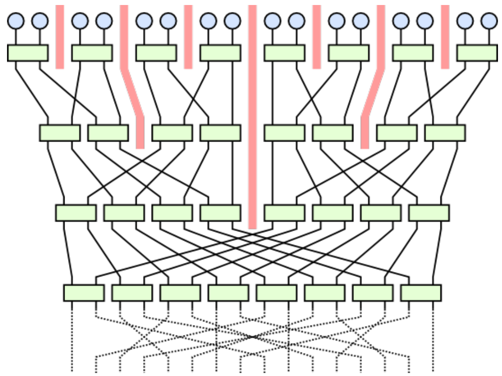
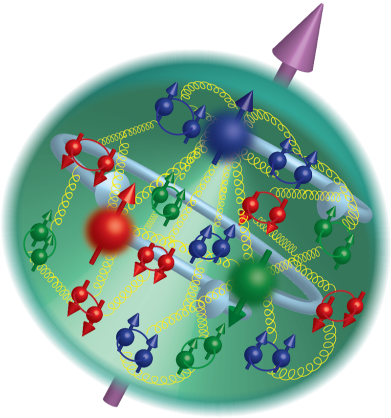
Tree Tensor Networks approach for Lattice Gauge Theories





Giuseppe Magnifico
University of Padova

HPC and Quantum Computing Workshop – December 15, 2020 arXiv:2011.10658

in collaboration with:

TIMO FELSER – University of Padova

PIETRO SILVI – University of Innsbruck

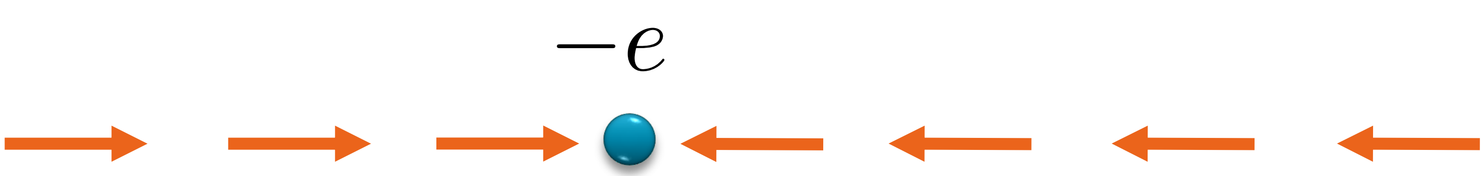
SIMONE MONTANGERO – University of Padova

Lattice Gauge Theories (LGT)

1) Quantum Matter and Quantum Fields

2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$



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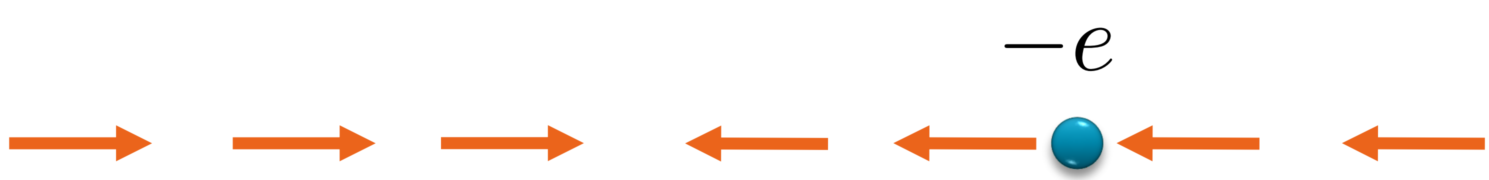

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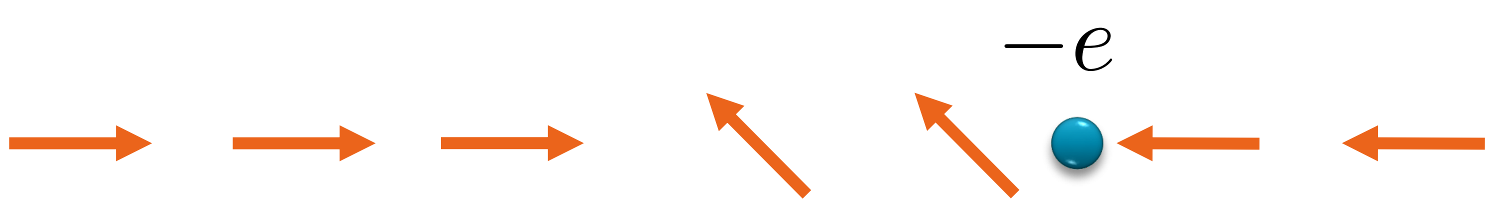

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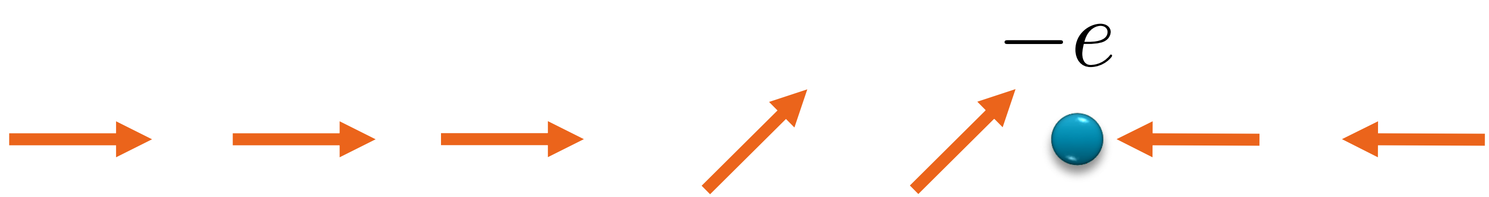

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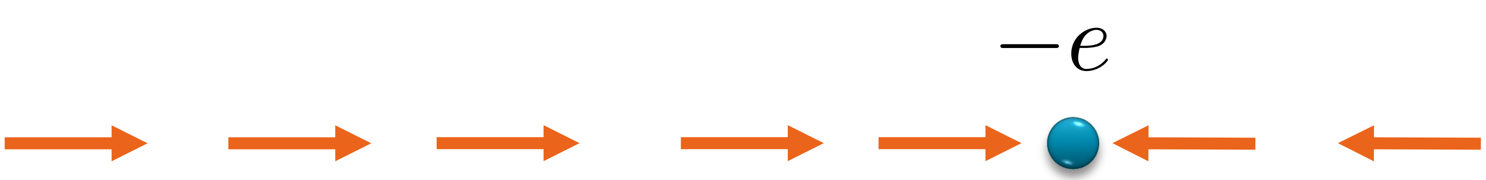
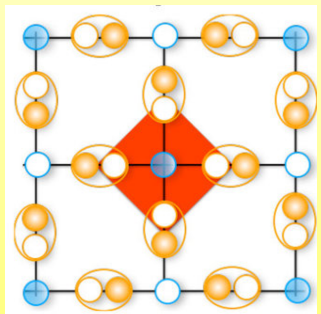
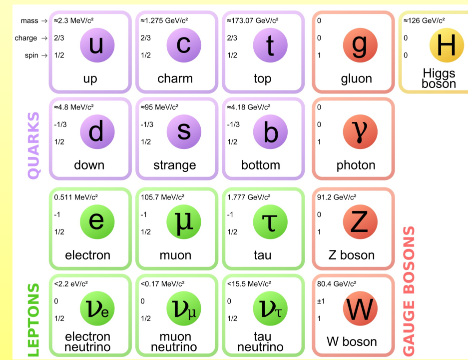


LGT are ubiquitous theoretical framework!

As emergent theories in condensed matter: **high-Tc superconductors, frustrated systems, spin liquids.**



As fundamental description in particle physics: **Standard Model**



- They are <u>**extremely demanding**</u> from a numerical point of view.
- Ideal goal for HPC and Quantum Simulation/Computation!

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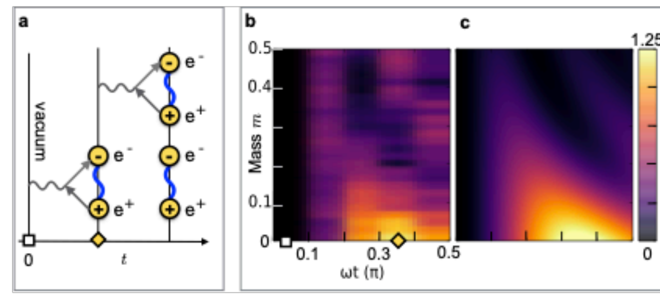
2016: U(1) on quantum computers



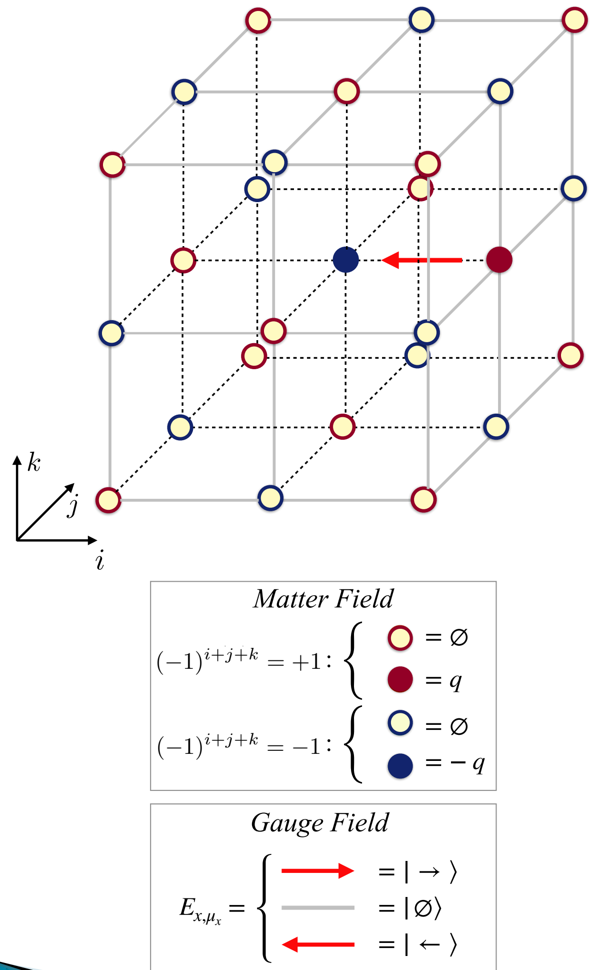
Nature **534,** 516–519 (2016).

Simulating Lattice Gauge Theories within Quantum Technologies

M.C. Bañuls[1,2], R. Blatt[3,4], J. Catani[5,6,7], A. Celi[3,8], J.I. Cirac[1,2], M. Dalmonte[9,10], L. Fallani[5,6,7], K. Jansen[11], M. Lewenstein[8,12,13], S. Montangero[7,14 a], C.A. Muschik[3], B. Reznik[15], E. Rico[16,17 b], L. Tagliacozzo[18], K. Van Acoleyen[19], F. Verstraete[19,20], U.-J. Wiese[21], M. Wingate[22], J. Zakrzewski[23,24], and P. Zoller[3]

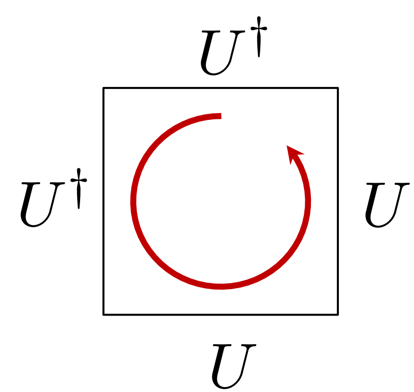
Eur. Phys. J. D **74,** 165 (2020)

Lattice QED in (3+1)D

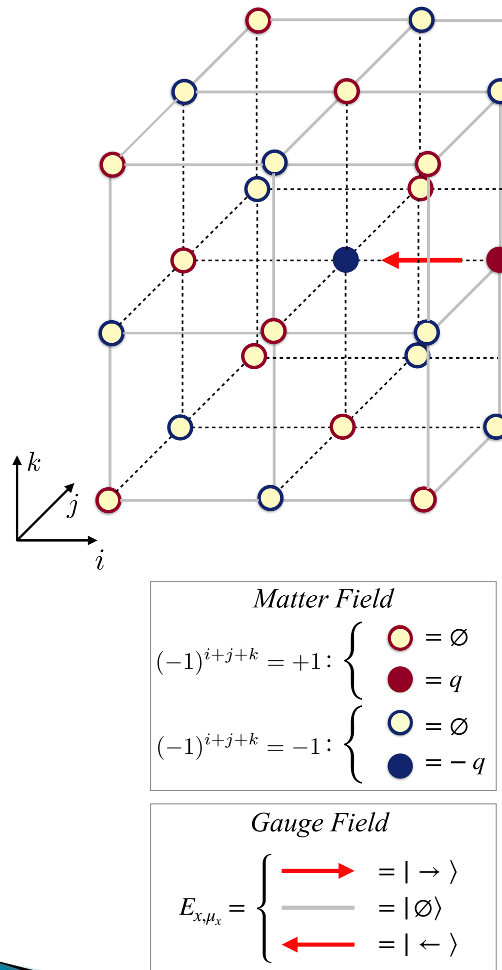


$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right) + m \sum_{x} (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_{x} \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

$$\Box_{\mu_x,\mu_y} = U_{x,\mu_x} U_{x+\mu_x,\mu_y} U^\dagger_{x+\mu_y,\mu_x} U^\dagger_{x,\mu_y}$$



Lattice QED in (3+1)D

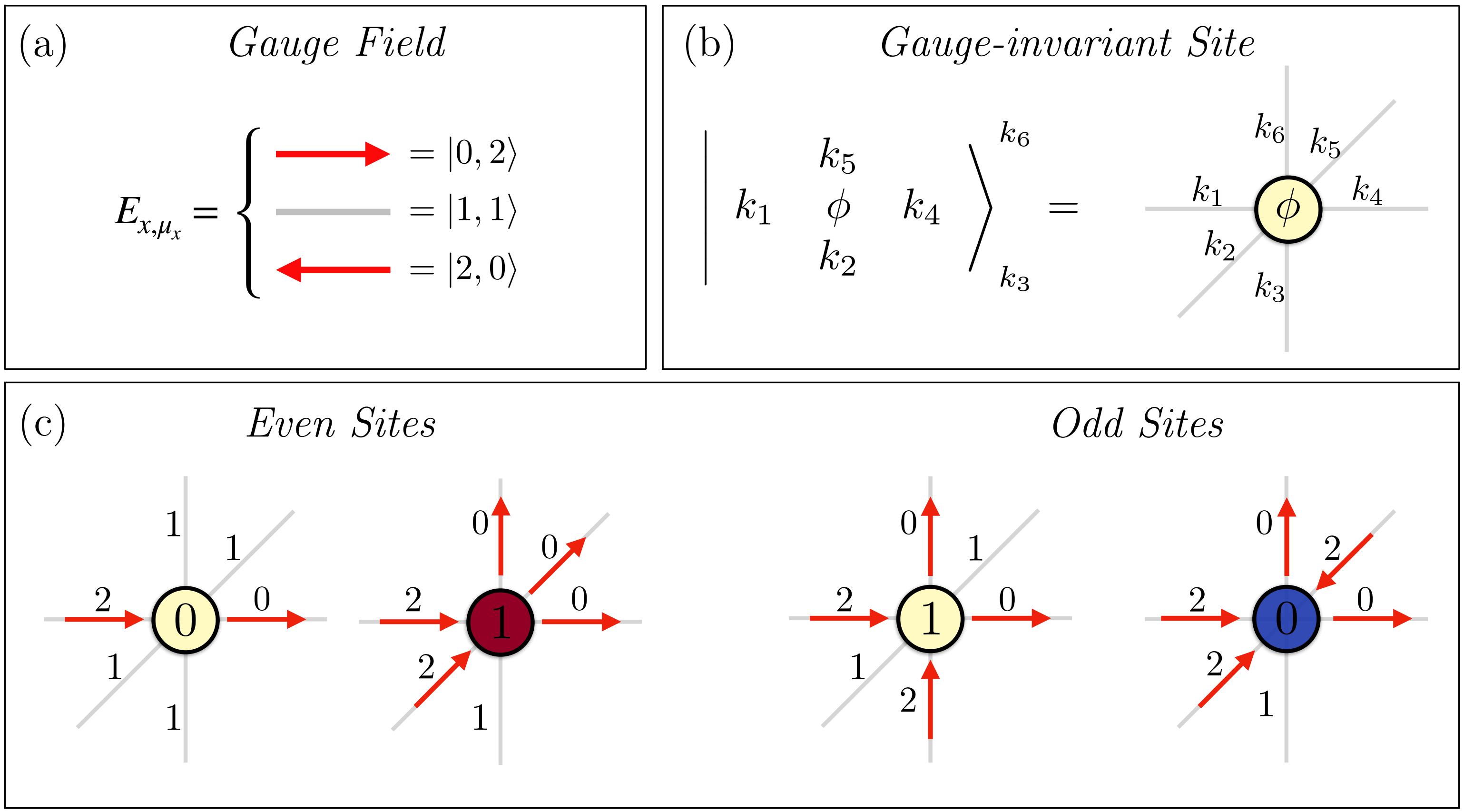


$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$
$$+m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$
$$-\frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

Quantum Link Model
discretization of Gauge Fields

$$\hat{E}_{x,\mu} \rightarrow \hat{S}^z_{x,\mu}$$
$$\hat{U}_{x,\mu} \rightarrow \hat{S}^+_{x,\mu}/s,$$

$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1-p_x}{2} - \sum_\mu \hat{E}_{x,\mu} \qquad \hat{G}_x|\Phi\rangle = 0 \ \forall x$$

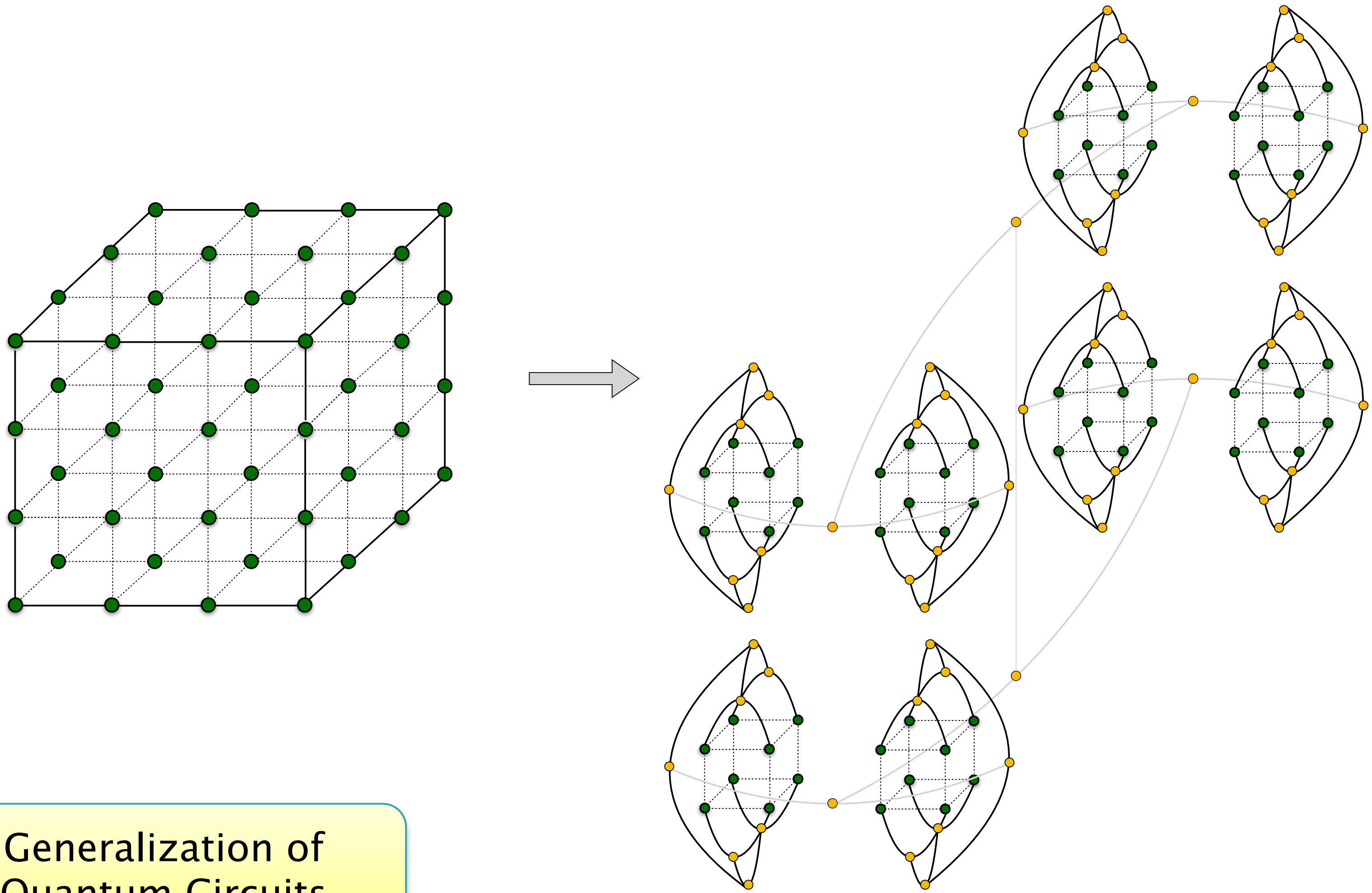


Quantum Simulation model

$$dim H_x = 267$$

just for comparison, like a spin system with $s = 133$

Tree Tensor Networks in 3D

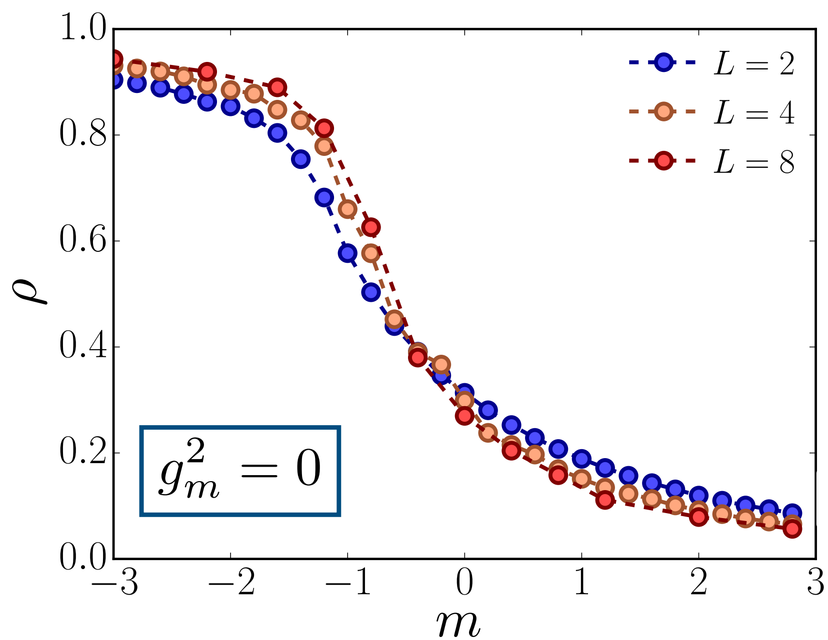


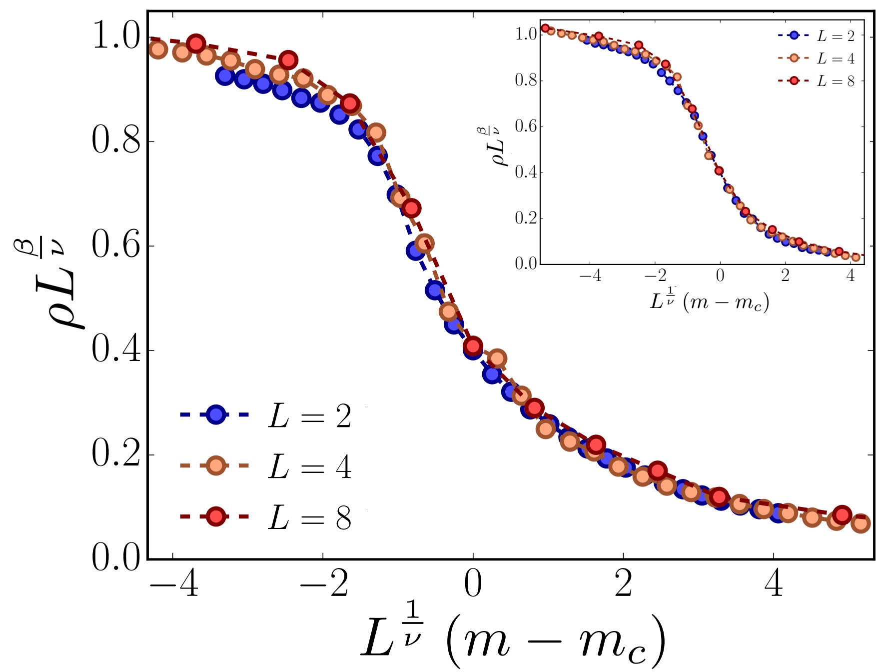
Generalization of
Quantum Circuits

Ground state properties for $g_m^2 = 0$

$$\rho = \frac{1}{L^3} \sum_x \langle GS|\hat{n}_x|GS\rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda\left(L^{\frac{1}{\nu}}(m - m_c)\right)$$



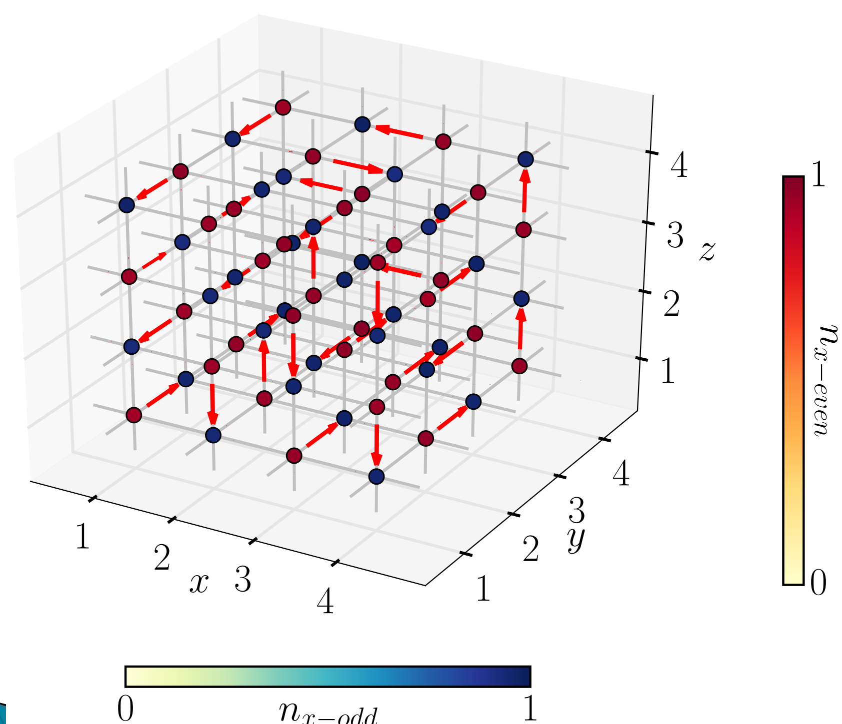


$$m_c = -0.39$$
$$\beta = 0.16 \quad \nu = 0.22$$

Local configurations of matter and gauge fields

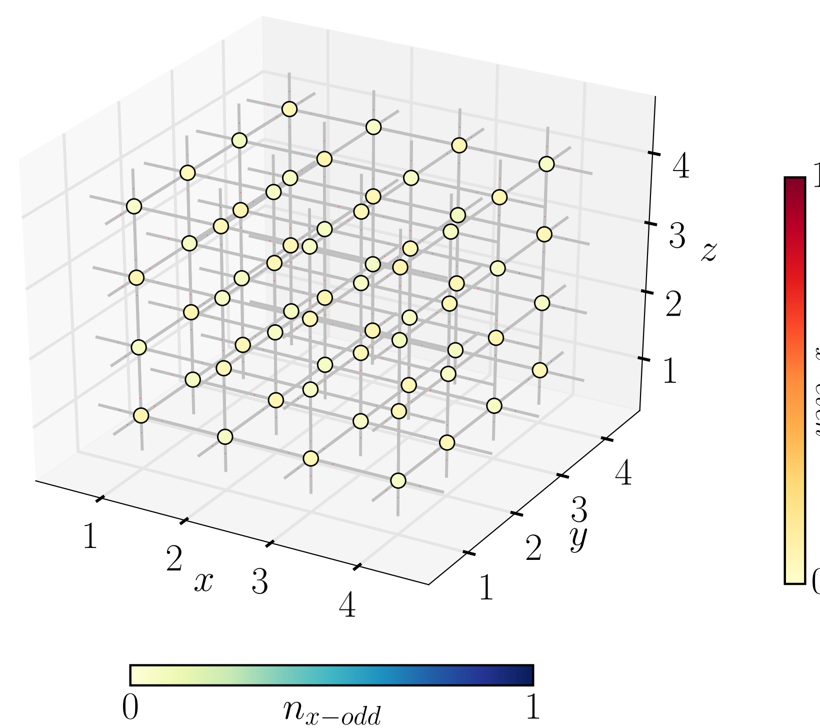
$$-2m \gg g_e^2/2 > 0$$

Charge–Crystal Phase: particle–antiparticle dimers



$$g_e^2/2 \gg 2|m|$$

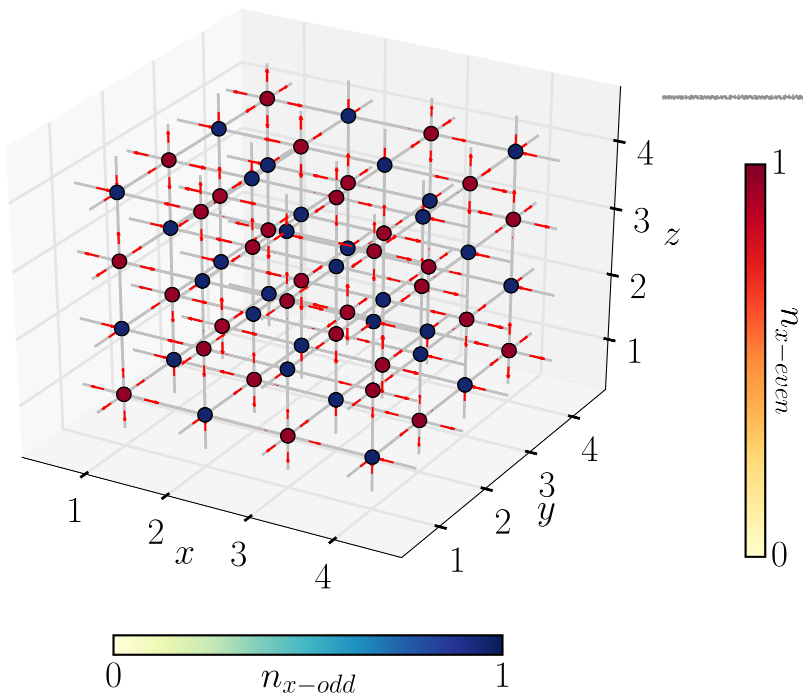
Vacuum phase: no particles, no field excitations

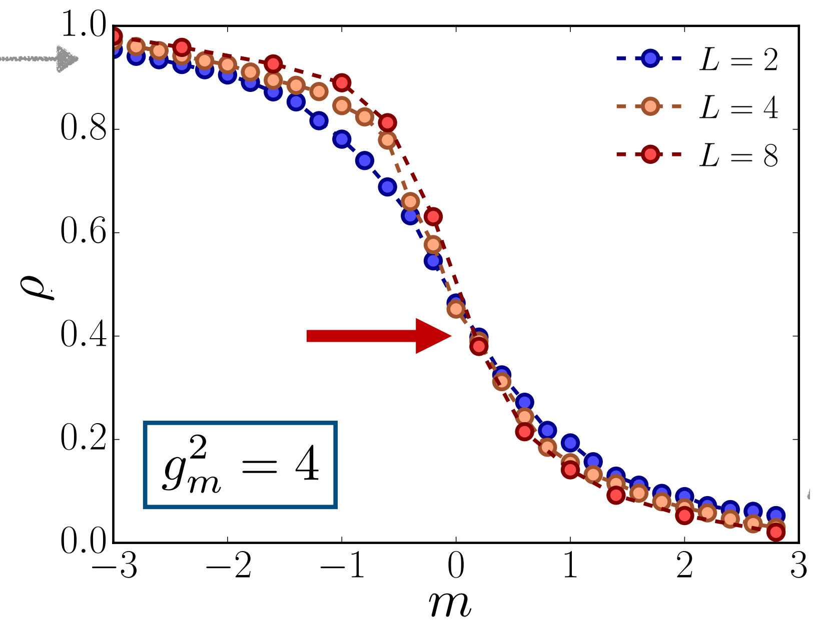


Ground state properties for $g_m^2 = \frac{8}{g_e^2} = 4$

$$\rho = \frac{1}{L^3} \sum_x \langle GS|\hat{n}_x|GS\rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda\left(L^{\frac{1}{\nu}}(m - m_c)\right)$$



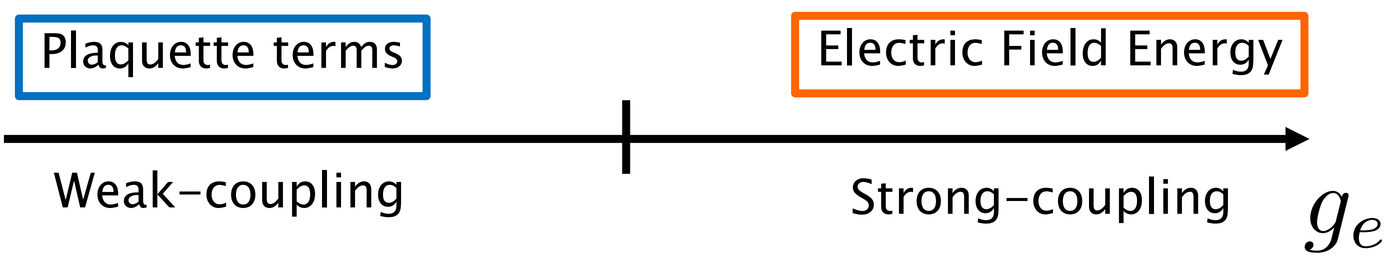


$$m_c = +0.22$$
$$\beta = 0.16 \quad \nu = 0.22$$

Confinement Properties

$$g_m^2 = 8/g_e^2$$

$$\hat{H} = -t\sum_{x,\mu}\left(\hat{\psi}_x^\dagger\,\hat{U}_{x,\mu}\,\hat{\psi}_{x+\mu} + \text{H.c.}\right)$$
$$+m\sum_x(-1)^x\hat{\psi}_x^\dagger\hat{\psi}_x + \frac{g_e^2}{2}\sum_{x,\mu}\hat{E}_{x,\mu}^2$$
$$-\frac{g_m^2}{2}\sum_x\left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.}\right)$$



Confinement Properties

Plaquette terms

Electric Field Energy

Weak-coupling

Strong-coupling

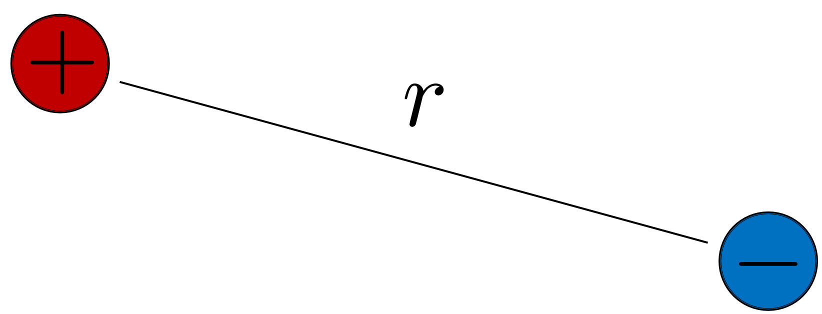
g_e

$$V(r) \propto 1/r$$

Deconfined phase

$$V(r) \propto r$$

Confined phase



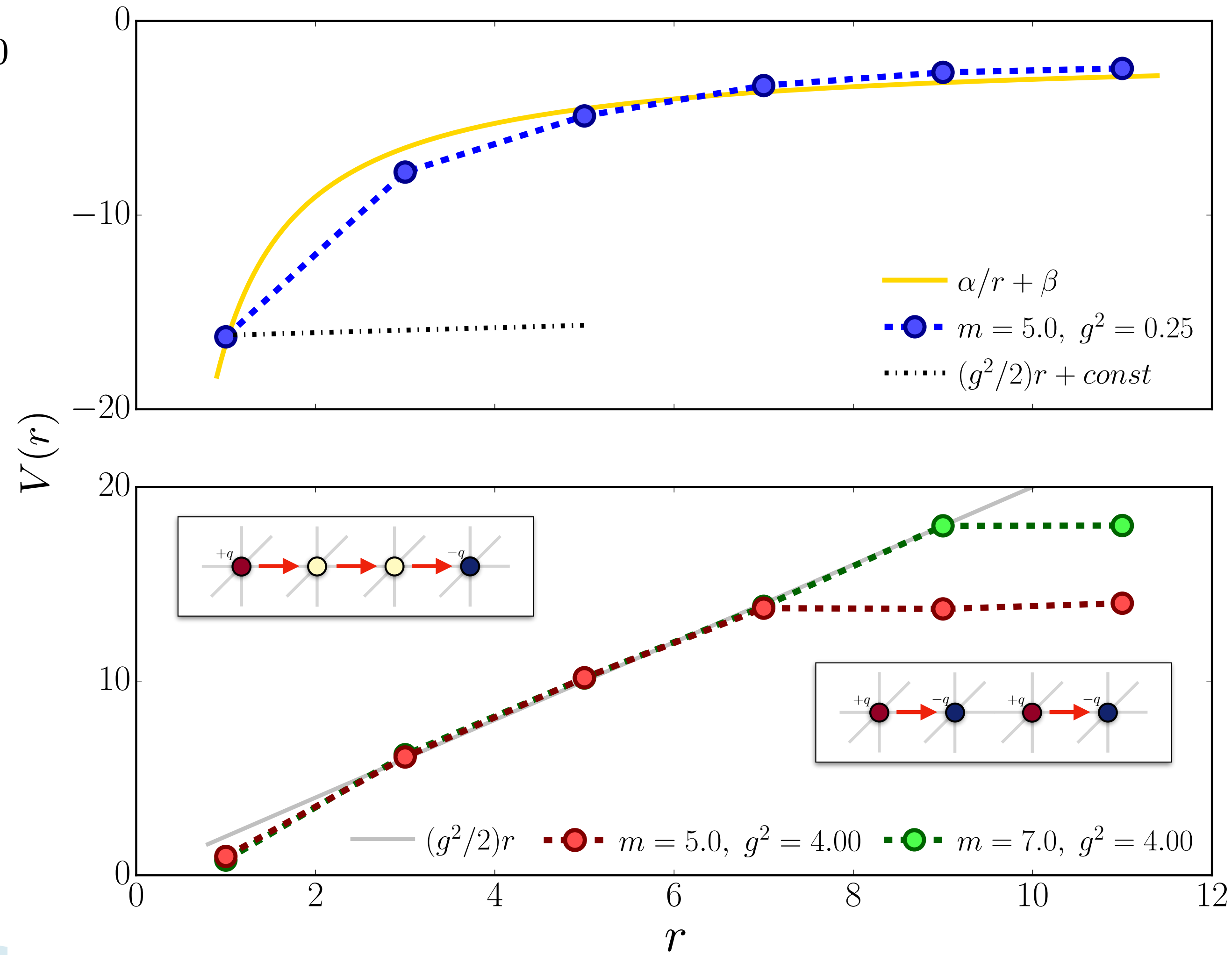
Kei-Ichi Kondo, Phys. Rev. D **58**, 085013 (1998)

Confinement Properties

$$V(r) = E(r) - E_0$$

Weak–coupling

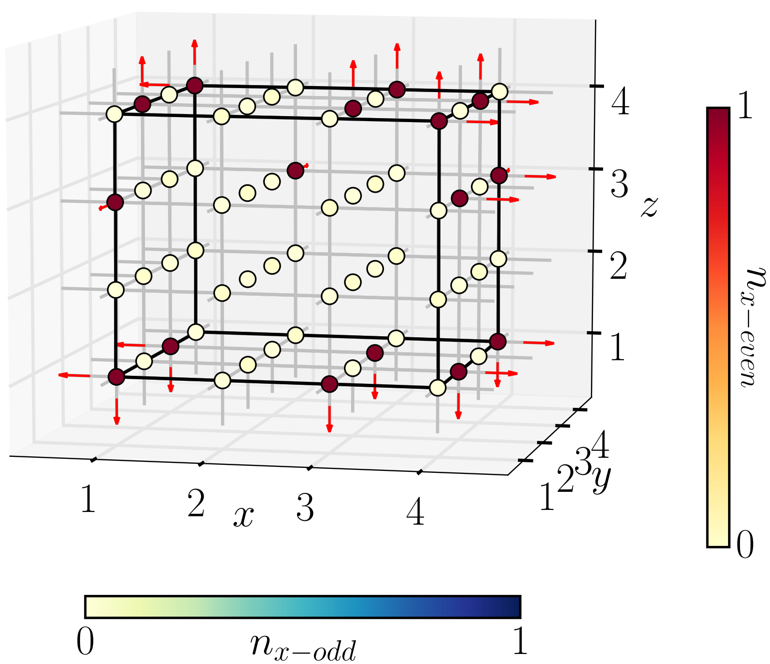
Strong–coupling

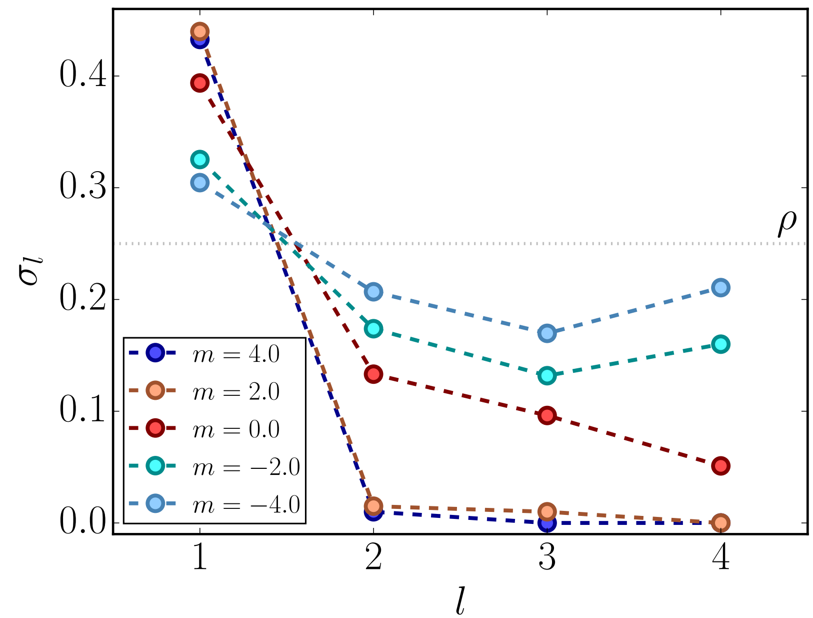


Finite Density

$L = 4\,, Q = 16\,, \rho = 1/4$

$L = 8\,, Q = 128\,, \rho = 1/4$



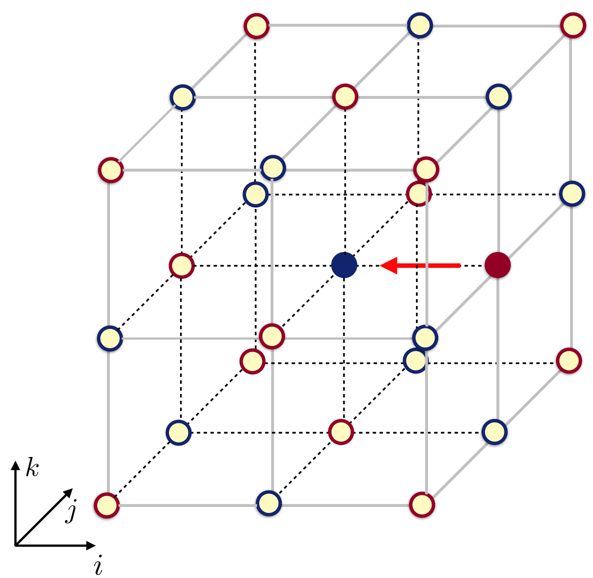


$$\sigma(l) = \frac{1}{A(l)} \sum_{x \in A(l)} \left\langle \hat{\psi}_x^\dagger \hat{\psi}_x \right\rangle$$

What's next?

Quantum Matter Physics ⟷ *High-Energy Physics domain*

- Testing and validating experimental implementations on quantum hardware.
- Non-Abelian symmetries (QCD).
- Exploit TTN algorithms to simulate phenomena of high-energy physics relevance, such as real-time dynamics and scattering.



THANK YOU