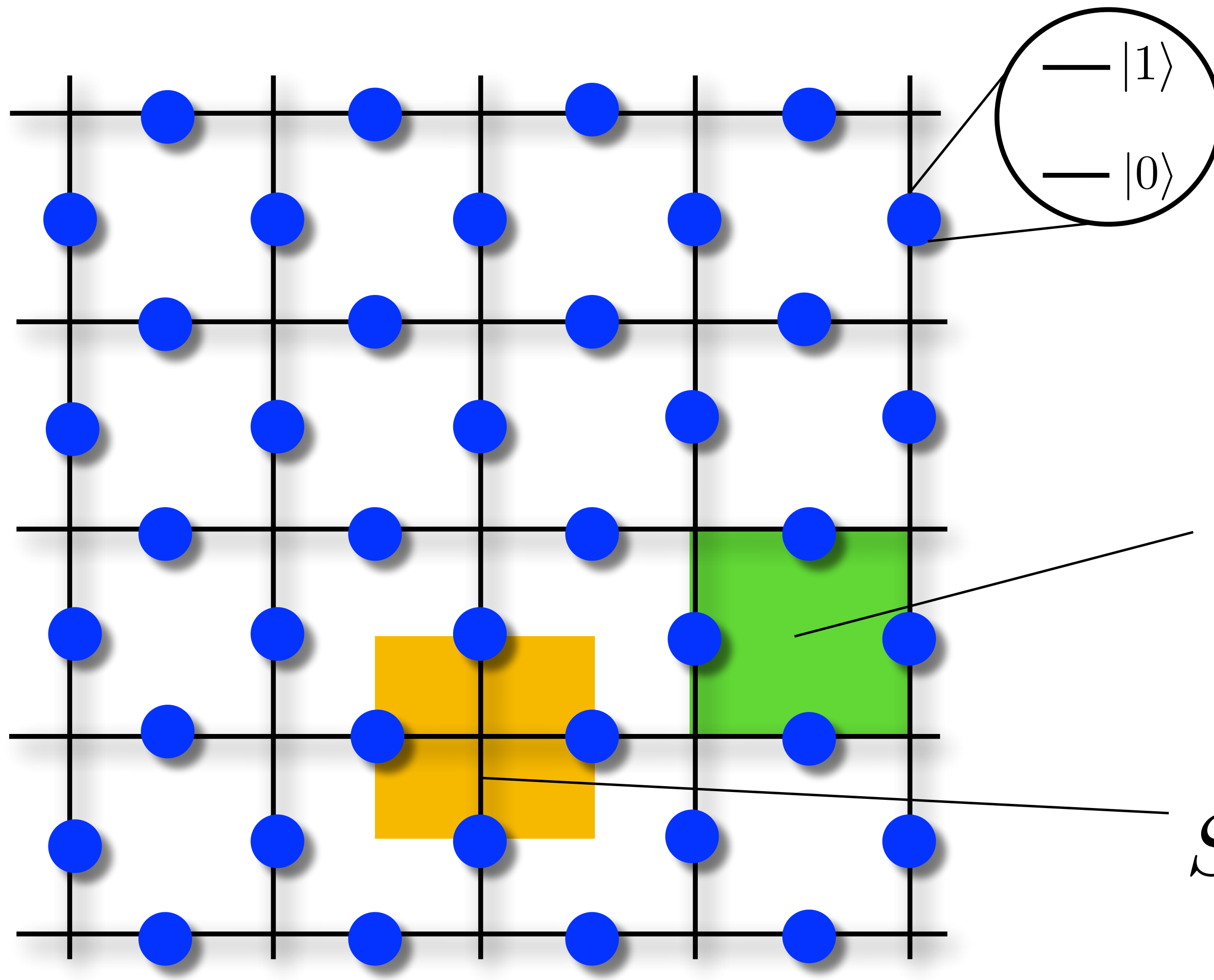


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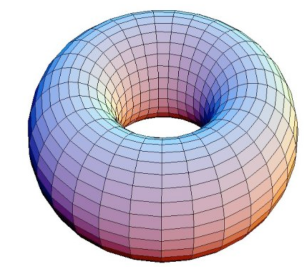

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Logical info

code space

$$S_z|\psi_L\rangle = +|\psi_L\rangle \qquad S_x|\psi_L\rangle = +|\psi_L\rangle$$

$|\bar{0}\rangle|\bar{0}\rangle$ $|\bar{0}\rangle|\bar{1}\rangle$ $|\bar{1}\rangle|\bar{0}\rangle$ $|\bar{1}\rangle|\bar{1}\rangle$

logical states

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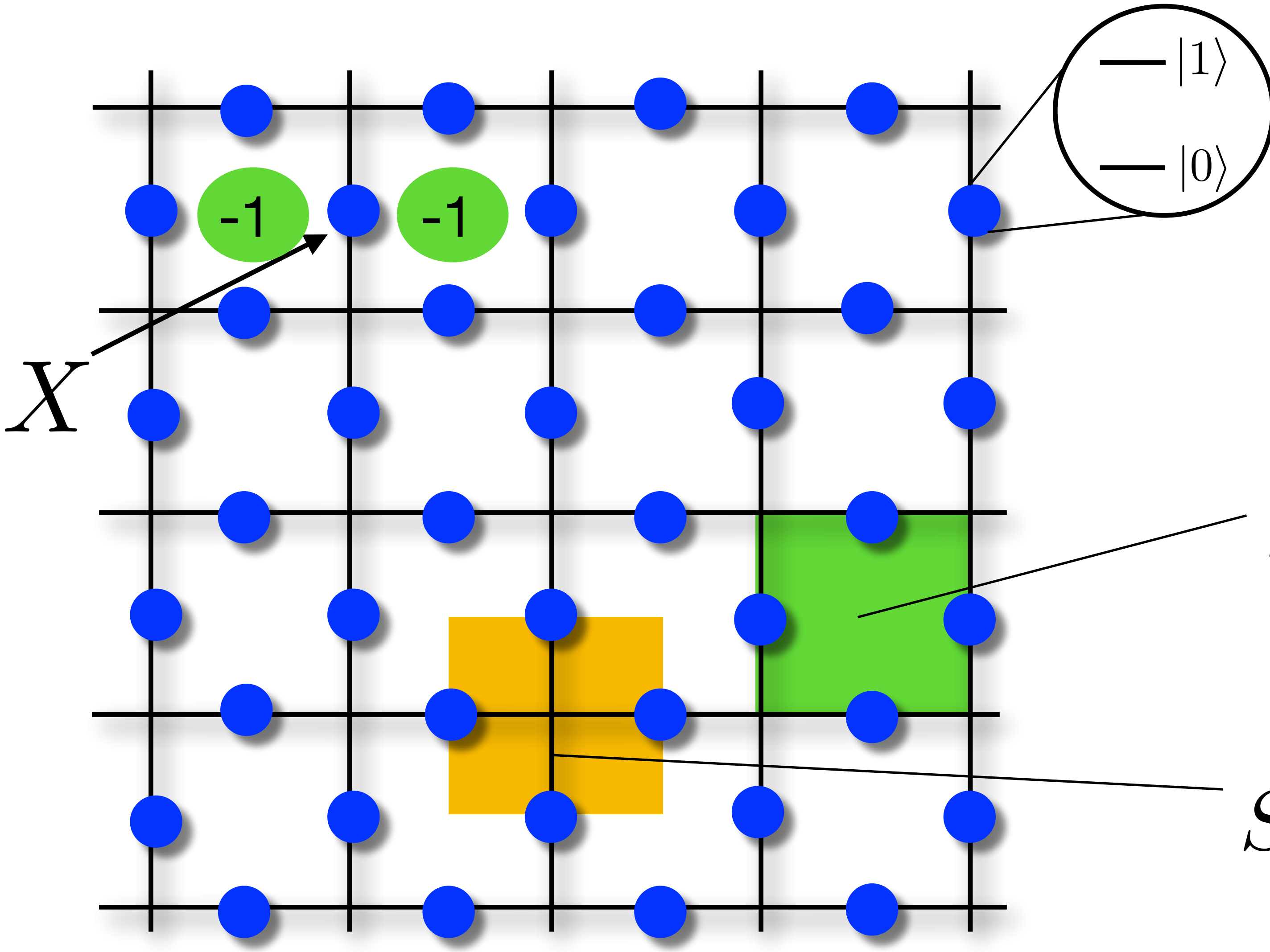


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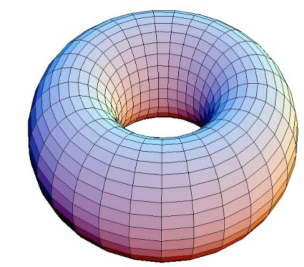

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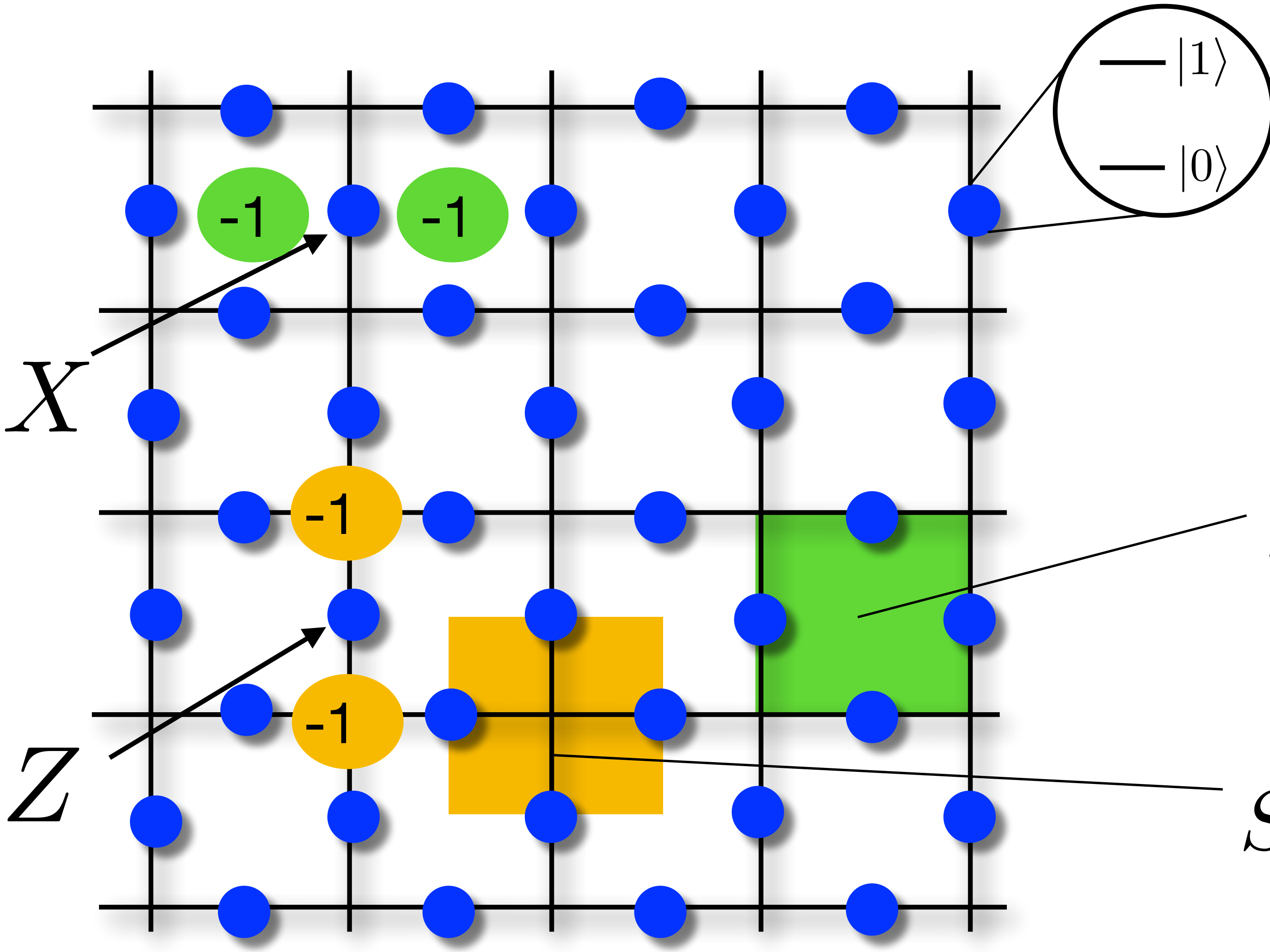


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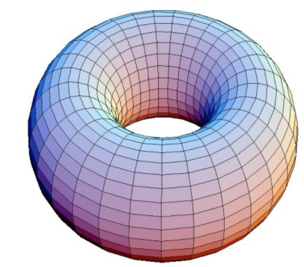

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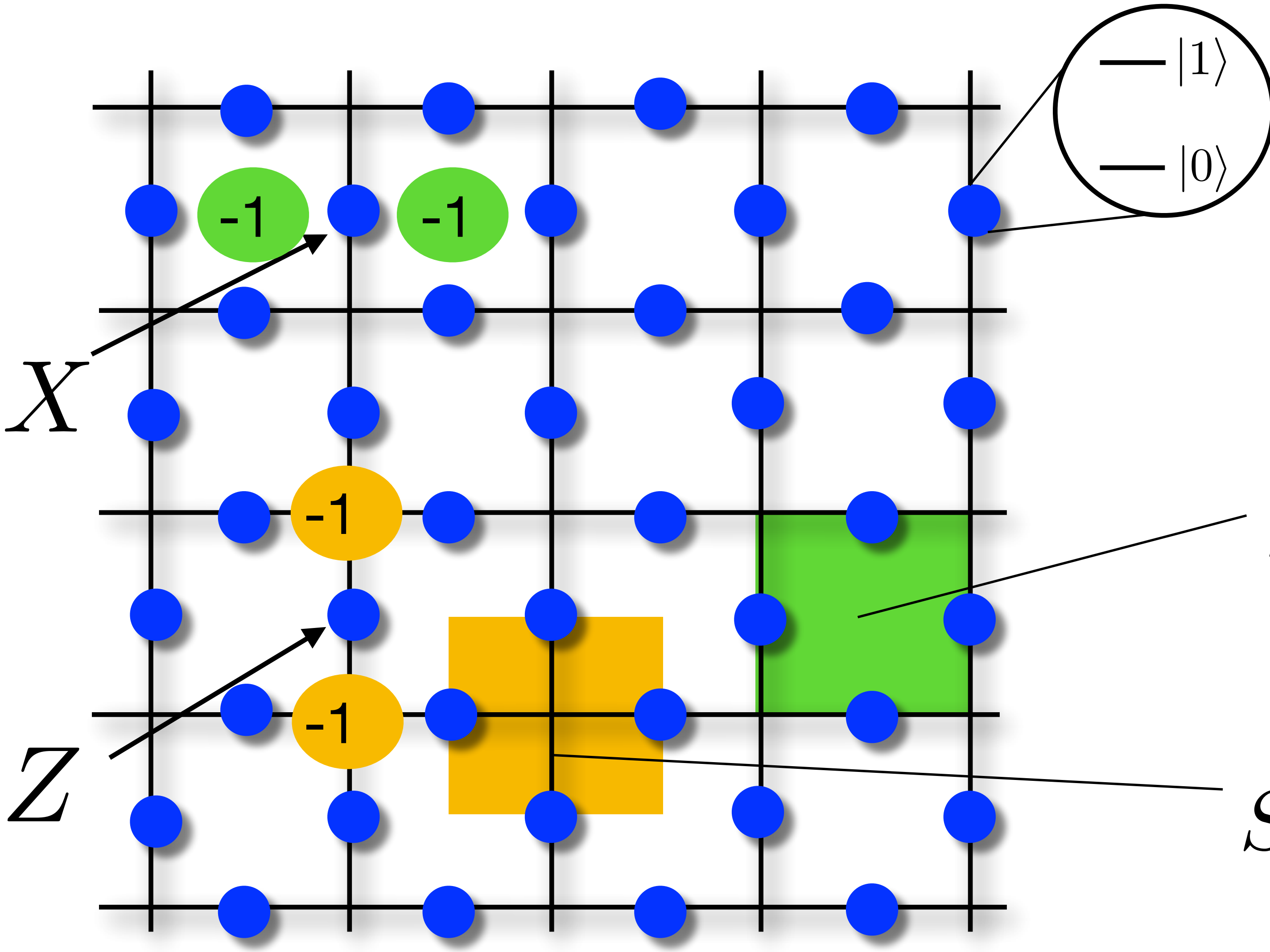


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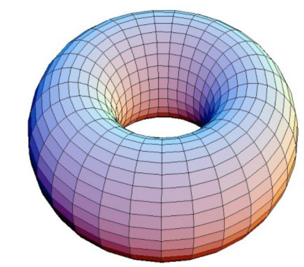

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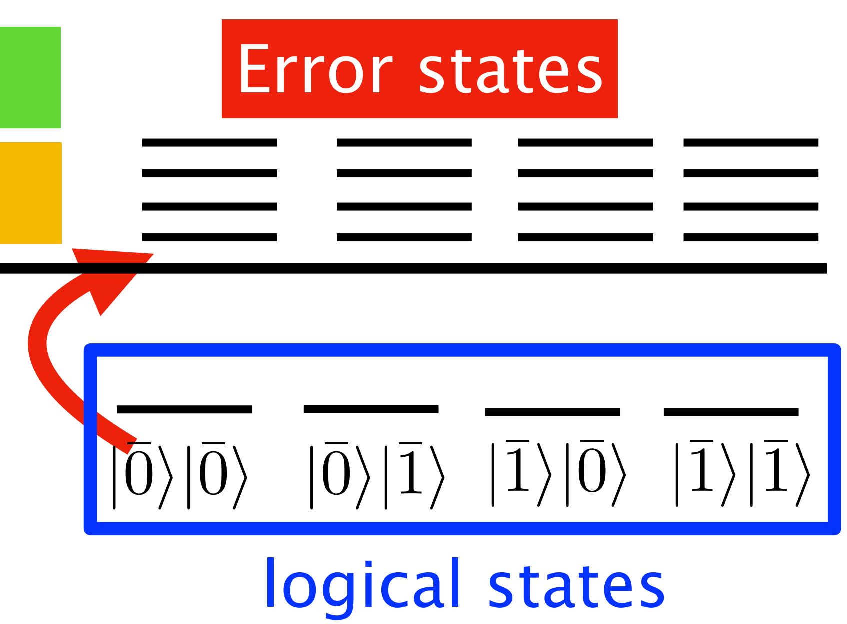
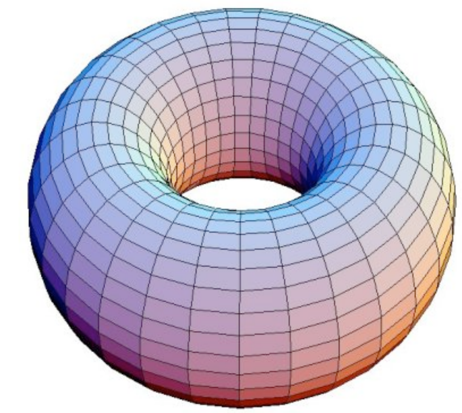


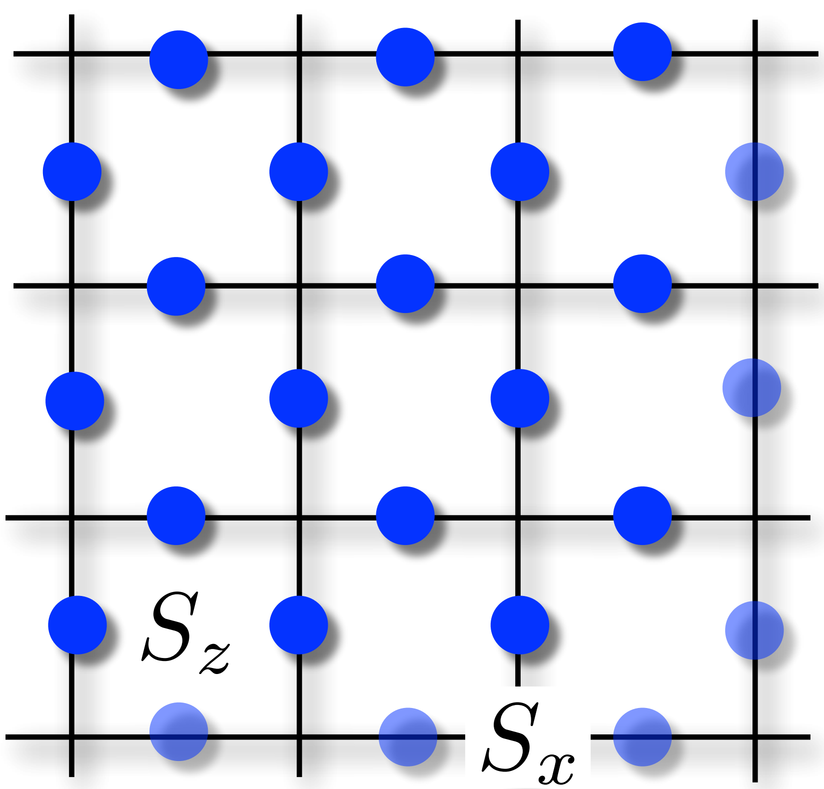
# Logical qubits

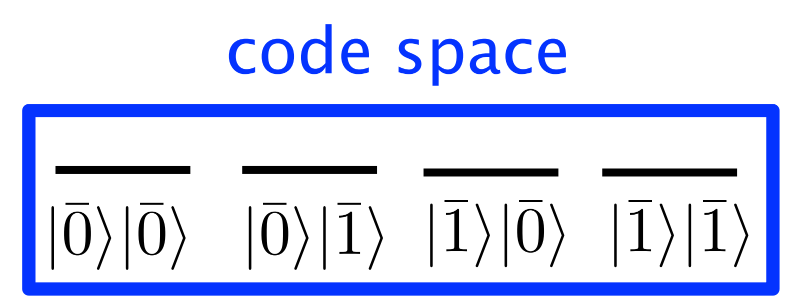
**Logical operators**

- must commute with all stabilisers
- must be independent
- must respect the anticommutation relations e.g.

$$\{\bar{X}_1, \bar{Z}_1\} = 0$$





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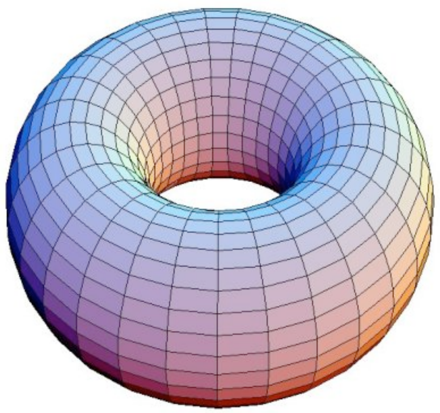

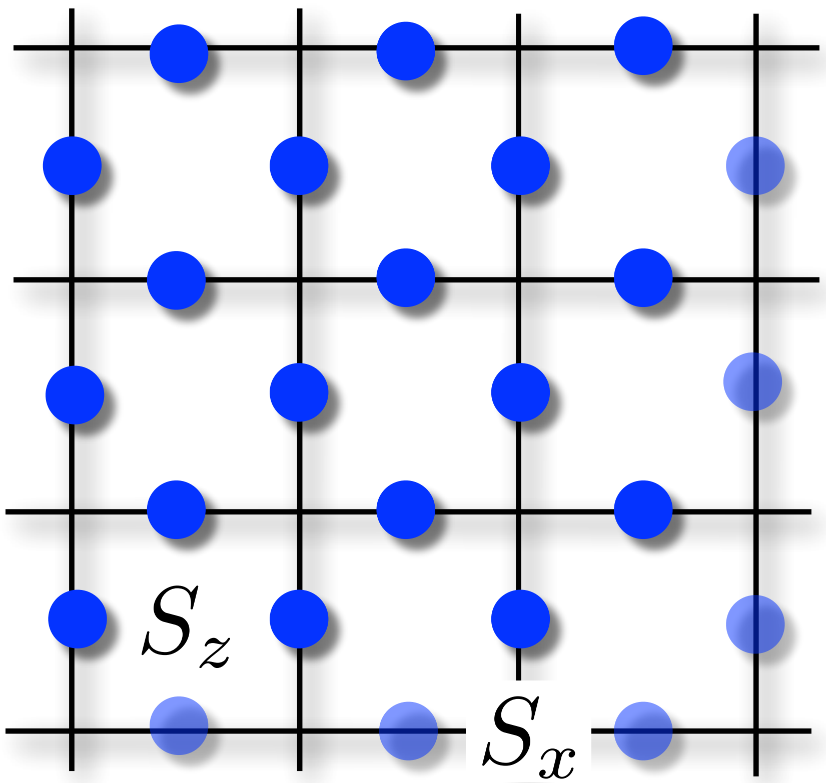


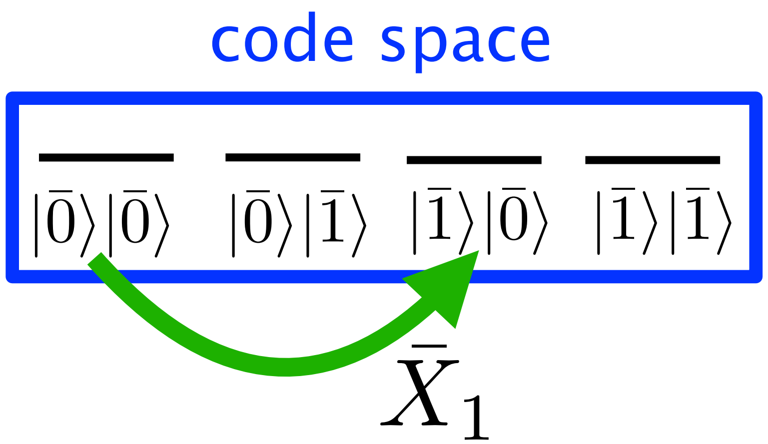

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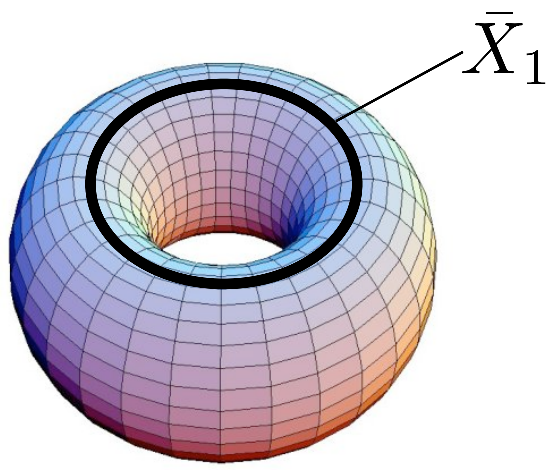


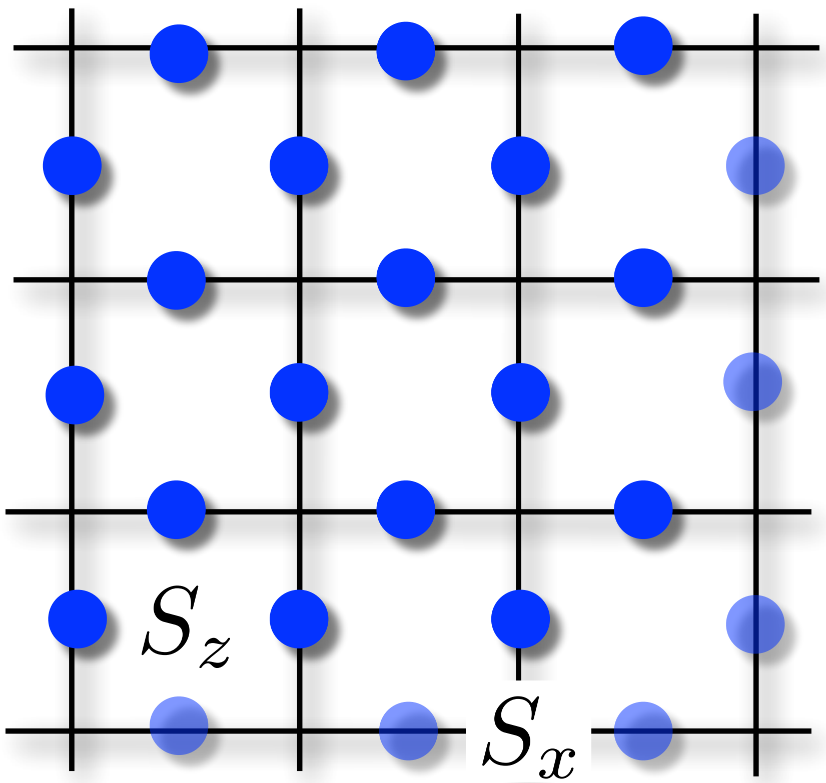


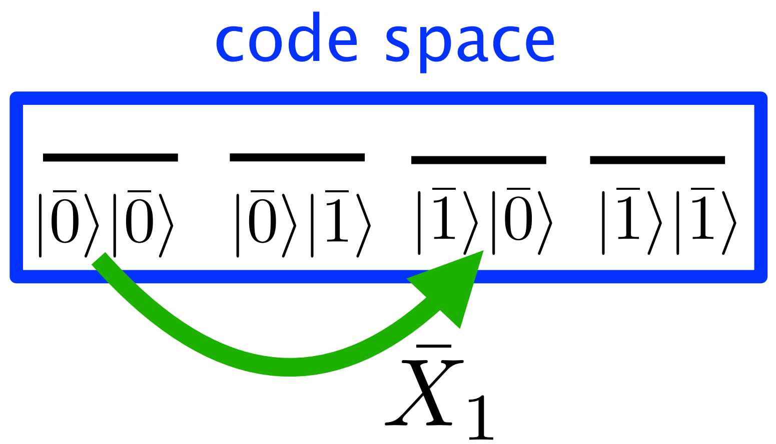

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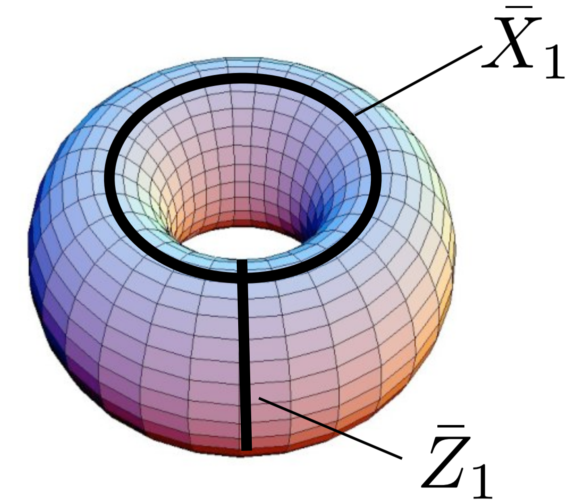


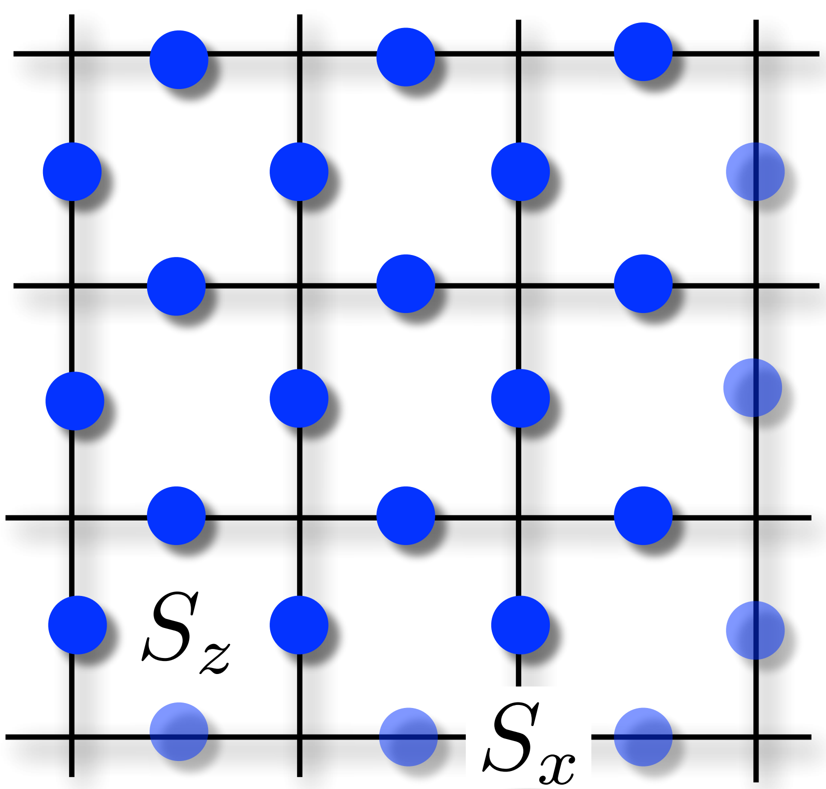


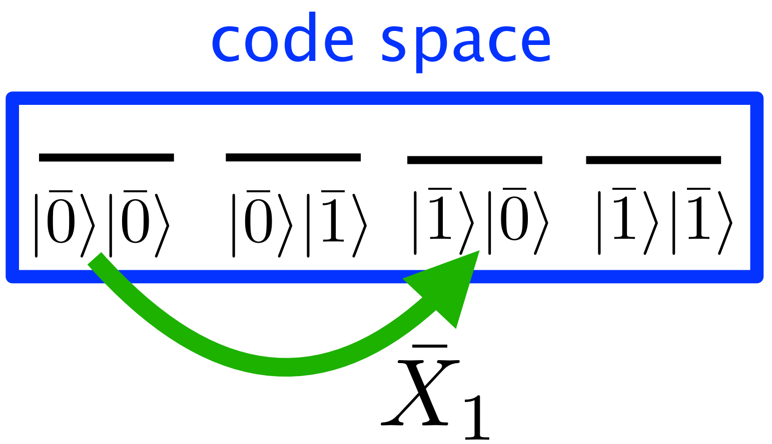

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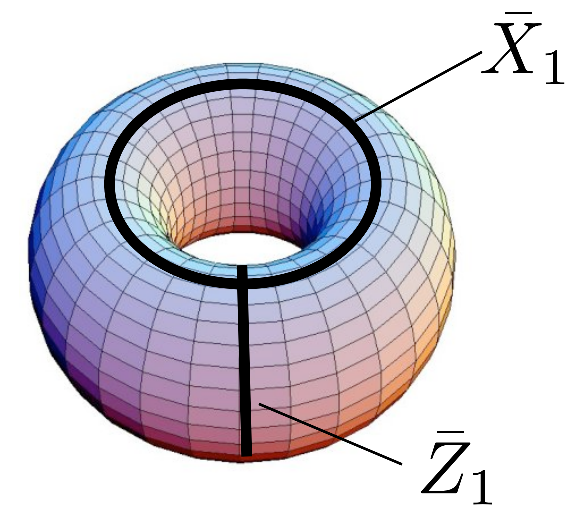
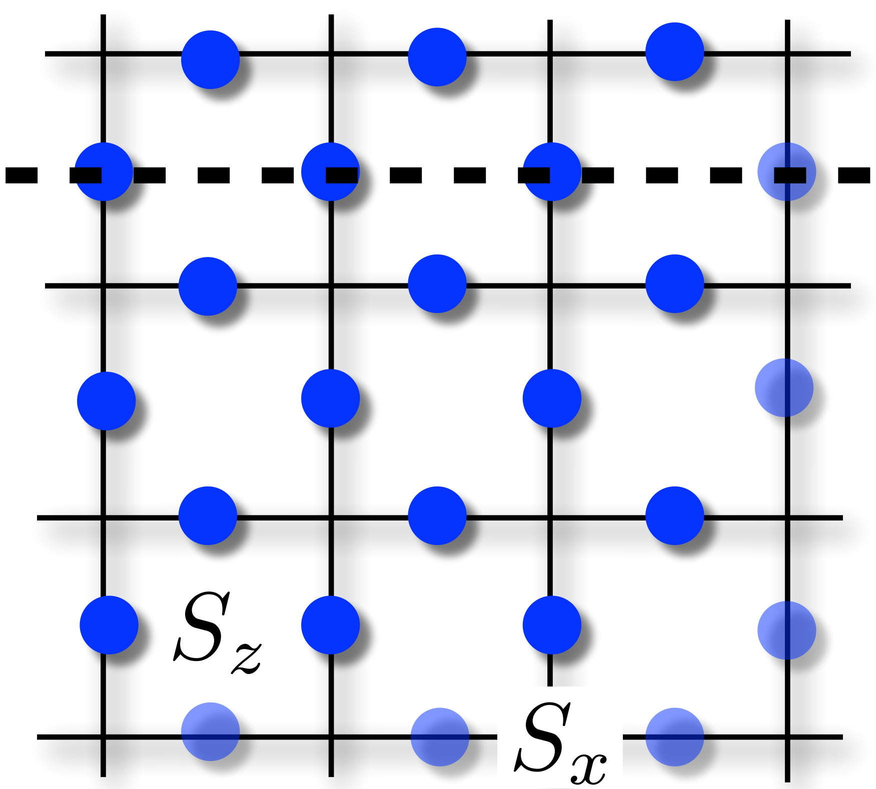
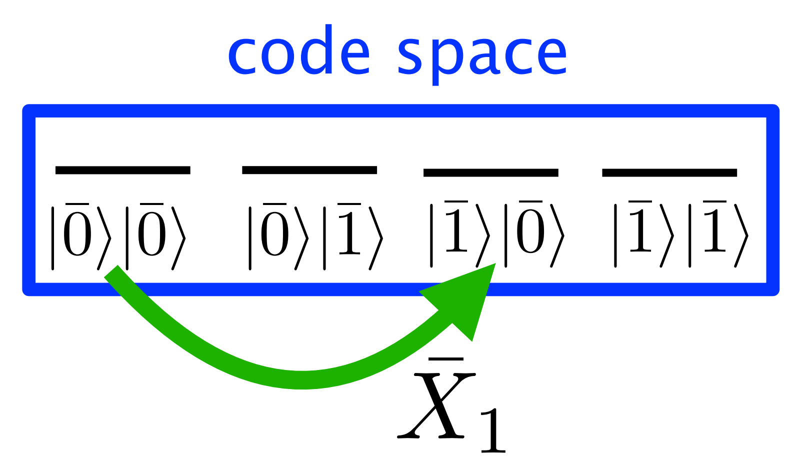



$$\bar{X}_1 = \sigma_1^x \sigma_2^x \ldots \sigma_n^x$$



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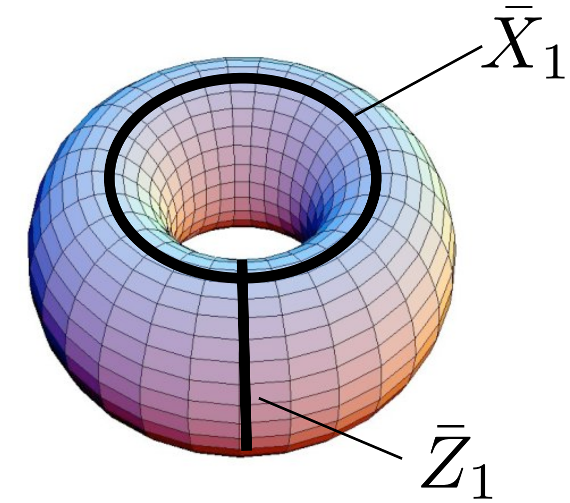


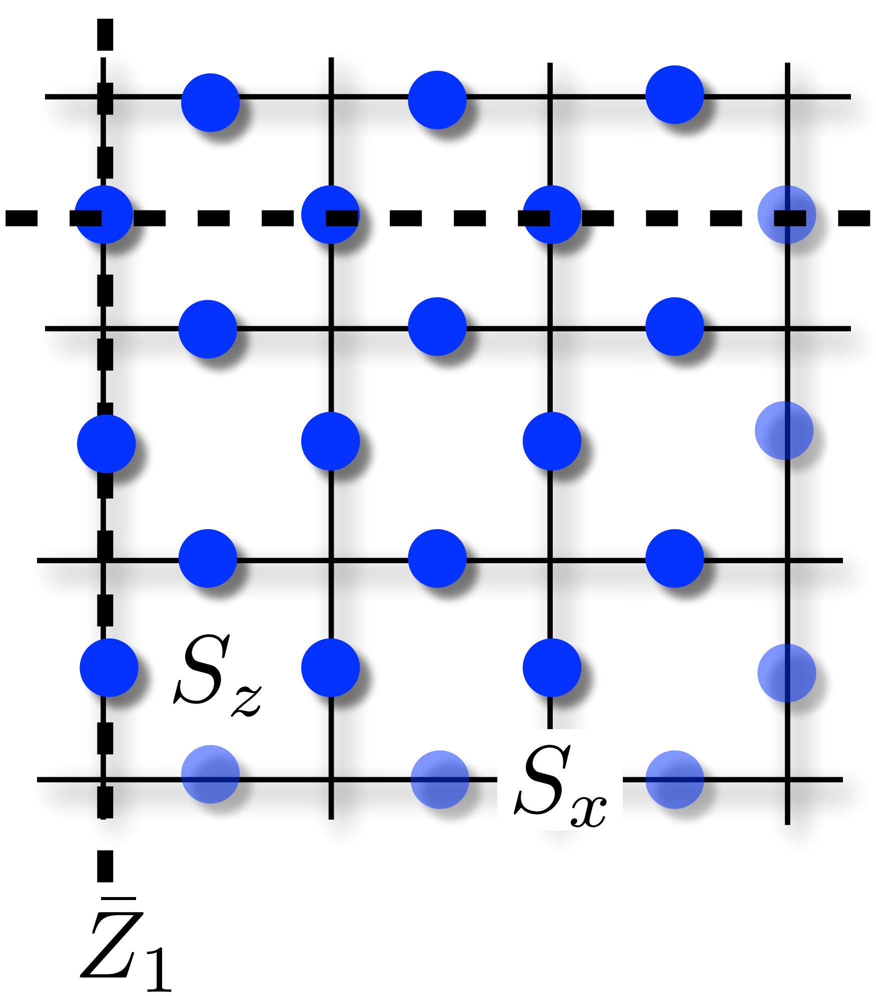


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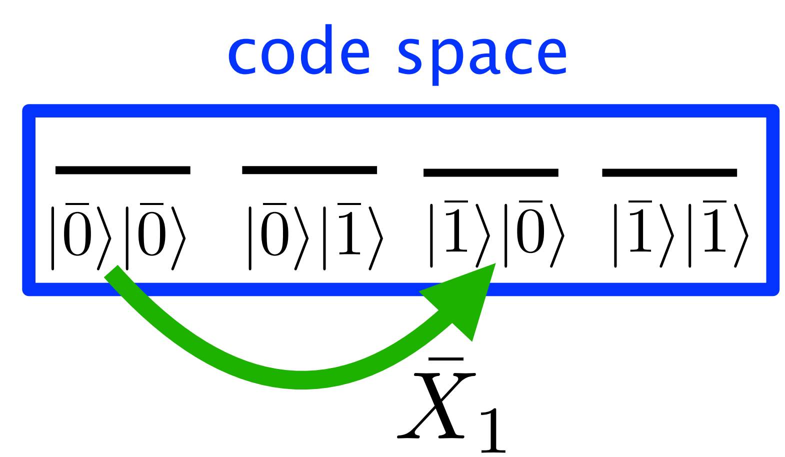
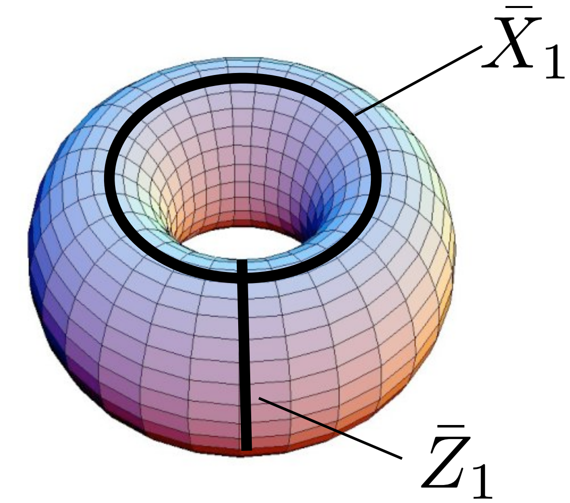


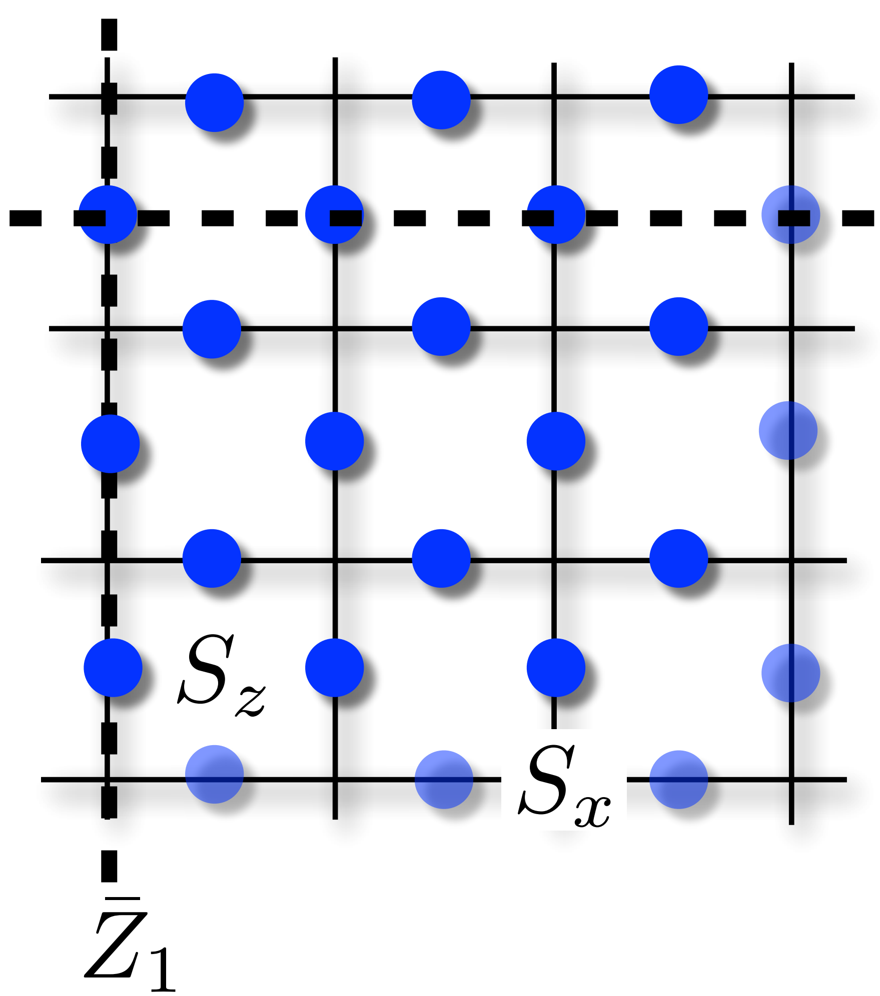
# Logical qubits

**Logical operators**

- must commute with all stabilisers
- must be independent
- must respect the anticommutation relations e.g.

$$\{\bar{X}_1, \bar{Z}_1\} = 0$$

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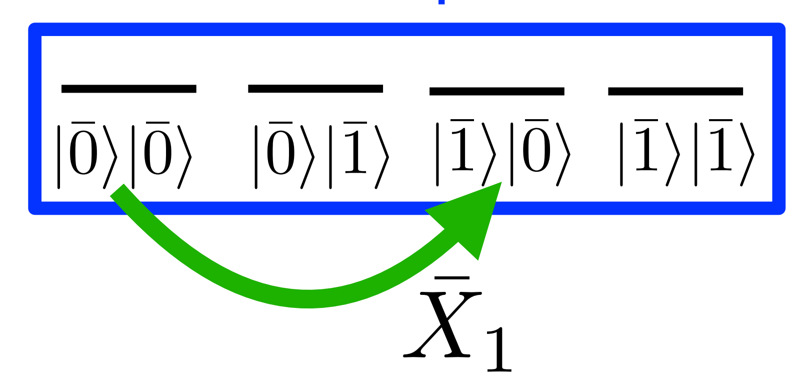
Logical operators = strings that percolate through the lattice and change the logical state in the code space



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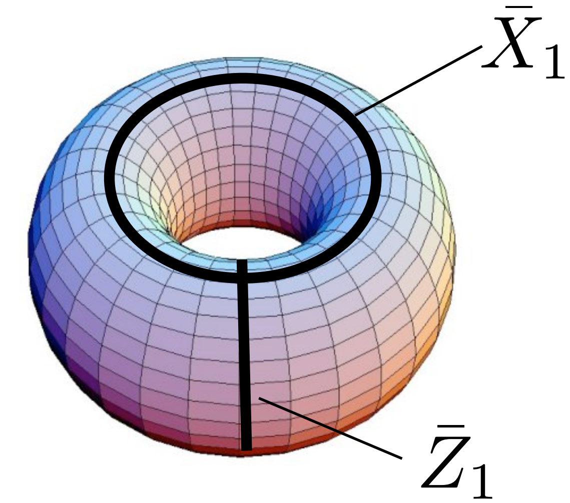


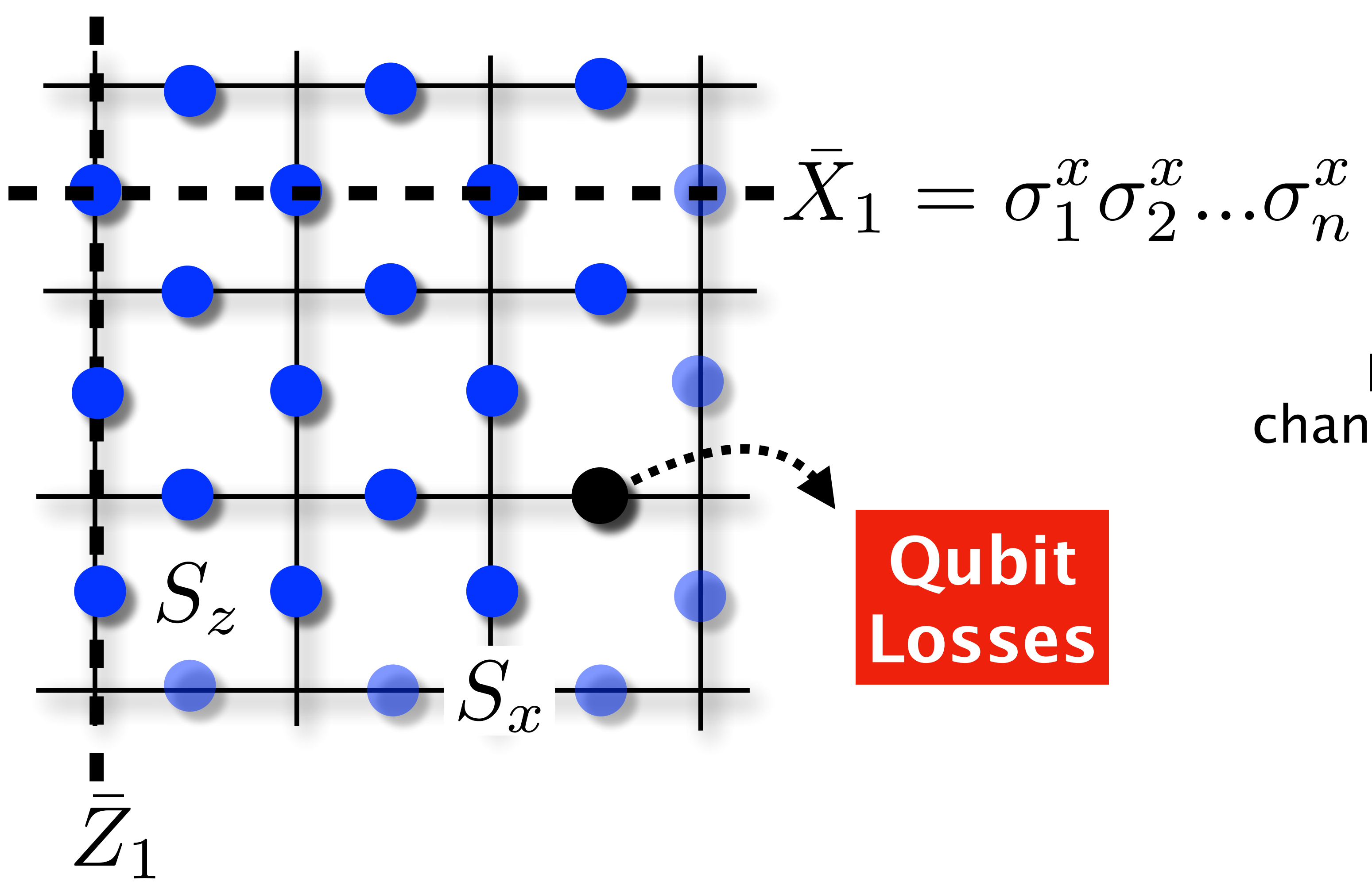


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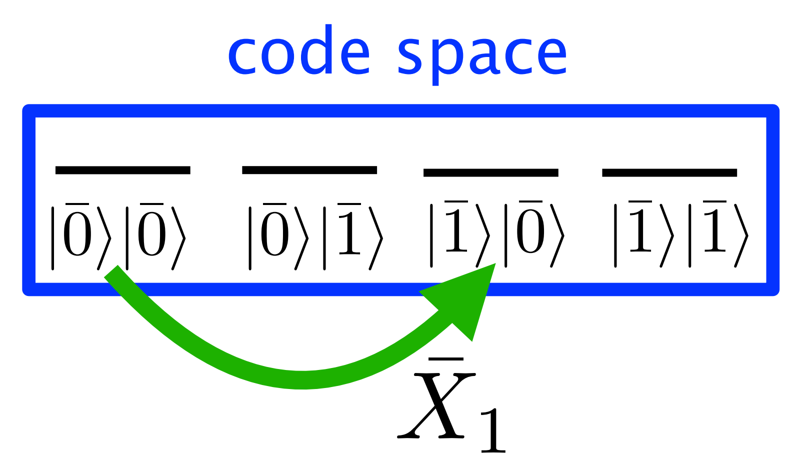
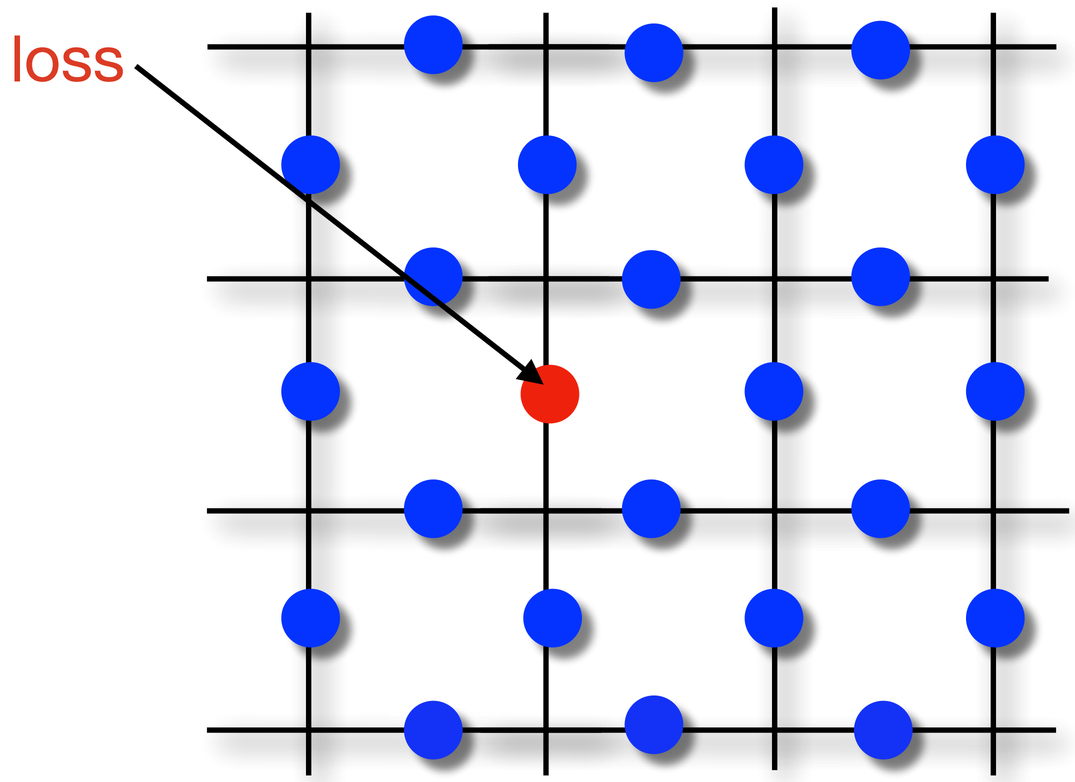


# Qubit losses in the toric code

T. Stace, S. Barrett, A. Doherty,
PRL **102**, 200501 (2009)
PRA **81**, 022317 (2010)

Goal

Redefine the plaquette/vertex and the logical operators



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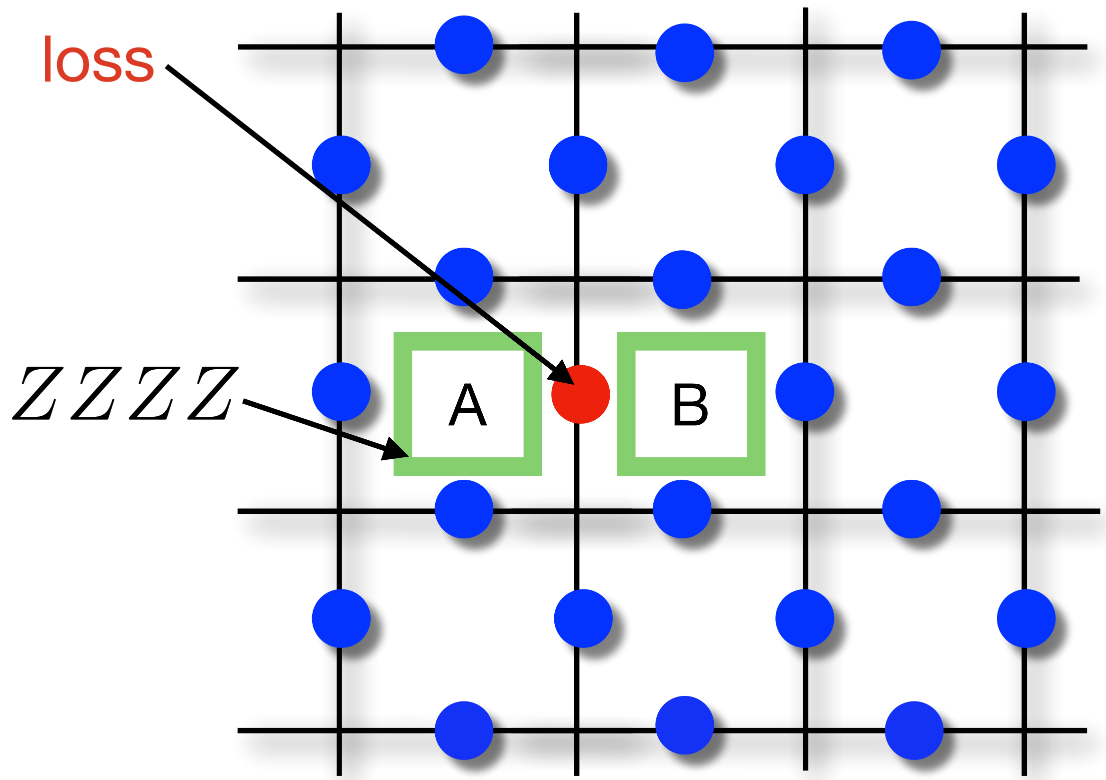


The loss affects

- two Z-stabilisers

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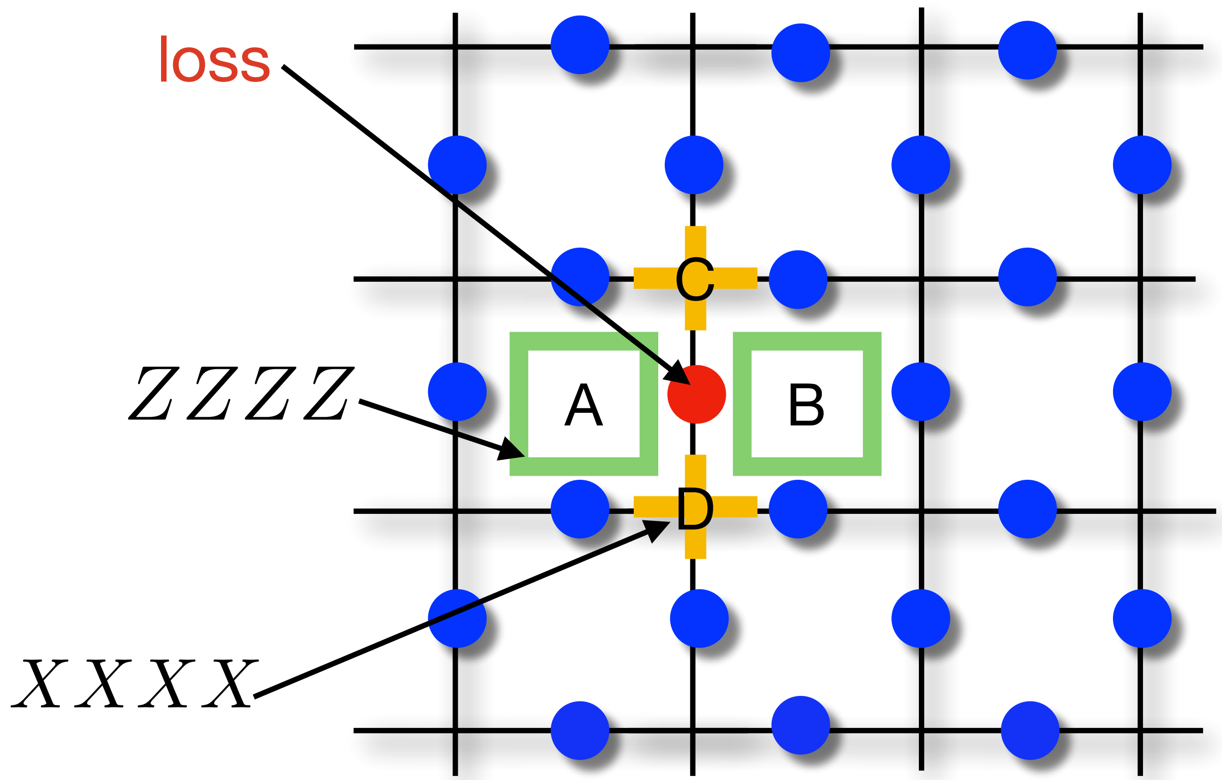


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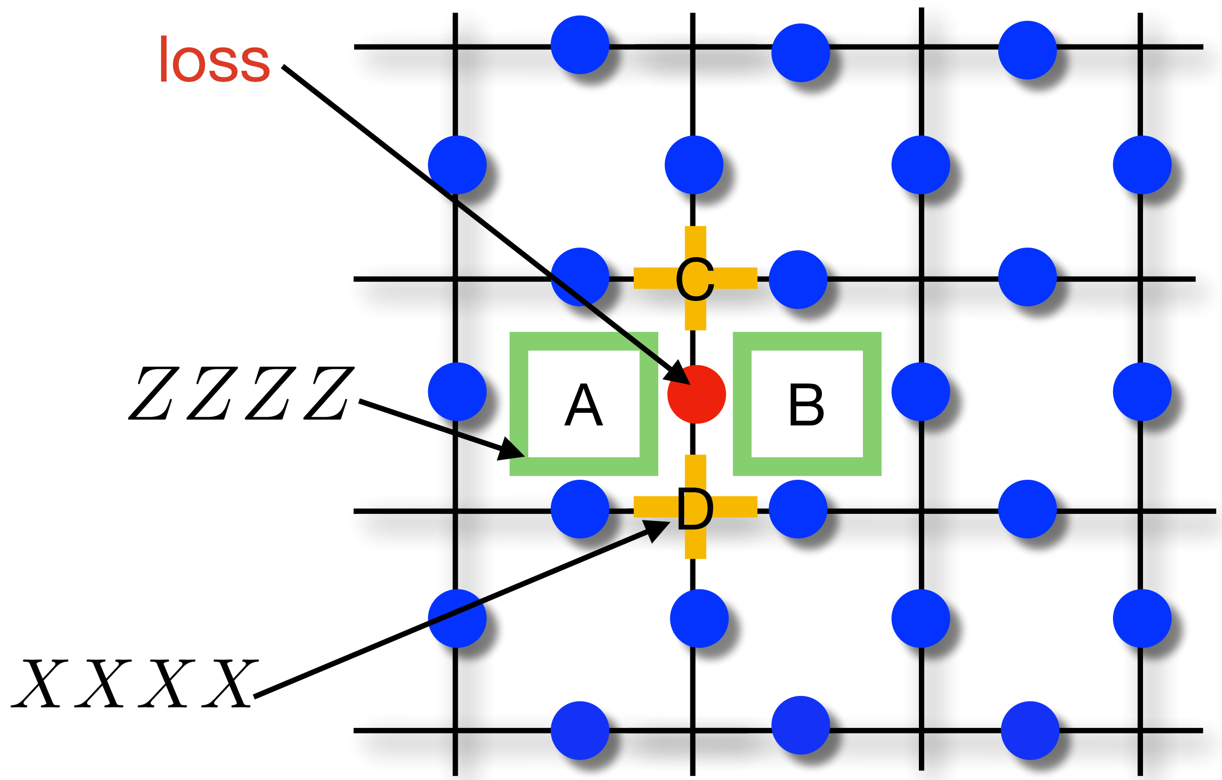


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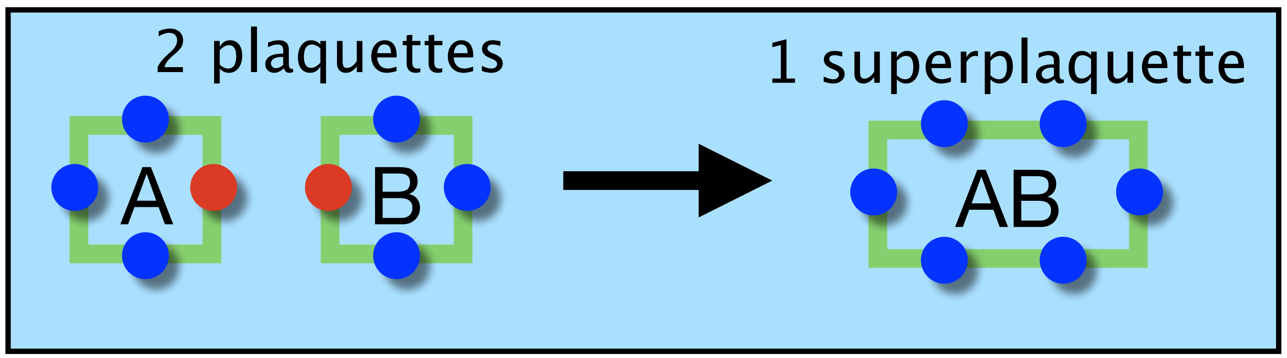

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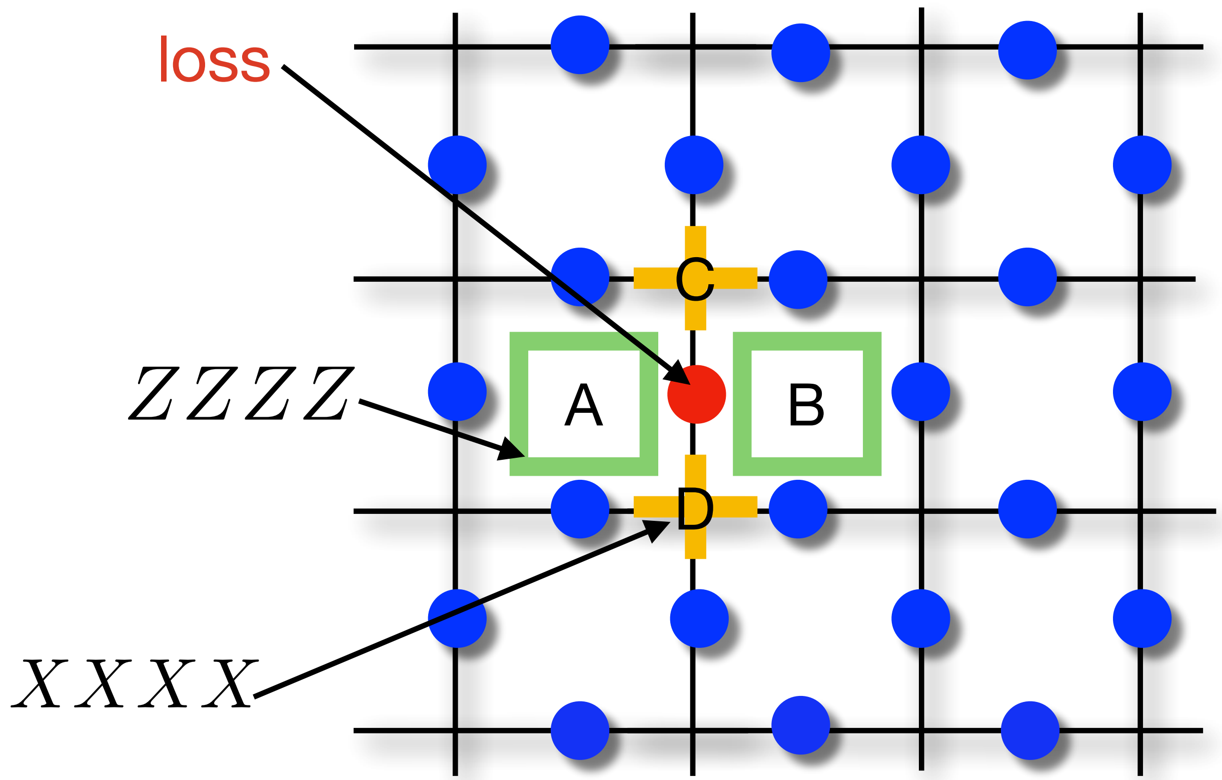


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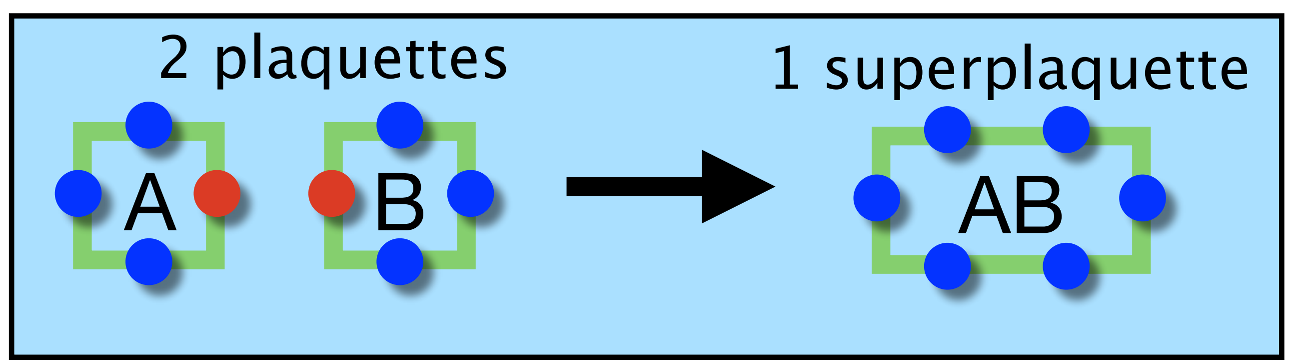


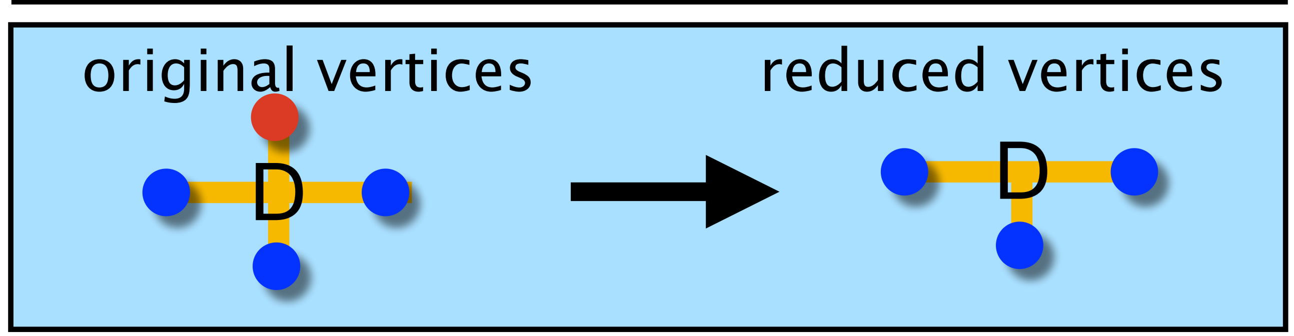

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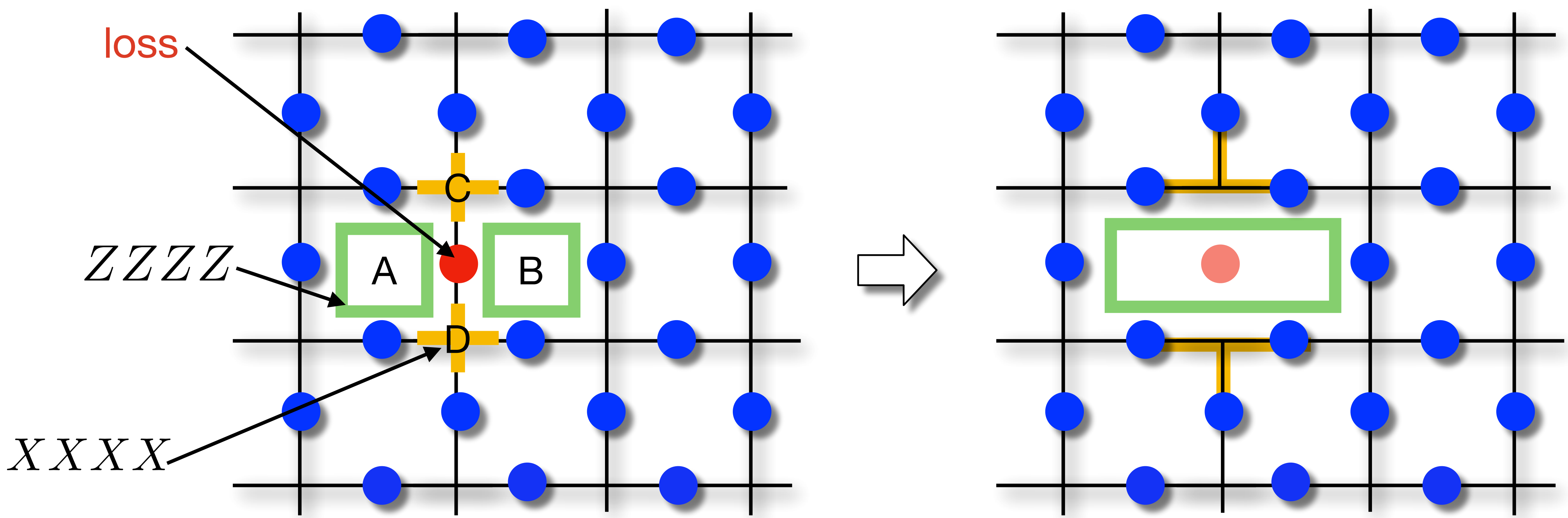


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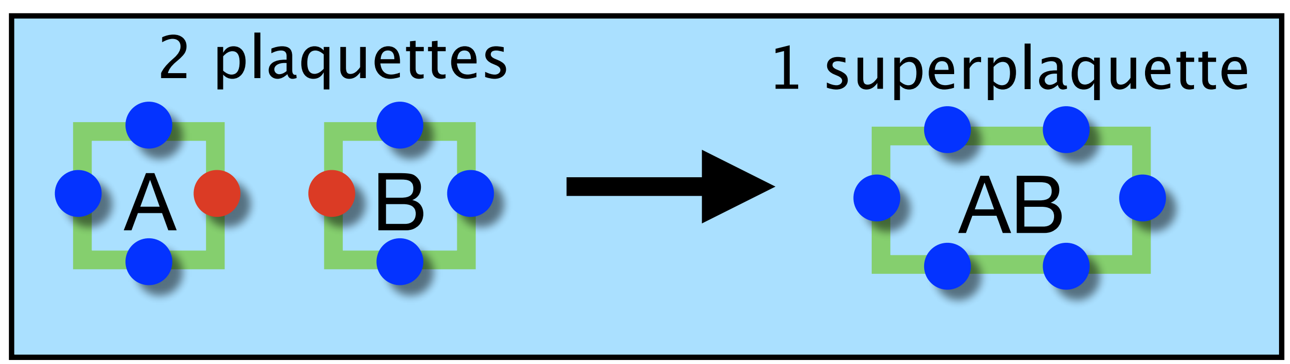


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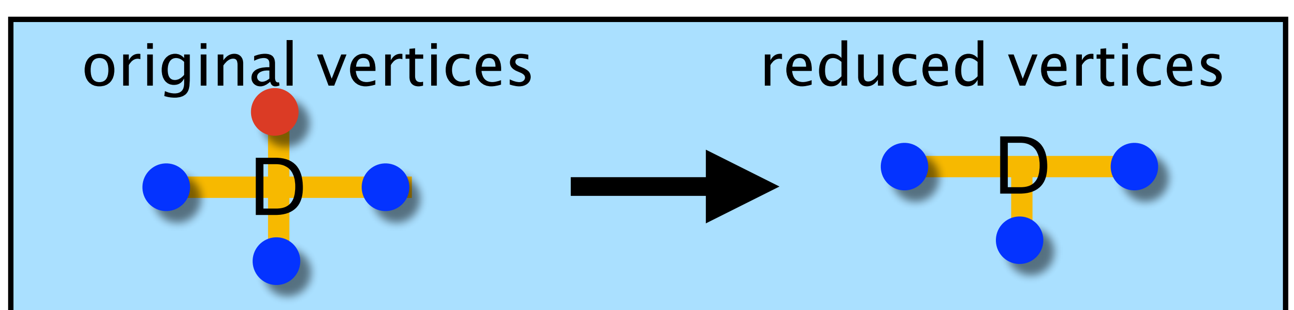
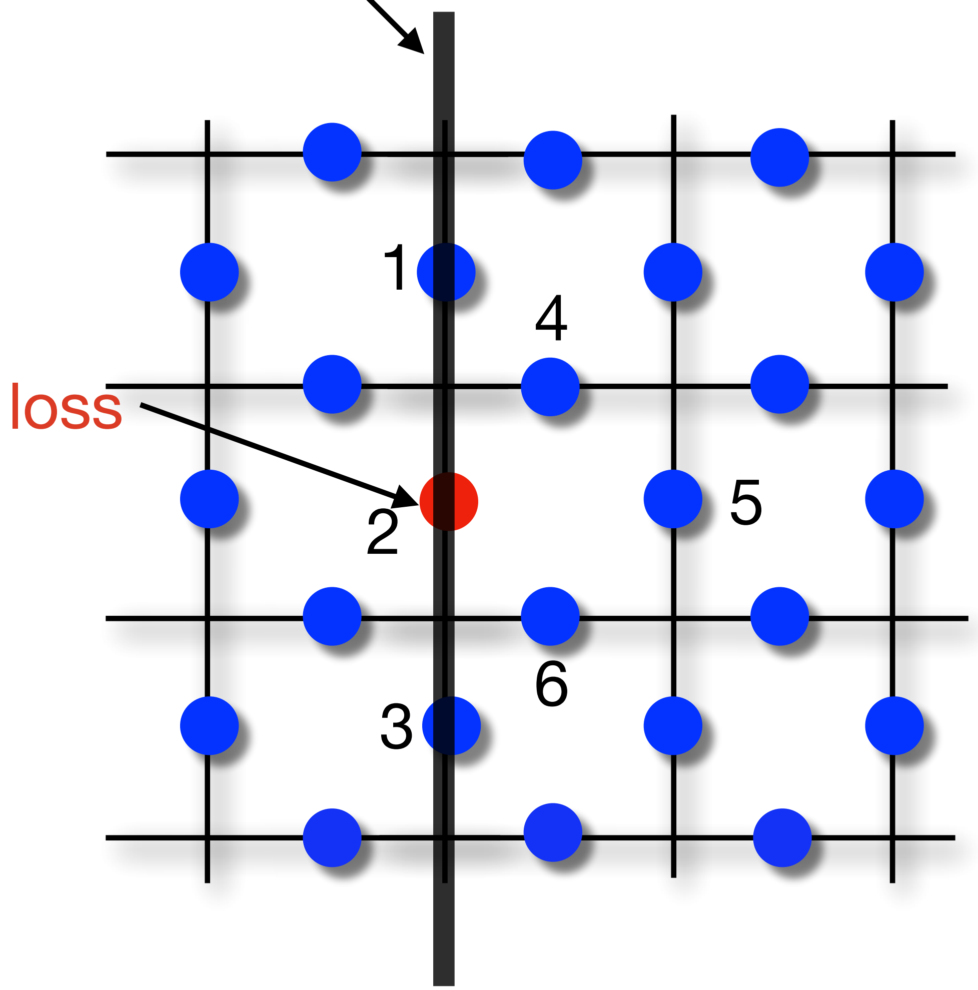


# Qubit losses in the toric code

$$\bar{Z} = \sigma_1^z \sigma_2^z \sigma_3^z$$




What about the logical operators?

Use stabilisers to **deform logical operators** that go through the lost qubits

⇨ avoid the positions of losses

**Action on logical states:**

$$\bar{Z}|\psi_L\rangle$$

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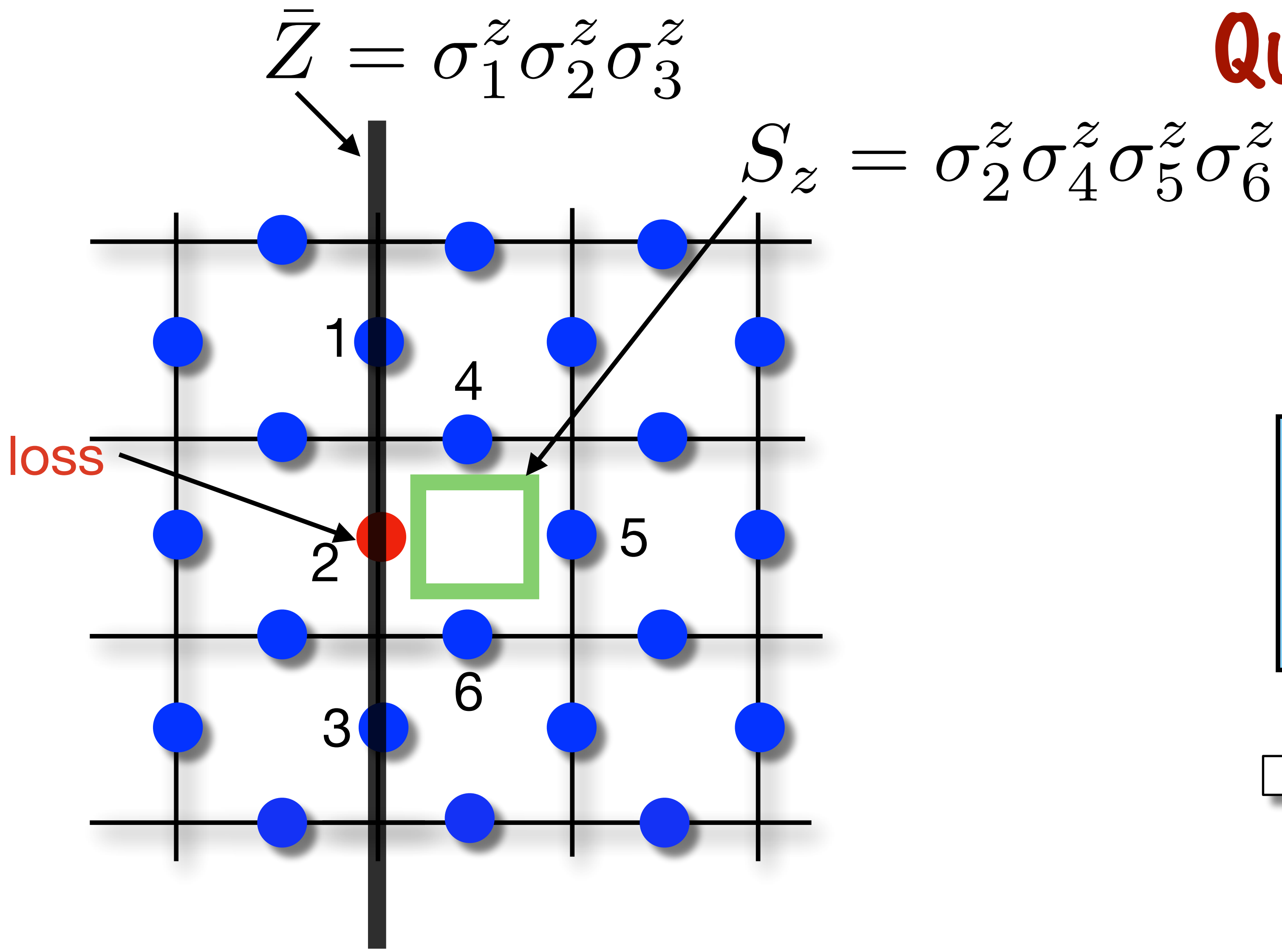


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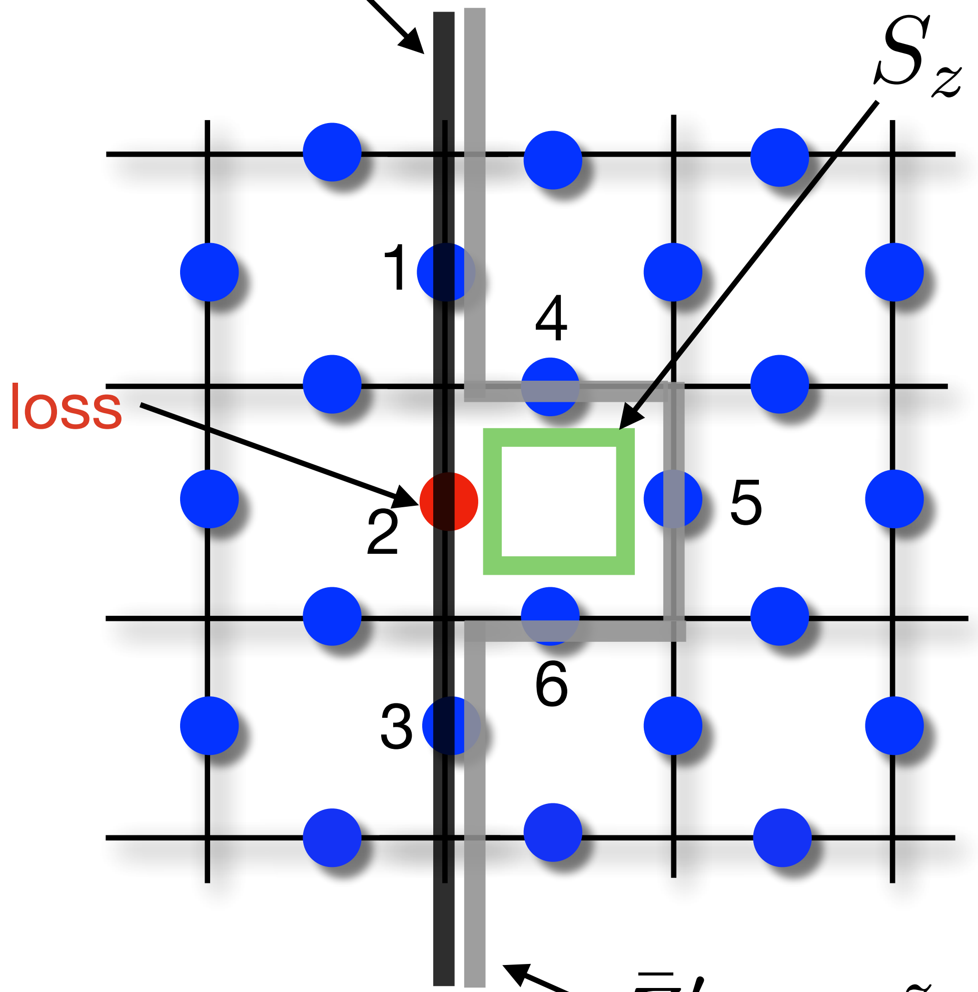


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$$= \bar{Z}' |\psi_L\rangle$$

**Example:**

$$\sigma_1^z \sigma_2^z \sigma_3^z |\psi_L\rangle$$

$$= (\sigma_1^z \cancel{\sigma_2^z} \sigma_3^z)(\cancel{\sigma_2^z} \sigma_4^z \sigma_5^z \sigma_6^z)|\psi_L\rangle$$

$$= \sigma_1^z \sigma_4^z \sigma_5^z \sigma_6^z \sigma_3^z |\psi_L\rangle$$

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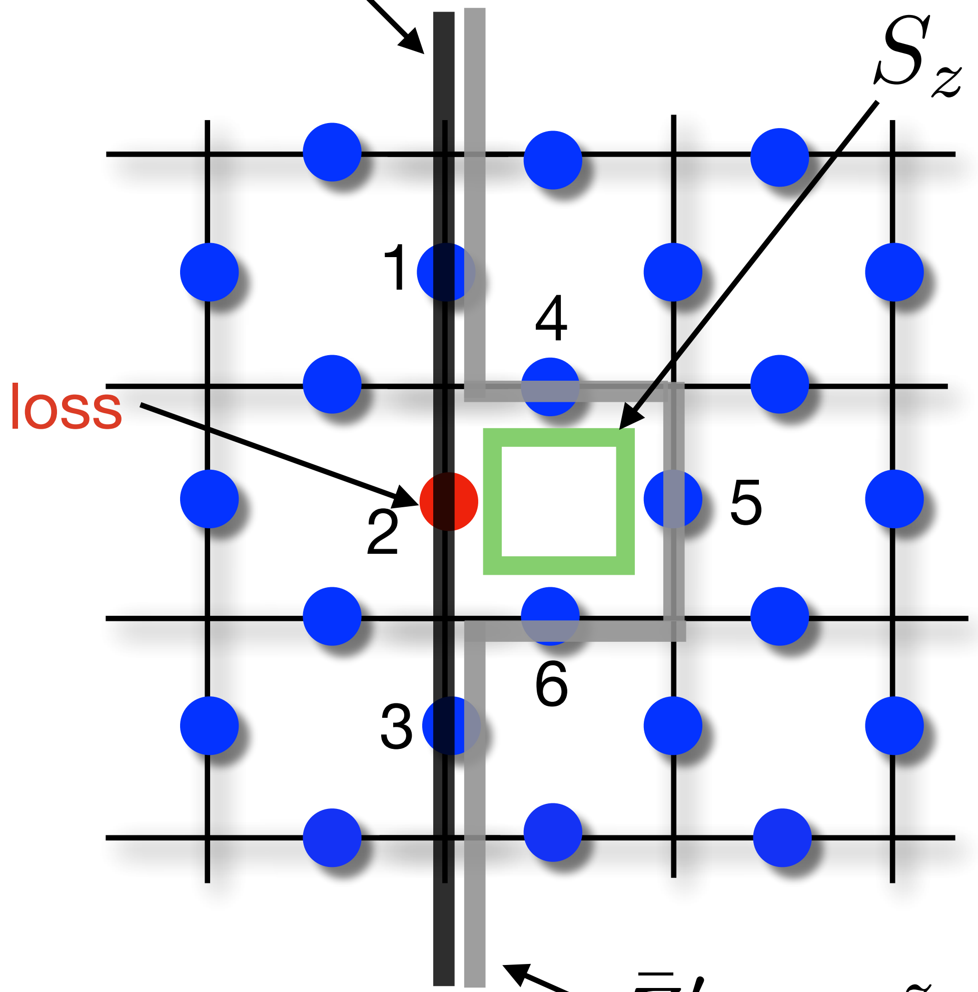


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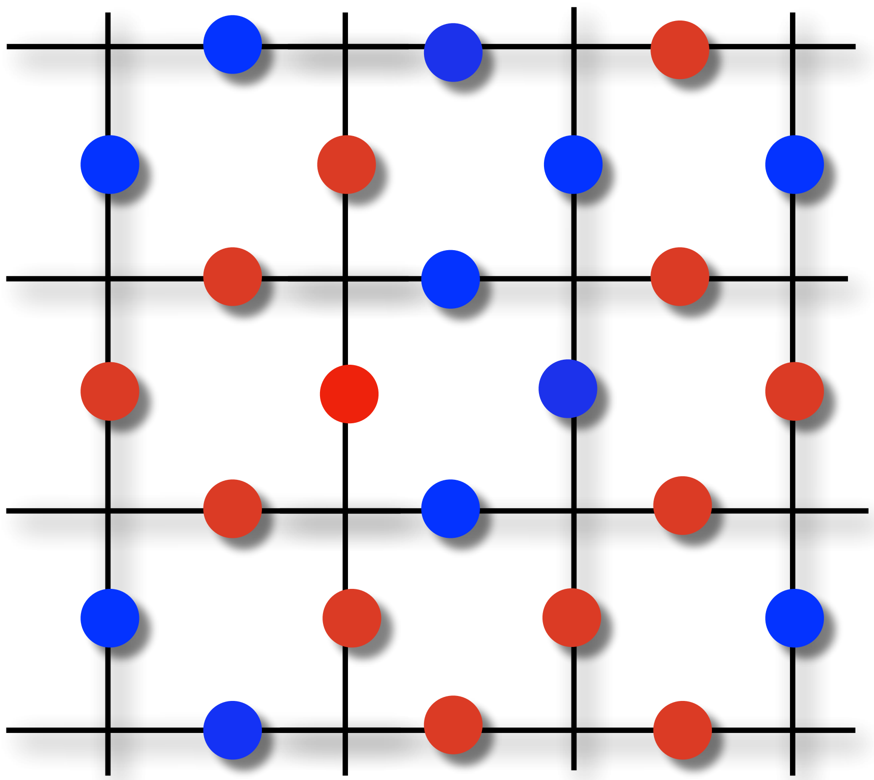
How many losses can be tolerated?

# Qubit losses in the toric code



no **percolating** path ⟷ no logical operator

Encoded qubit lost

The threshold for losses is given by the **bond percolation threshold**

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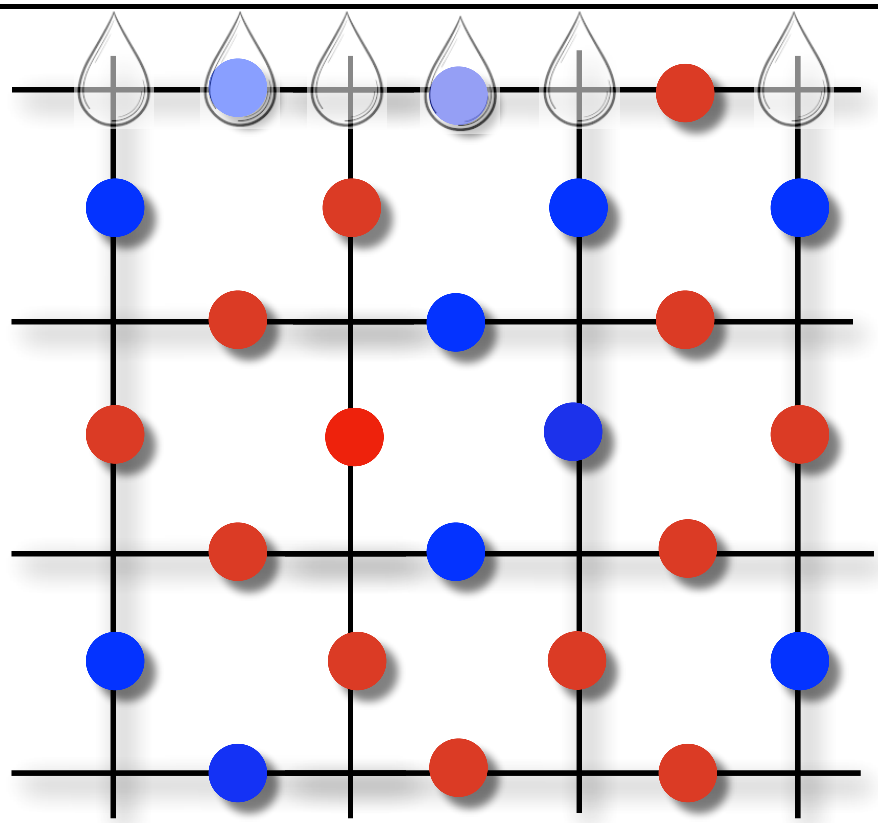

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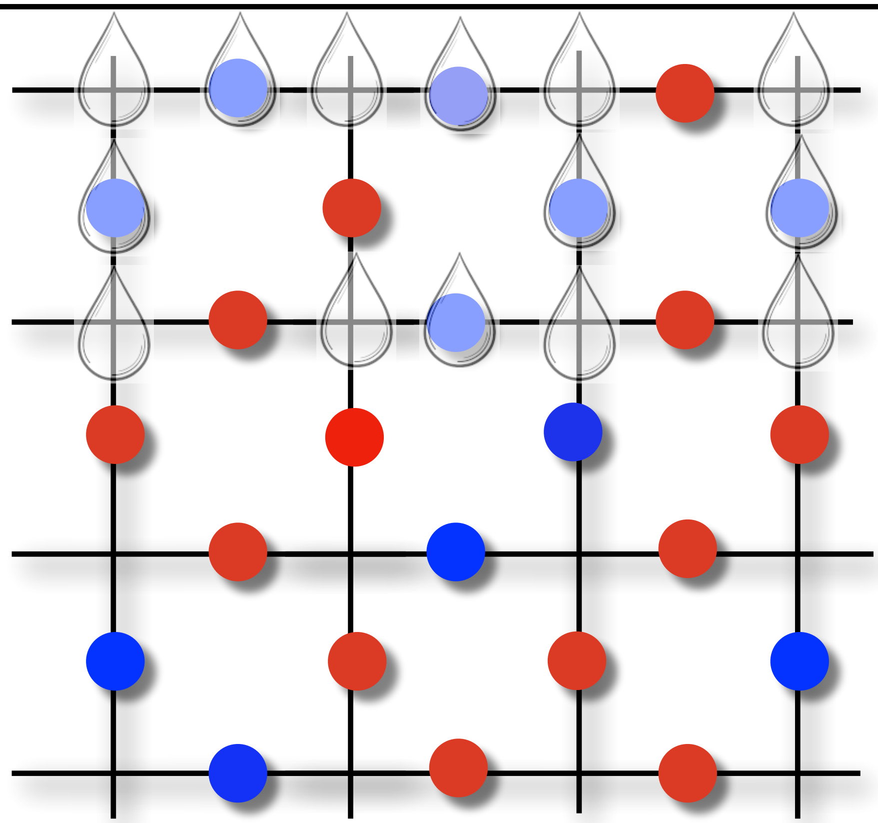

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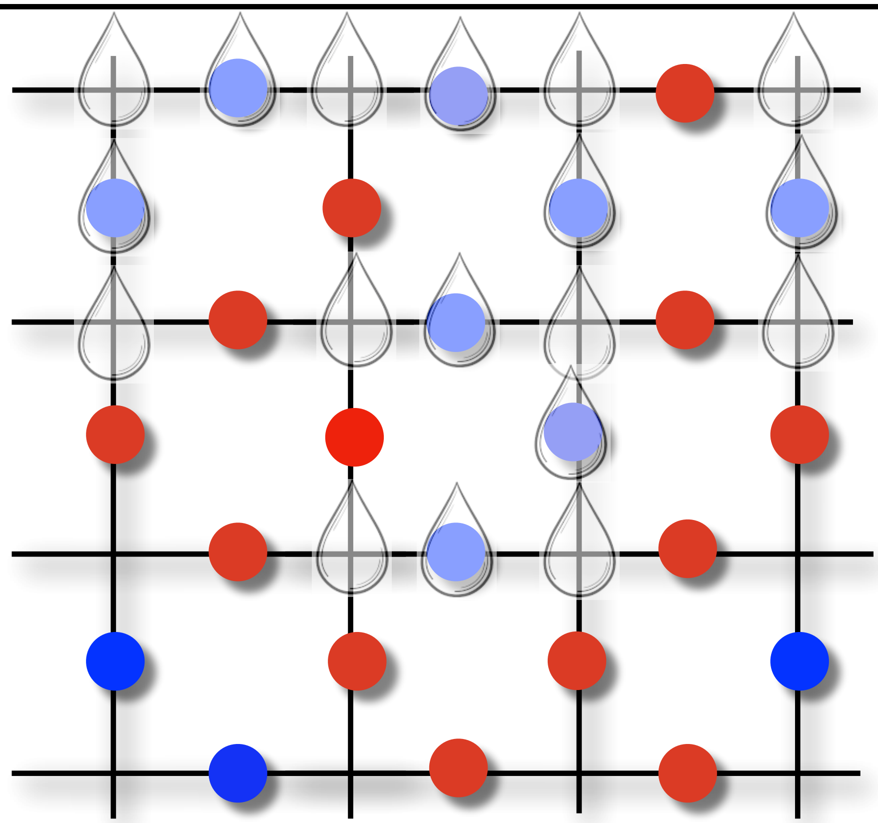

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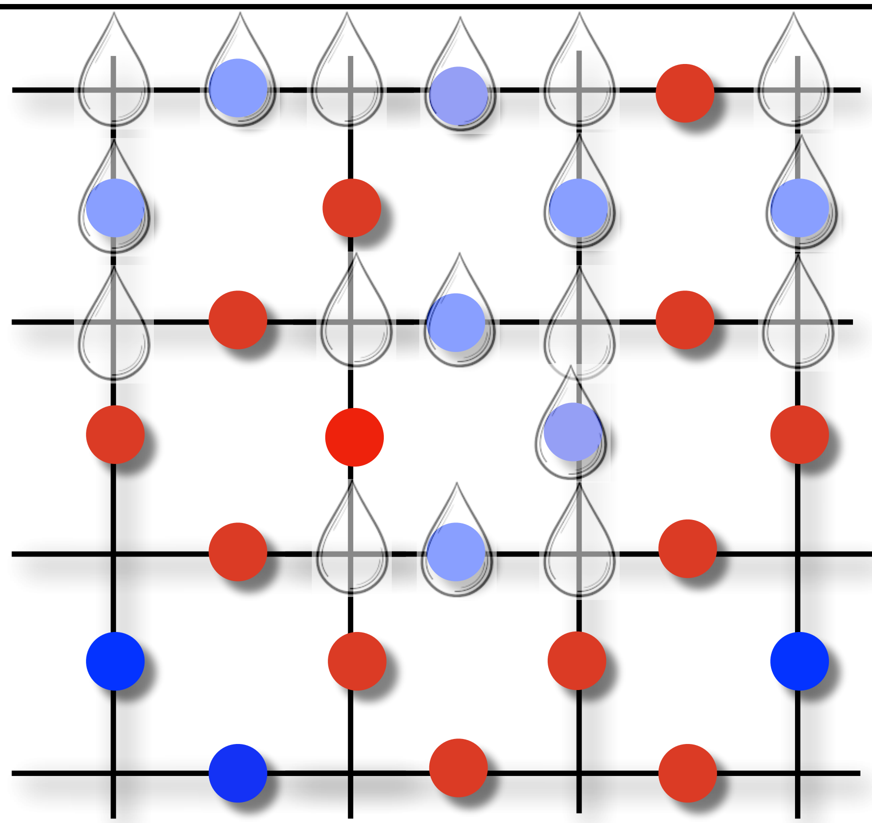

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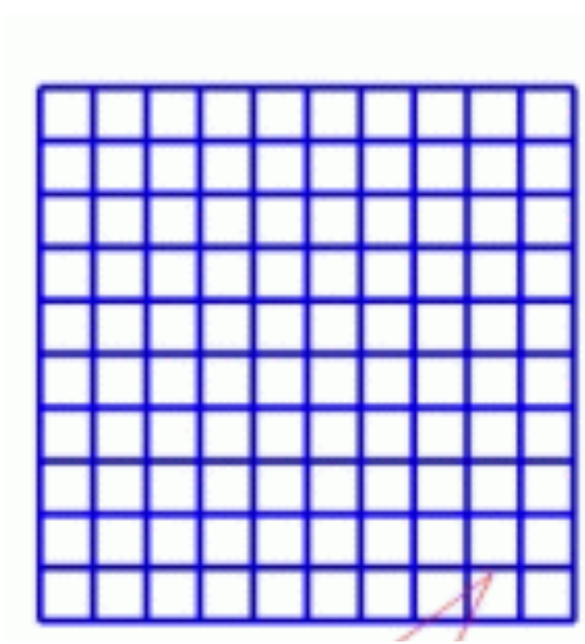

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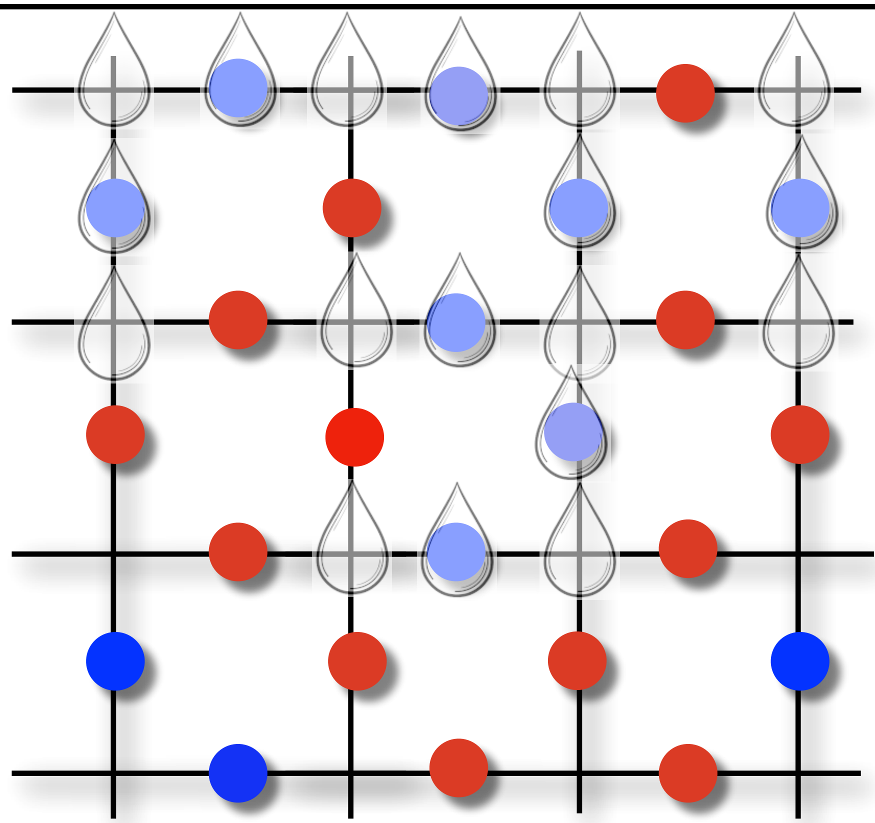

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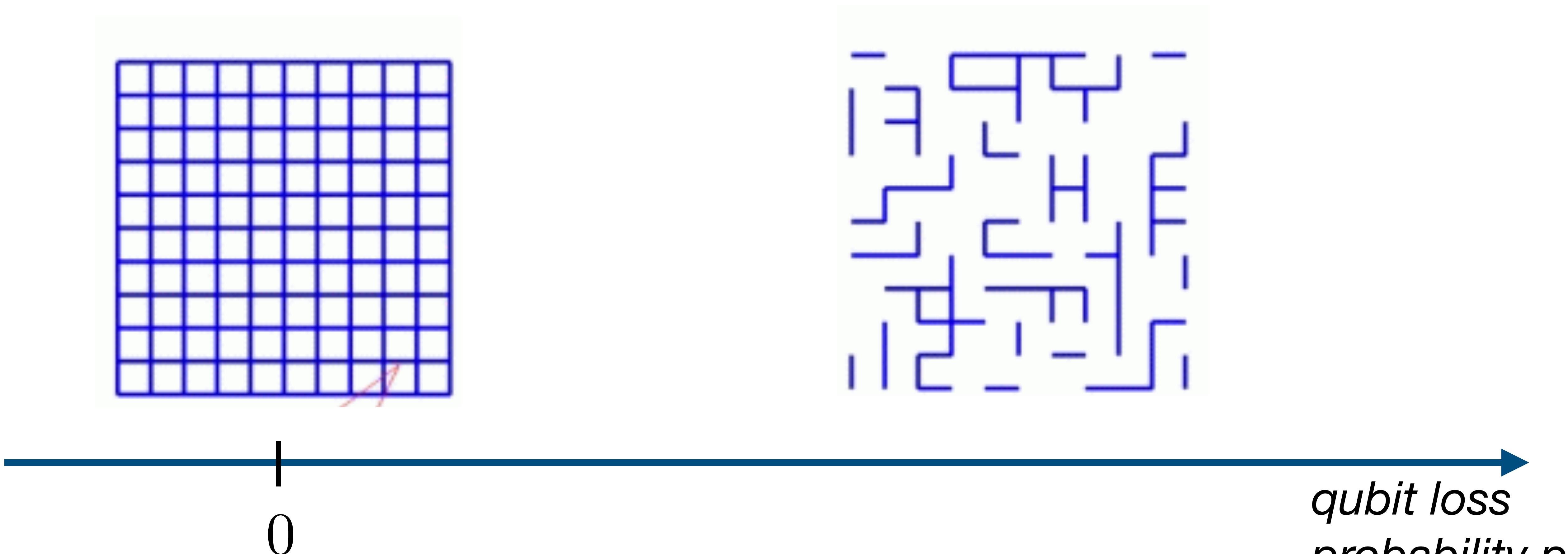

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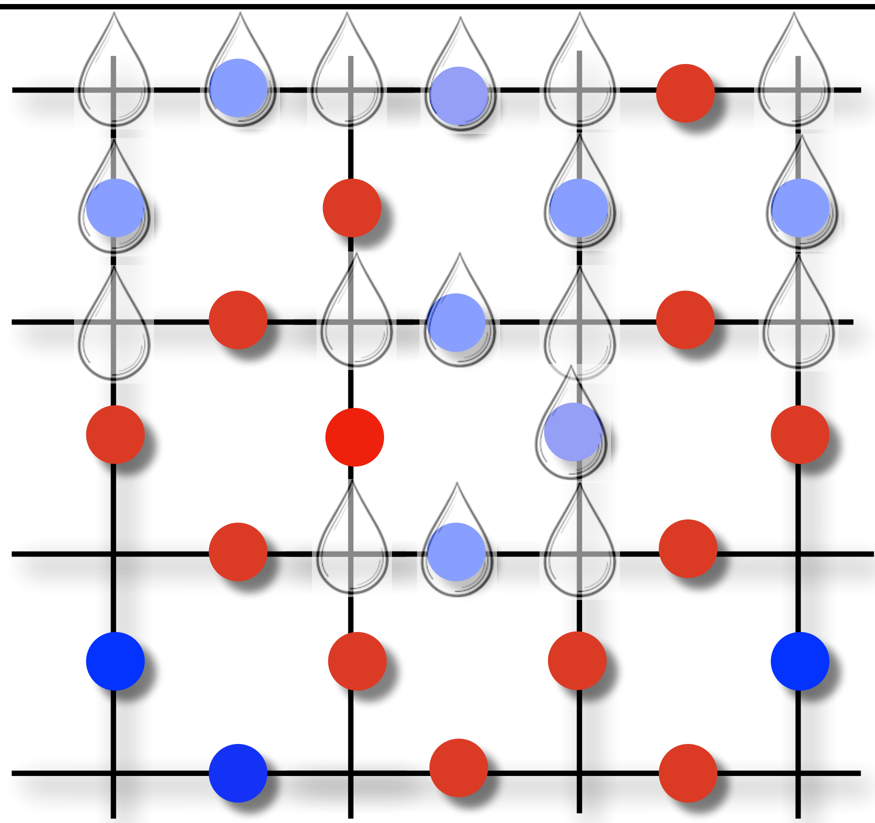

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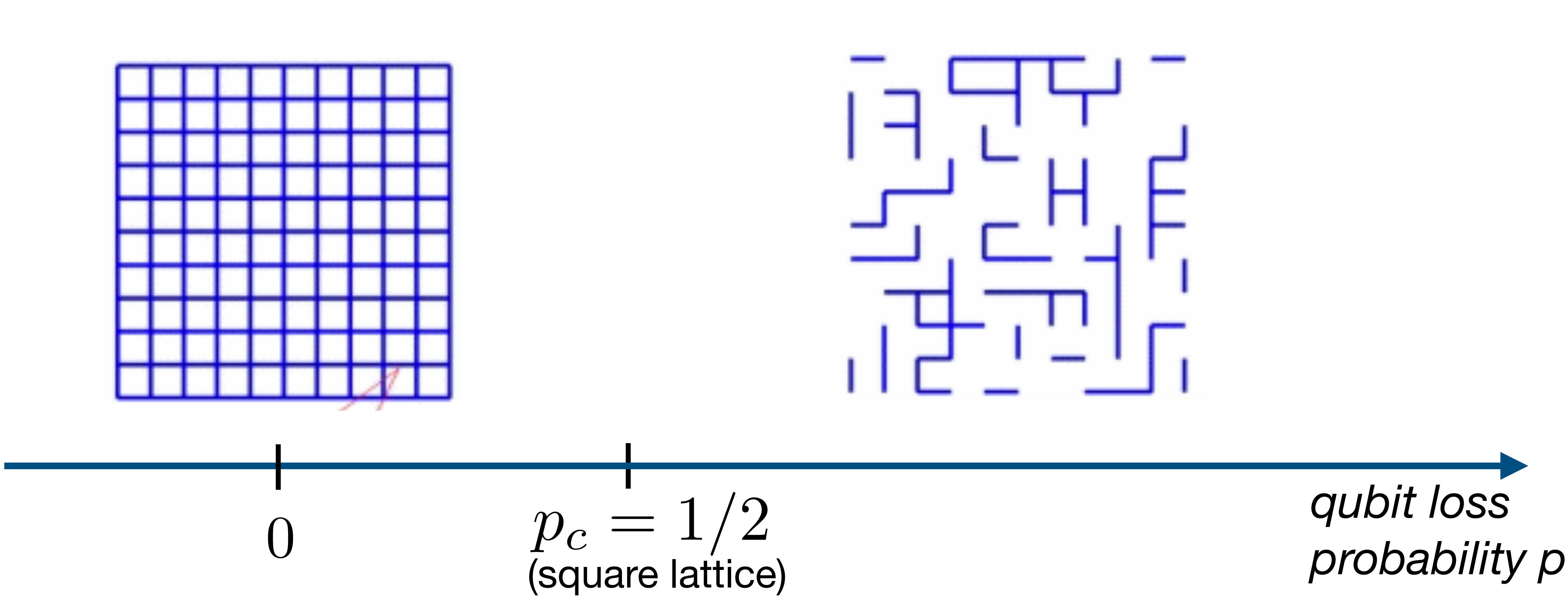
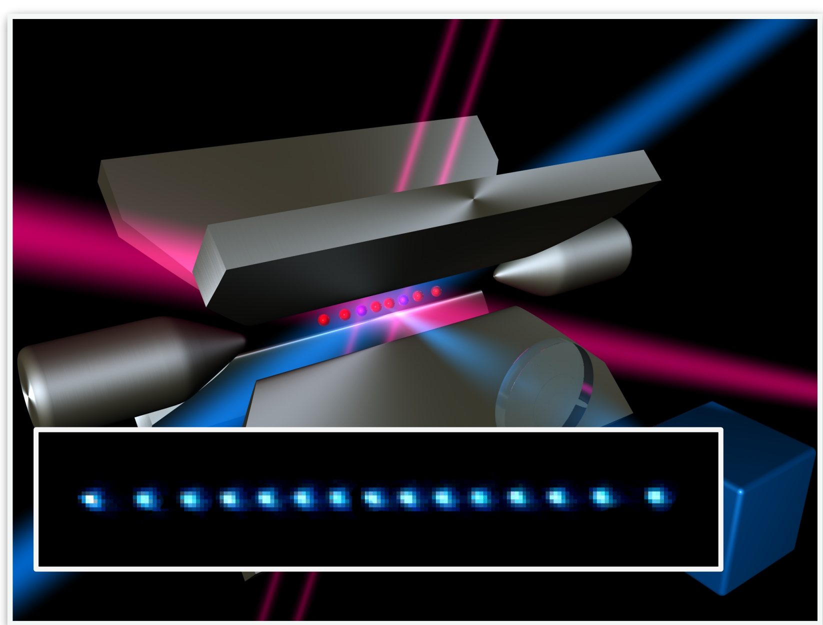
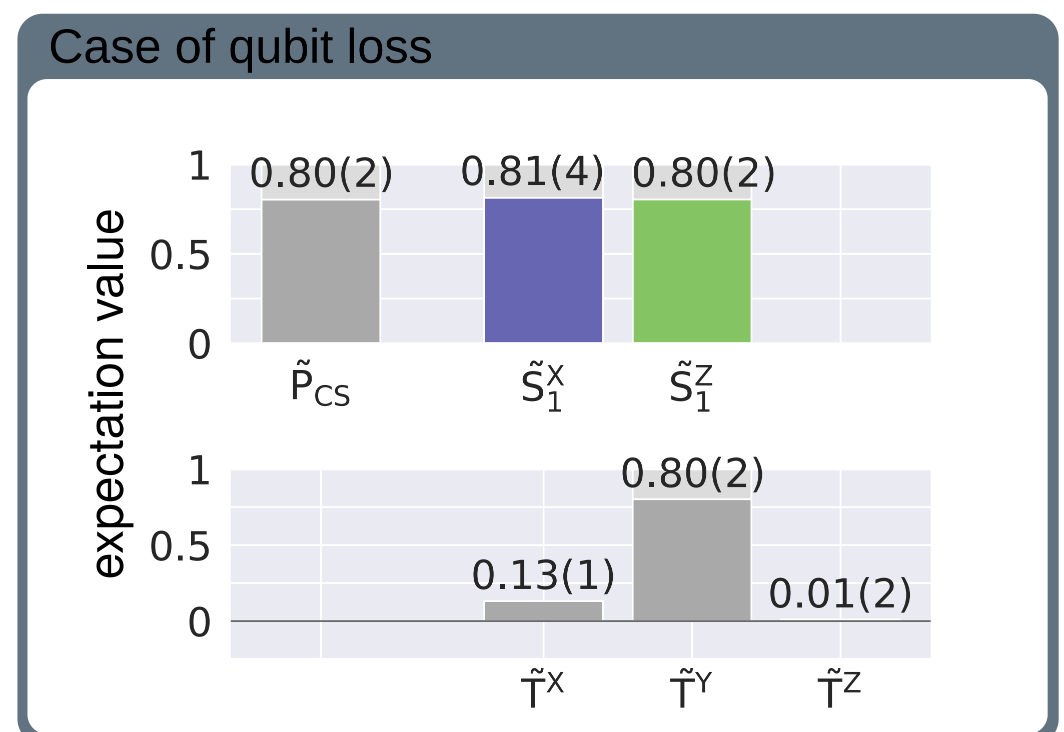


# 2 -
# Qubit Loss Error Correction: Theory and Experiment





# 2 - Qubit Loss Error Correction: Theory and Experiment

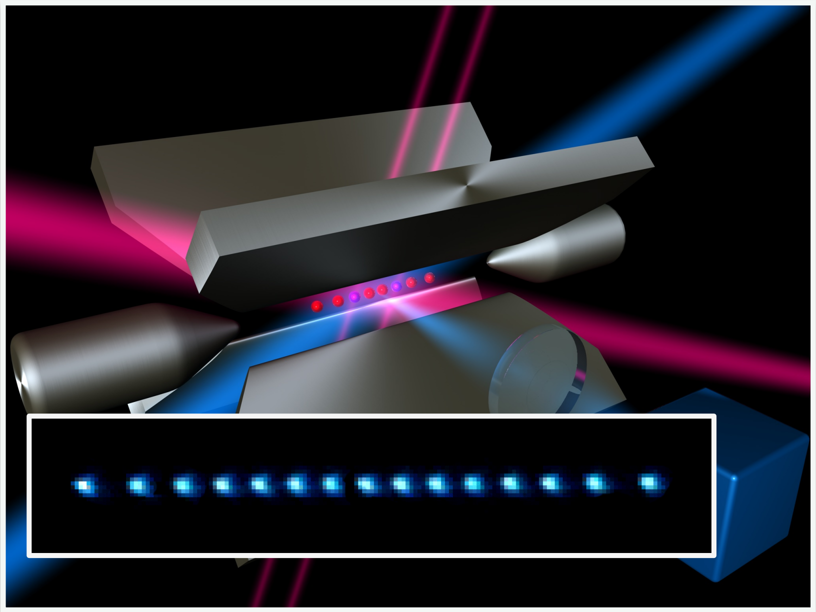
**Goal**

- Provide a toolbox for correcting losses in generic quantum codes

    Detect if the loss has happened

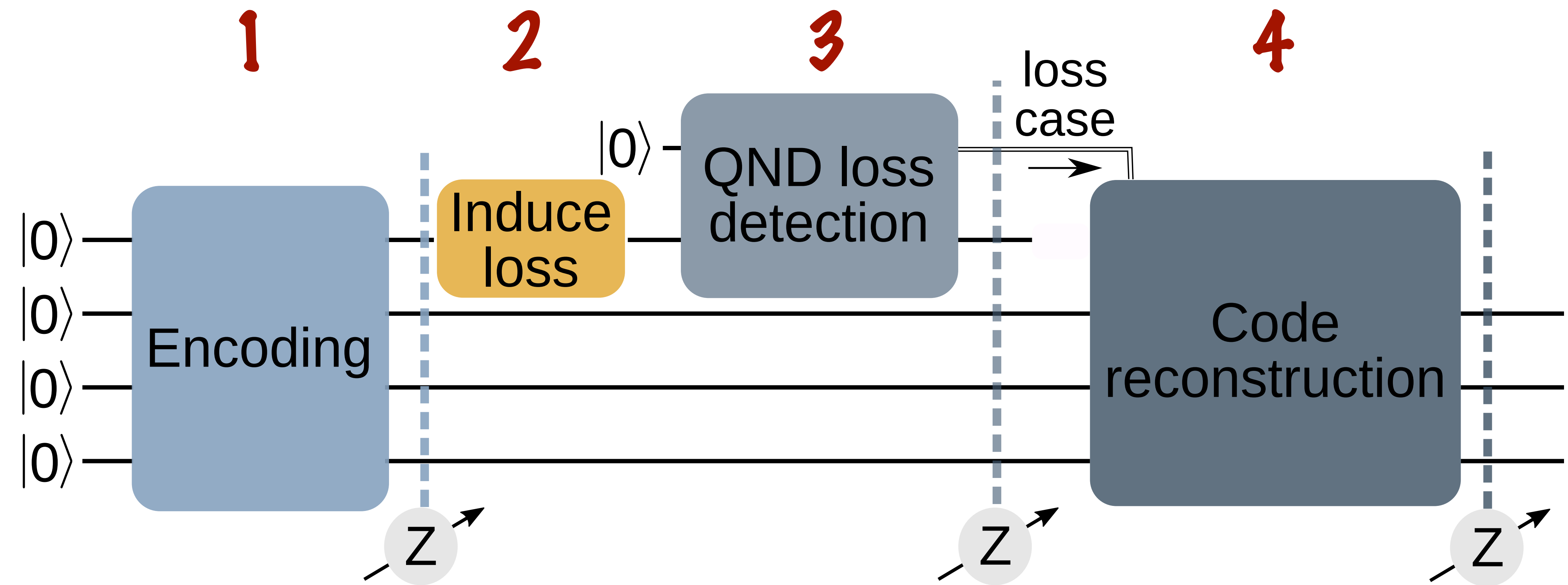
    Decide if correcting or not the code

- Devise the smallest example in a trapped ion setup



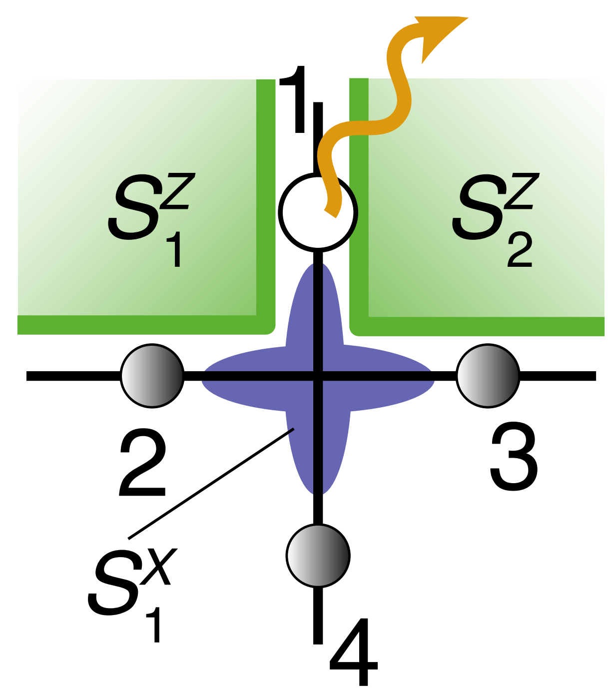
R. Stricker, DV, M. Ringbauer, P. Schindler, T. Monz, M. Müller, R. Blatt
*Deterministic correction of qubit loss*, in preparation

# Experimental qubit loss detection and correction: The whole picture




**Minimal example**

**4 physical qubits**



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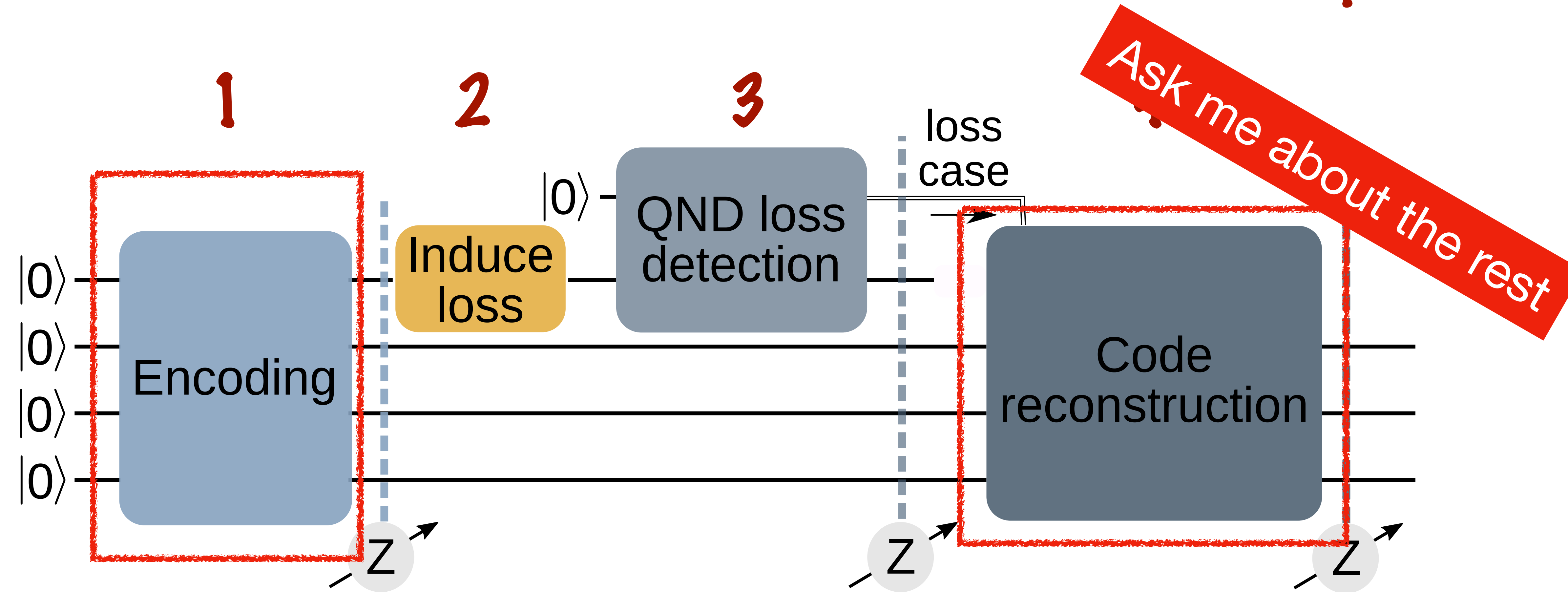


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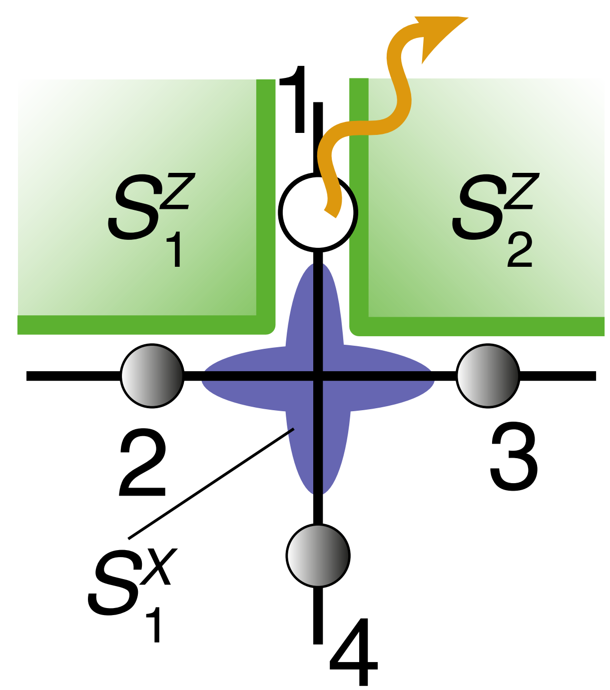


# 1 - Experimental encoding

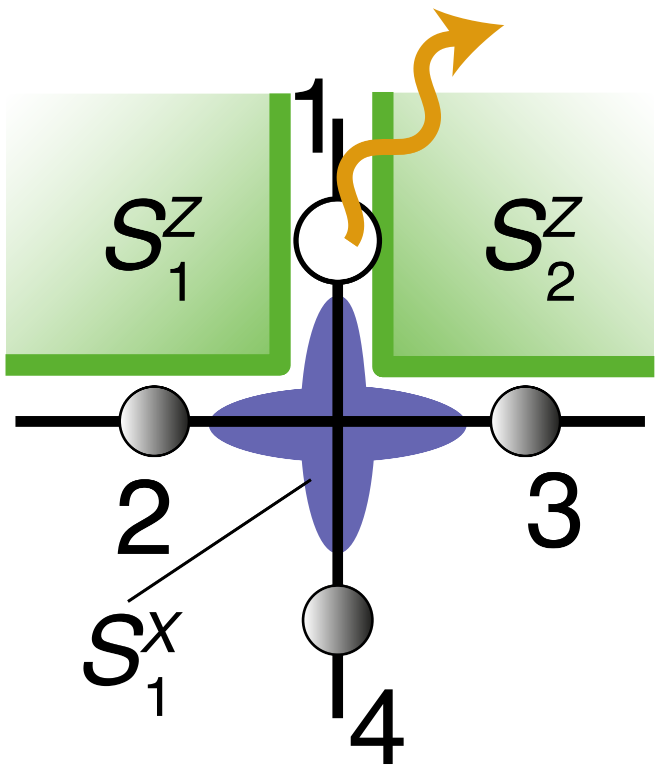
**4 physical qubits**

**3 stabilisers**

$$S_1^Z = Z_1 Z_2$$

$$S_2^Z = Z_1 Z_3$$

$$S_1^X = X_1 X_2 X_3 X_4$$




**1 logical qubit**

**Logical Z- and X-operators**

$$T^Z = Z_1 Z_4$$

$$T^X = X_4$$

**Logical basis states**

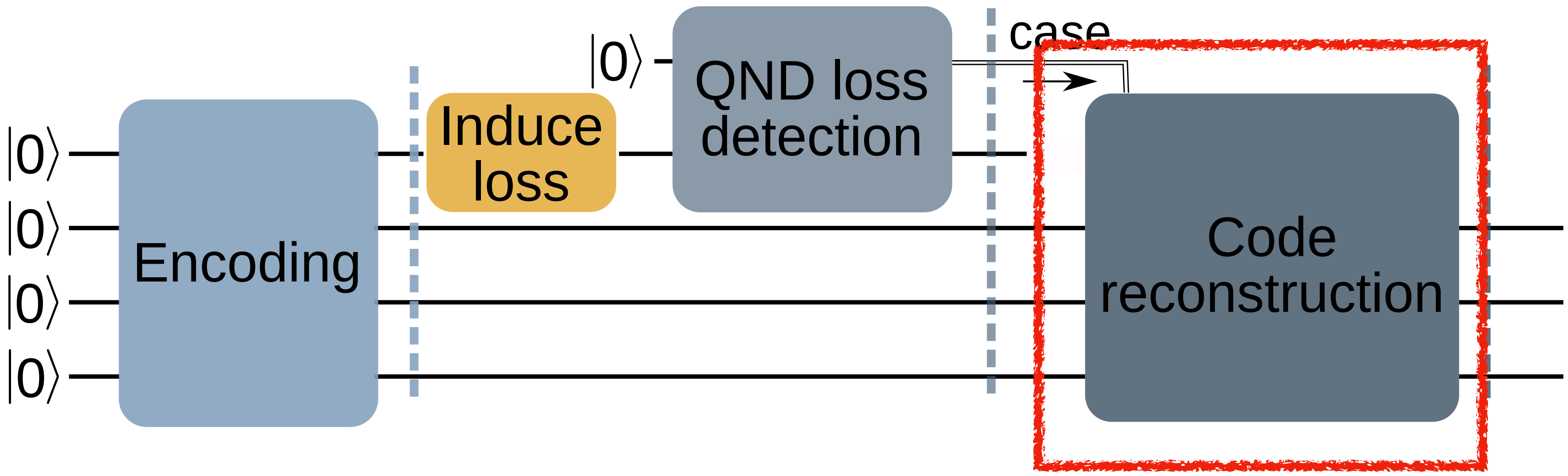
$$|0_L\rangle = |0000\rangle + |1111\rangle$$

$$|1_L\rangle = |0001\rangle + |1110\rangle$$

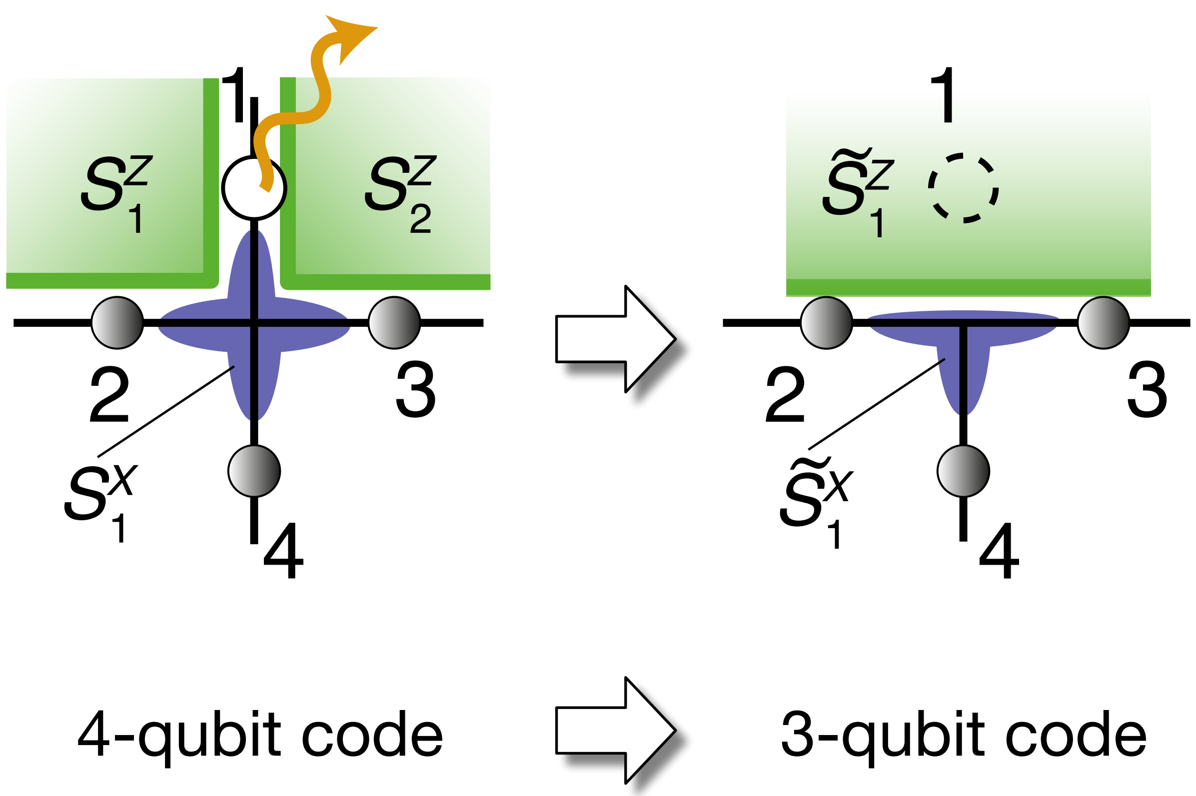
**Encoded superposition state**

$$|\psi_L\rangle = \cos(\alpha/2)\,|0_L\rangle + i\sin(\alpha/2)\,|1_L\rangle$$

# 4 - Recovery of the encoded qubit - code reconstruction




**Loss case:** Recover logical qubit by code-switching to a **reduced 3-qubit code**




**2 stabilisers**

$$\tilde{S}_1^Z = S_1^Z S_2^Z = Z_2 Z_3$$ ✔

$$\tilde{S}_1^X = X_1 X_2 X_3$$ undetermined ✘

**Logical Z- and X-operators**

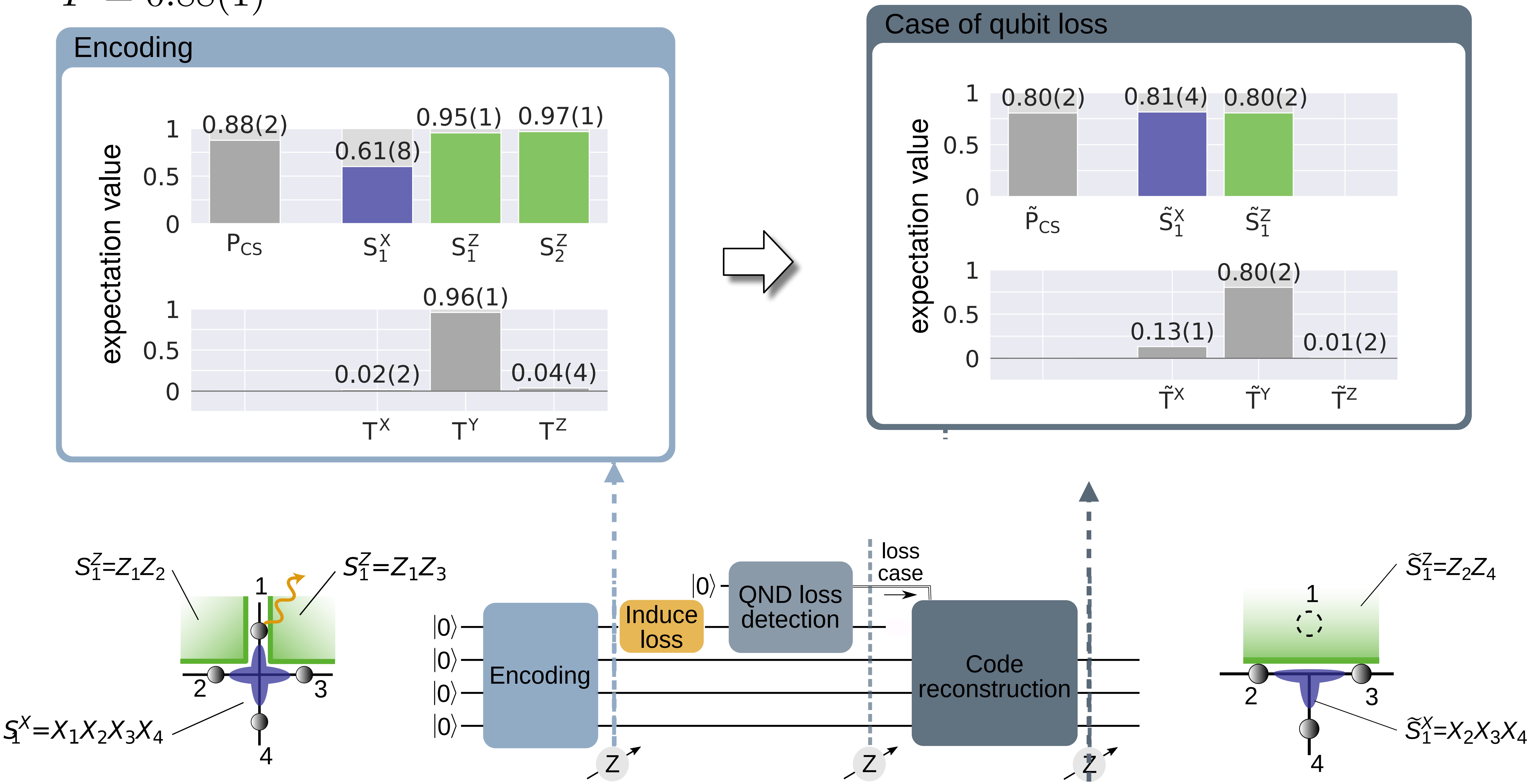
$$\tilde{T}^Z = T^Z S_1^Z = Z_2 Z_4$$ ✔

$$\tilde{T}^X = T^X = X_4$$ ✔

# Qubit loss and correction - the entire cycle

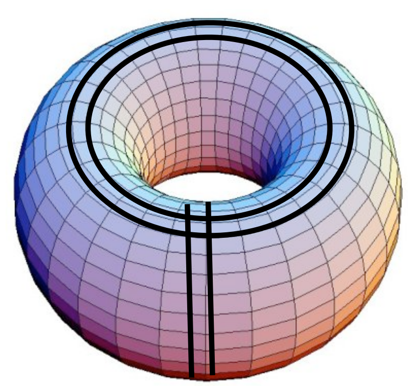
$$|\psi_L\rangle = \frac{1}{\sqrt{2}}(|0_L\rangle + i|1_L\rangle)$$

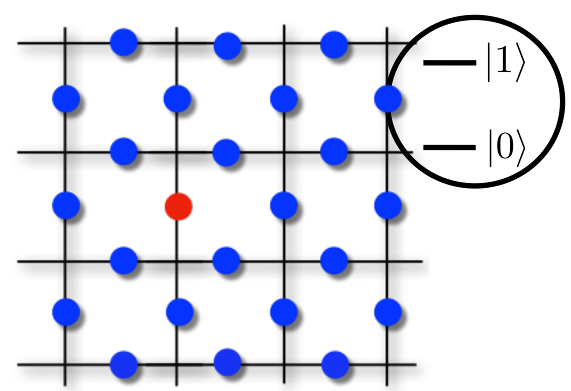
$F = 0.88(1)$ $F = 0.78(1)$

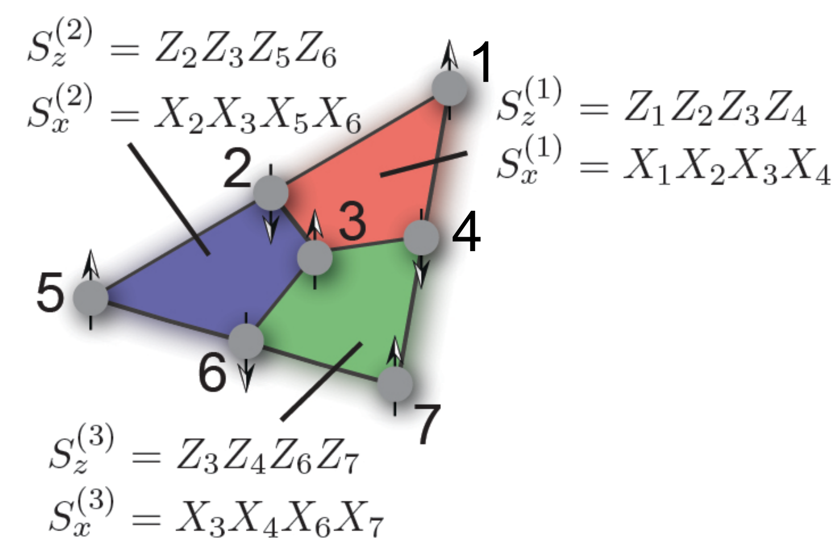


## What we've seen

- Quantum error correcting codes can be realised in topological systems
- Losses can affect quantum computers but can be cured with success
- We developed a scheme for detecting losses
  - Platform independent
  - Applicable to other codes

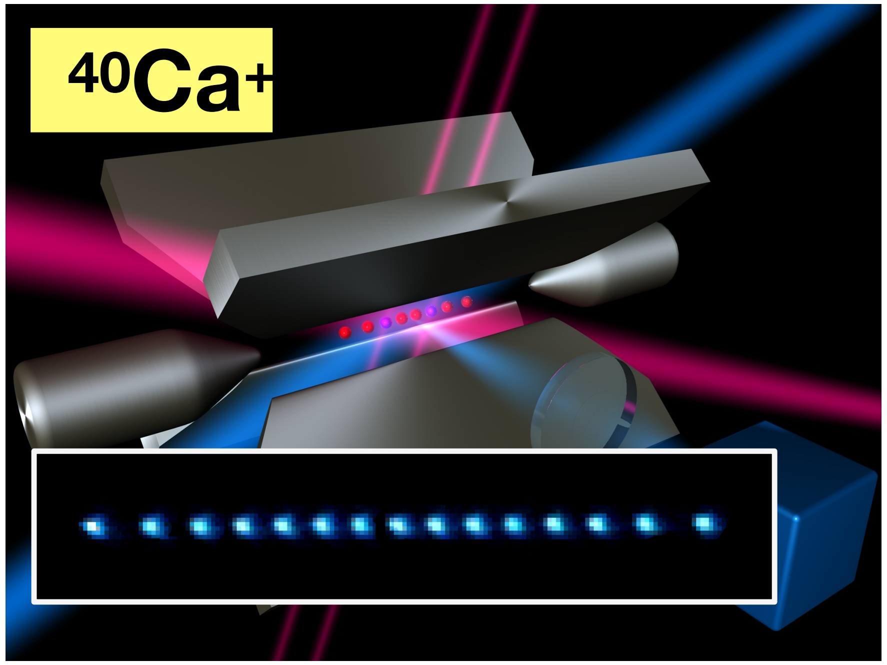




# Thank you!

# Innsbruck linear ion-trap QIP toolset



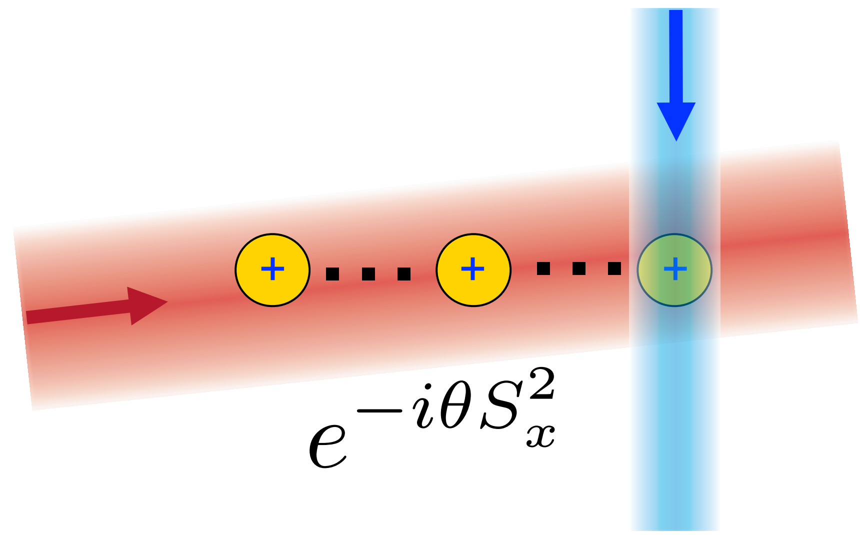

1. Individual light-shift gates

$$\sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)}$$

$$e^{-i\theta\sigma_z^{(j)}}$$

2. Collective rotations

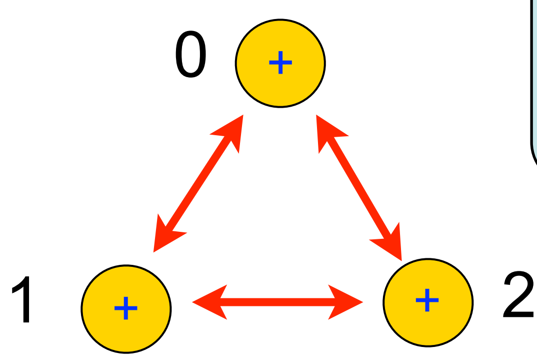
$$S_x, S_y$$



$$e^{-i\theta S_x^2}$$

$$e^{-i\theta S_x}$$

$$S_x = \sum_i \sigma_x^{(i)}$$




3. multi-ion Mølmer–Sørensen (MS) entangling gate

$$S_x^2 = \sigma_x^{(0)}\sigma_x^{(1)} + \sigma_x^{(1)}\sigma_x^{(2)} + \sigma_x^{(0)}\sigma_x^{(2)}$$

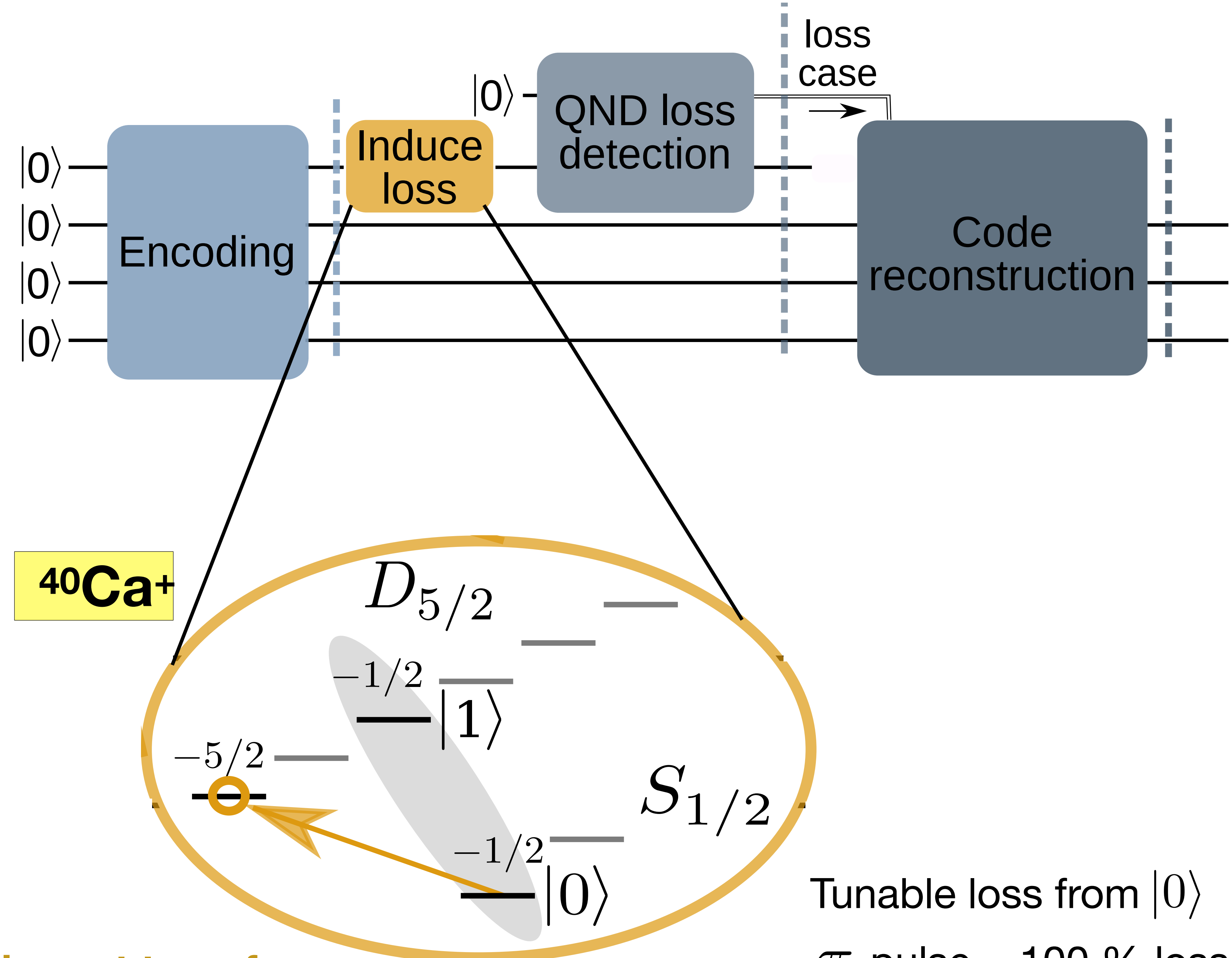
$$|00\rangle \xrightarrow{\text{MS}} |11\rangle$$

$$|11\rangle \xrightarrow{\text{MS}} |00\rangle$$

... fidelity > 99.3 % for 2 qubits, Benhelm *et al.* Nat. Phys. **4**, 463 (2008)

... 14-qubit entanglement, T. Monz *et al.* PRL **106**,130506 (2011)

# 2 - Qubit loss event

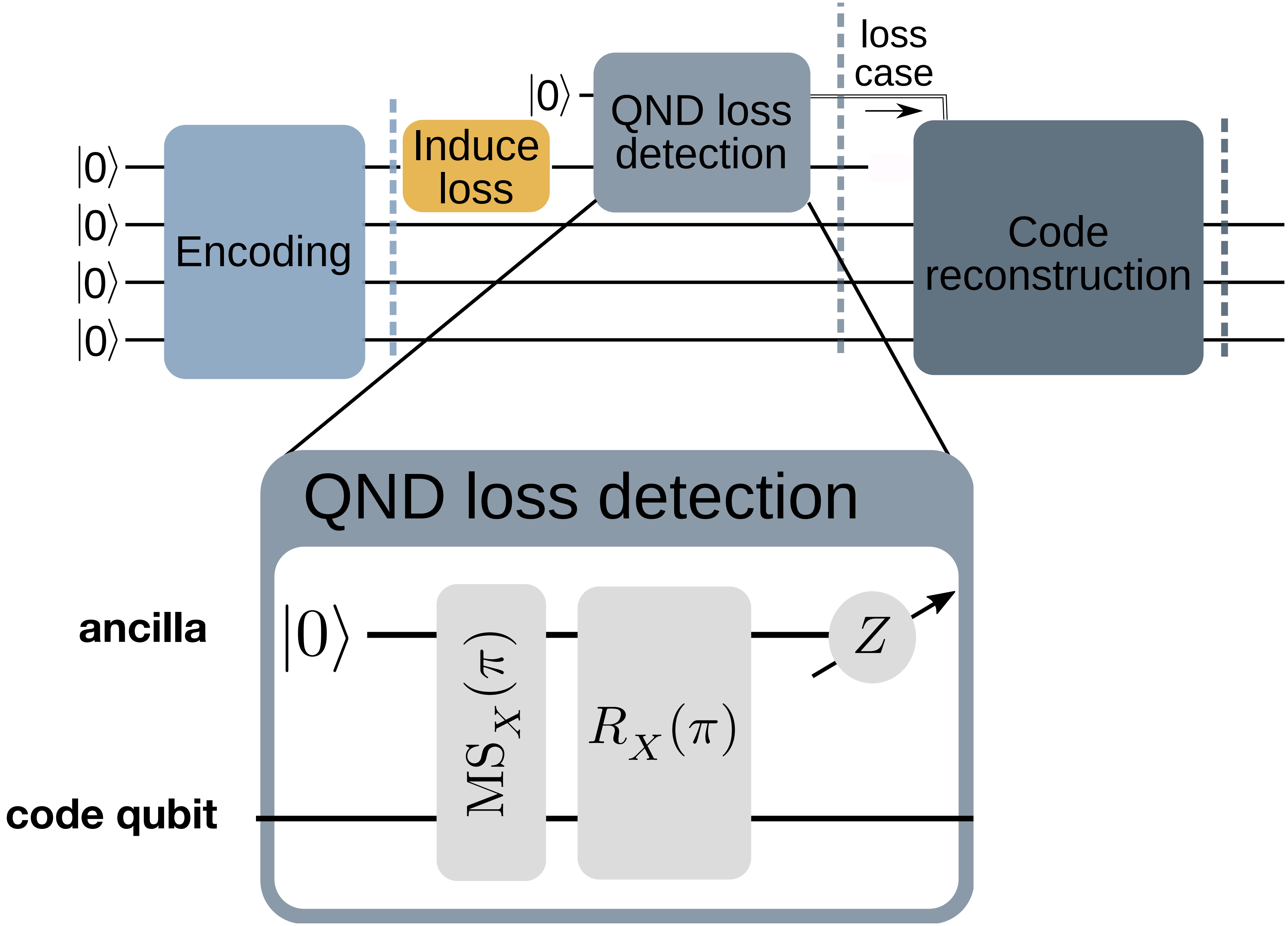



Coherent transfer on 'hiding' transition

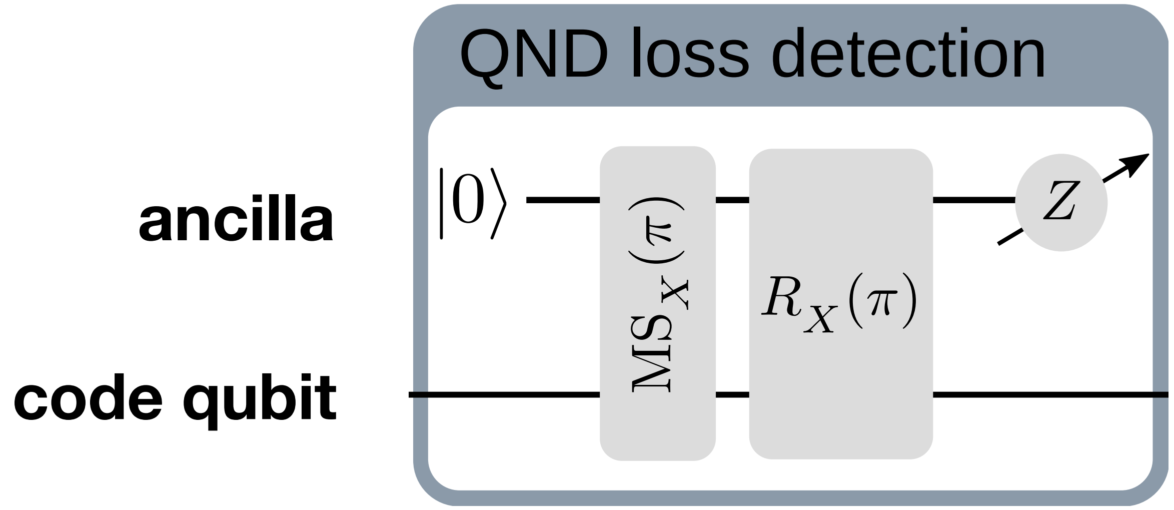
Tunable loss from $|0\rangle$

$\pi$-pulse = 100 % loss probability

# 3 - QND qubit loss detection

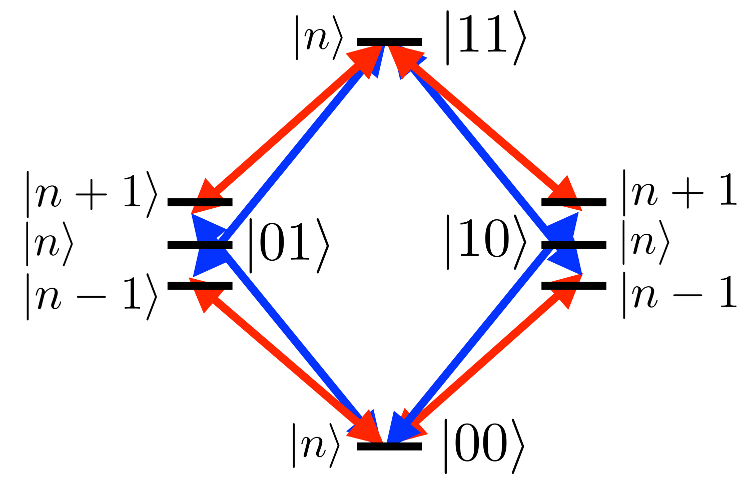


# 3 - QND qubit loss detection

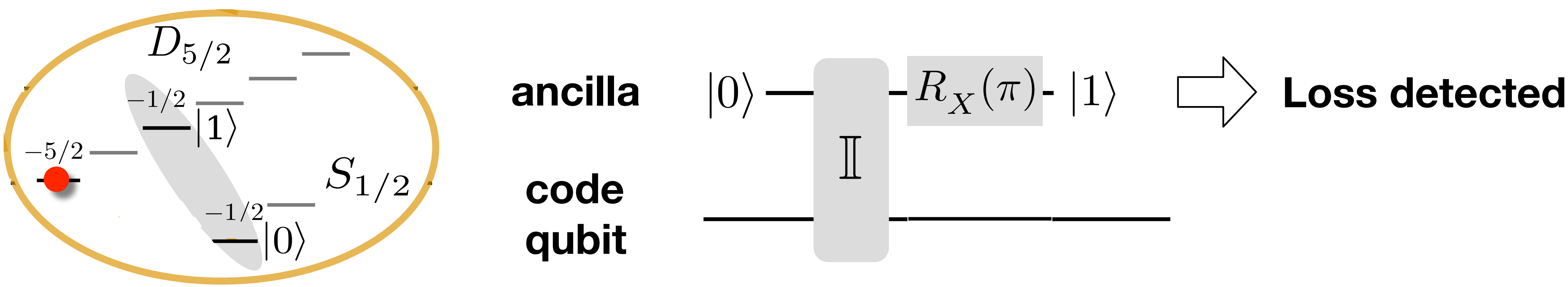



**Mølmer-Sørensen gate:**

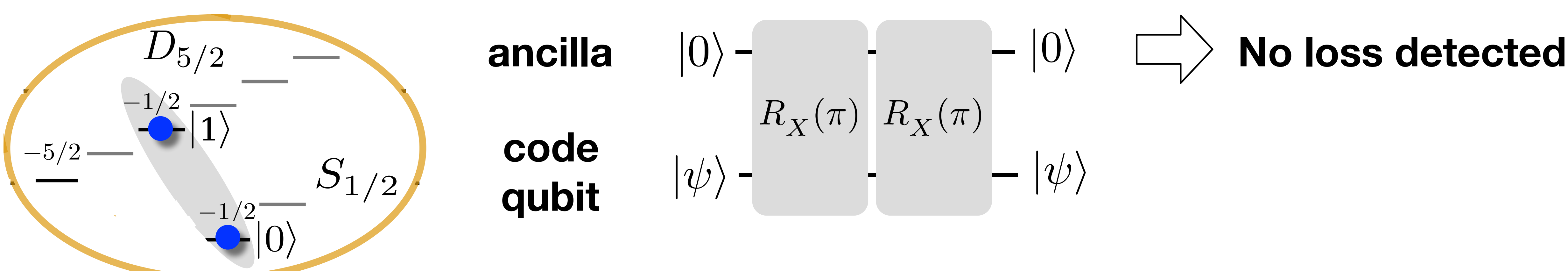
- Bichromatic laser field
- Two-photon resonant process



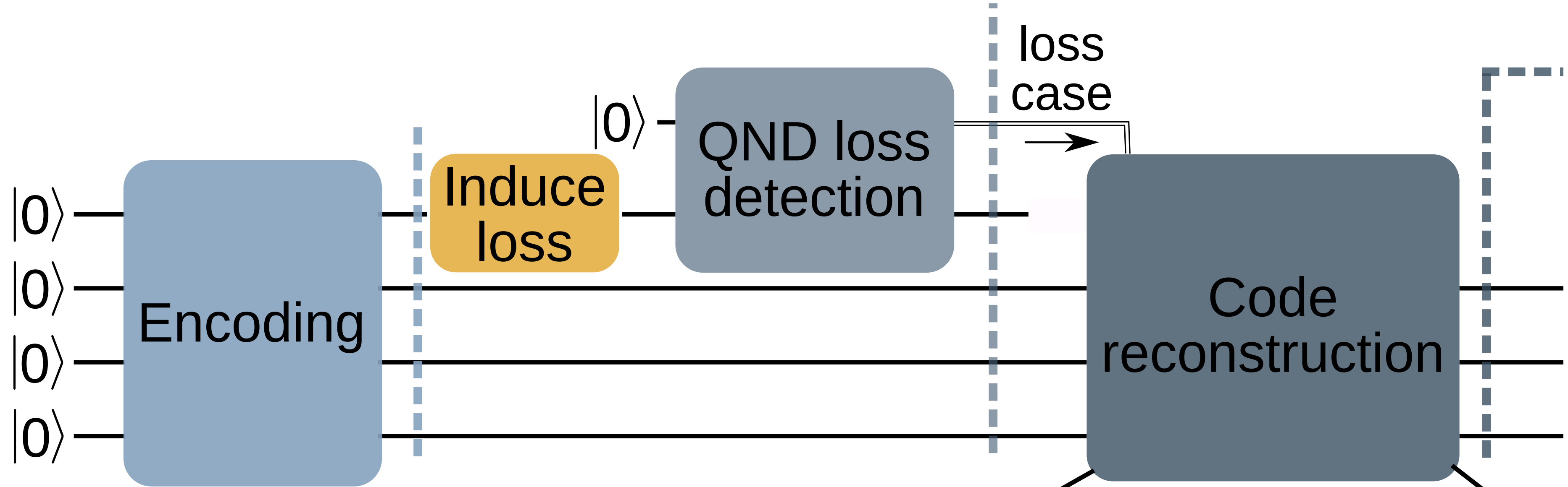

## If code qubit is lost



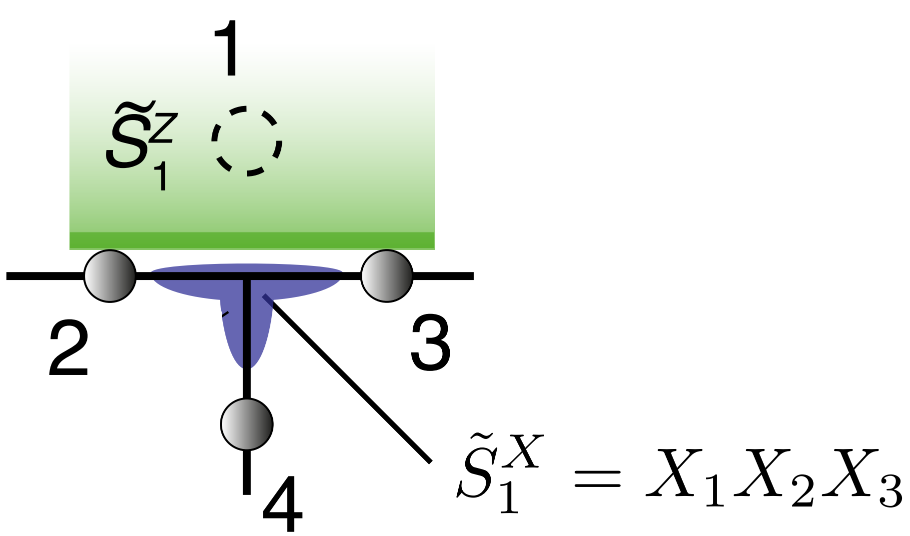

## If code qubit is not lost

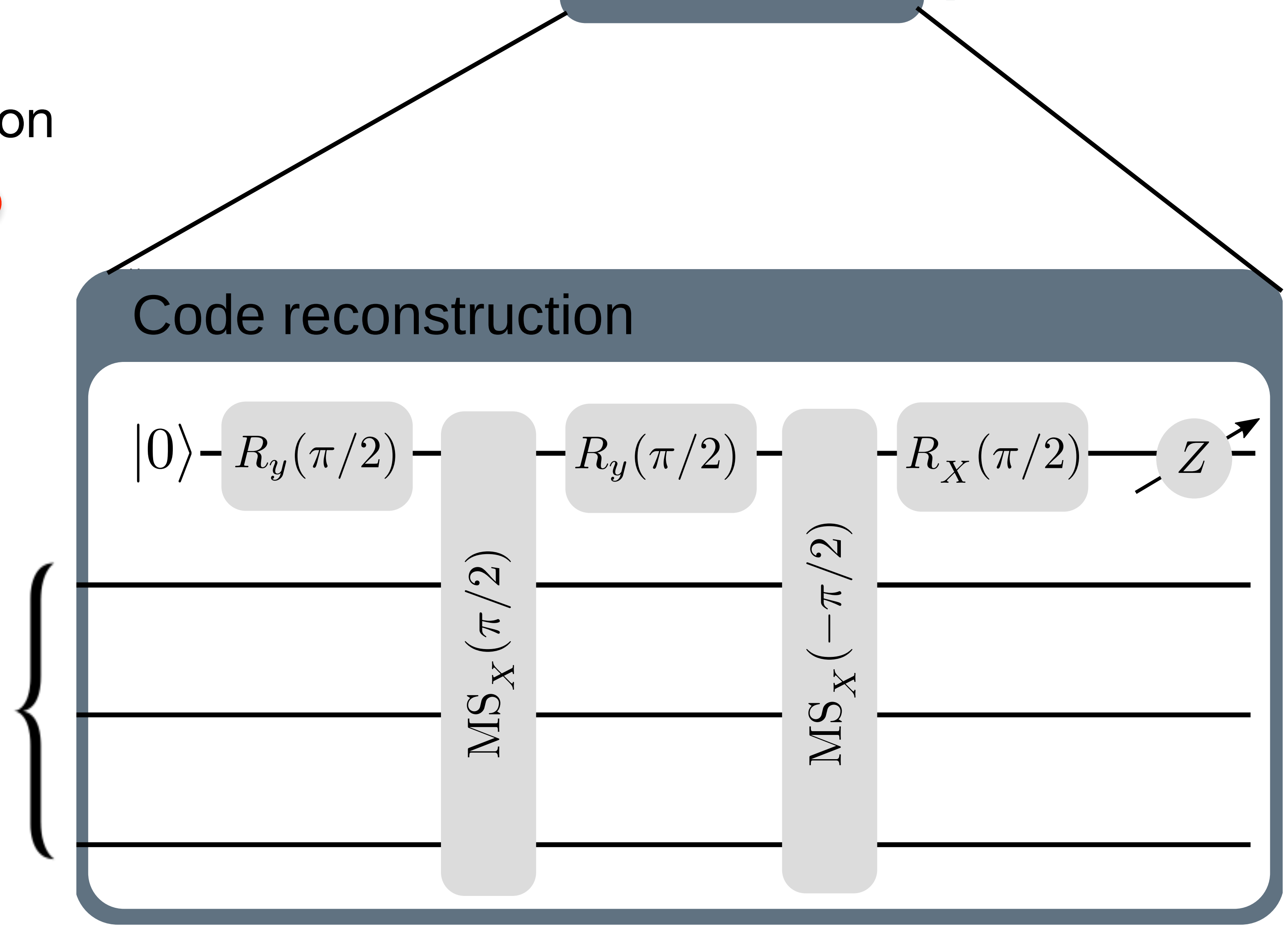


# 4 - Recovery of the encoded qubit - code reconstruction




If loss detected: Code reconstruction
by **measuring** $\tilde{S}_1^X = X_1 X_2 X_3$




Müller *et al.,* New J. Phys **13**, 085007 (2011)

# Qubit losses

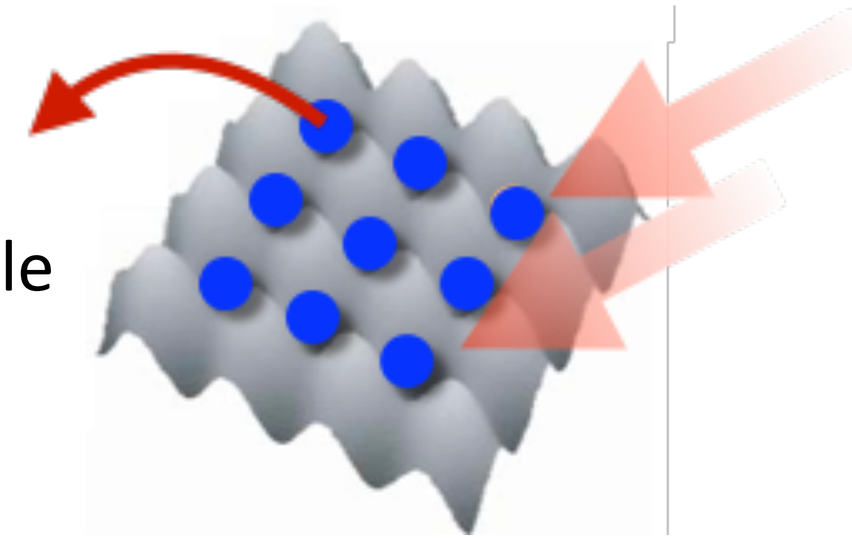
**Motivation:**

Losses and leakage can damage the performance of (topological) QEC codes

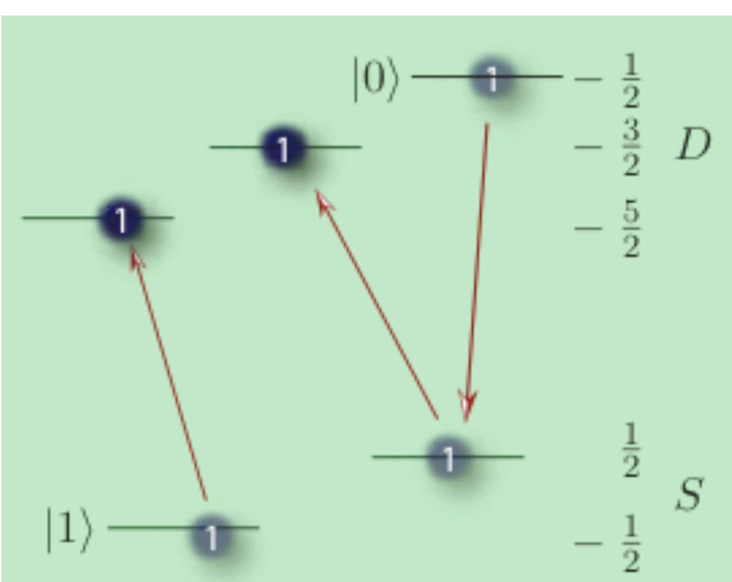
**Challenges:**

- Find protocols to deal with qubit loss
- Understand **robustness** of codes used
- Develop and experimentally test **in-situ leakage loss detection** and **correction** protocols

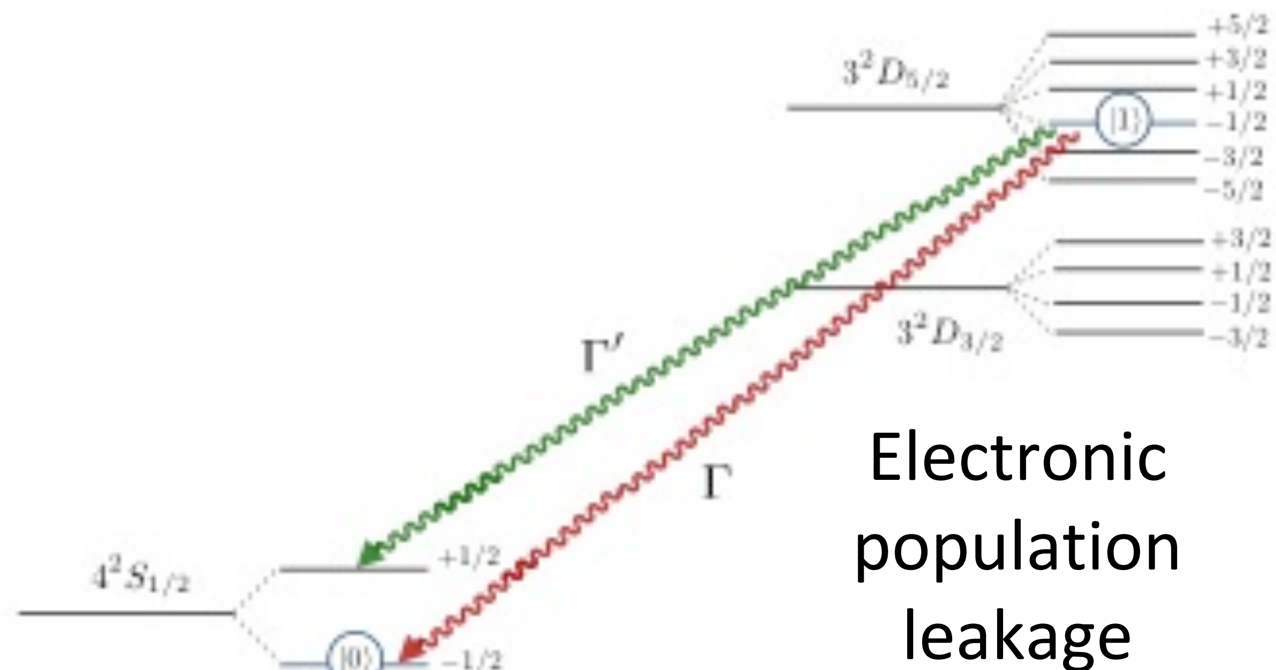
Particle loss



**Different incarnations of qubit loss:**




Imperfect spectroscopic decoupling ('hiding')




Electronic population leakage