QUBO FORMULATION FOR THE NUMBER PARTITIONING PROBLEM

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REPLY

ACKNOWLEDGEMENTS

1 Marco Magagnini, PhD – Reply Quantum computing practice leader

2 Davide Caputo, Blanca Silva, Giovanni Fazzi in Reply

3 Reply Quantum computing practice

OUTLINE

1 The Number Partitioning Problem (NPP): definition and motivation

2 QUBO formulation and problem instances

3 Results and conclusions

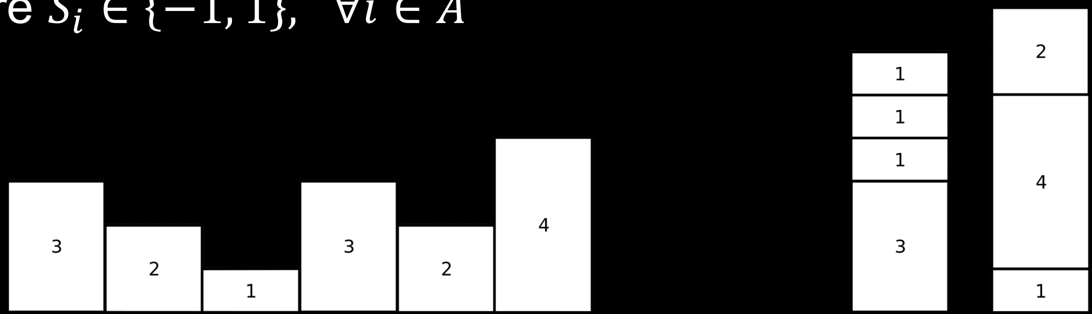
NUMBER PARTITIONING PROBLEM (NPP)

1 Given $A = \{a_i \in \mathbb{N}\}_{i=1}^{n}$, find A_1, A_2 such that $A_1 \cup A_2 = A$, $A_1 \cap A_2 = \emptyset$ and $\sum_{i \in A_1} a_i = \sum_{j \in A_2} a_j$

2 Equivalently, minimize the quadratic form:

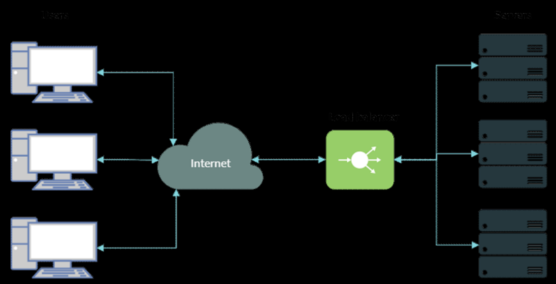
$$\left(\sum_{i \in A} a_i S_i \right)^2$$

where $S_i \in \{-1, 1\}, \quad \forall i \in A$

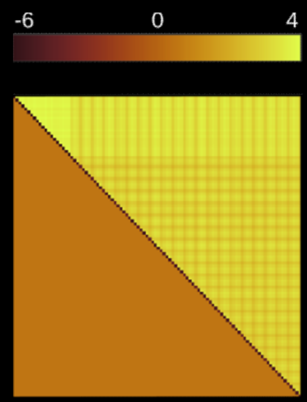


WHY THE NPP?

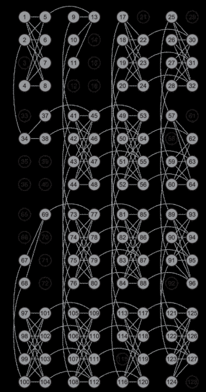
1. Recurrent optimization model
   - Load balancing problems



2. Great challenge for qubits connectivity
   - Fully connected architecture embedded on less connected graph



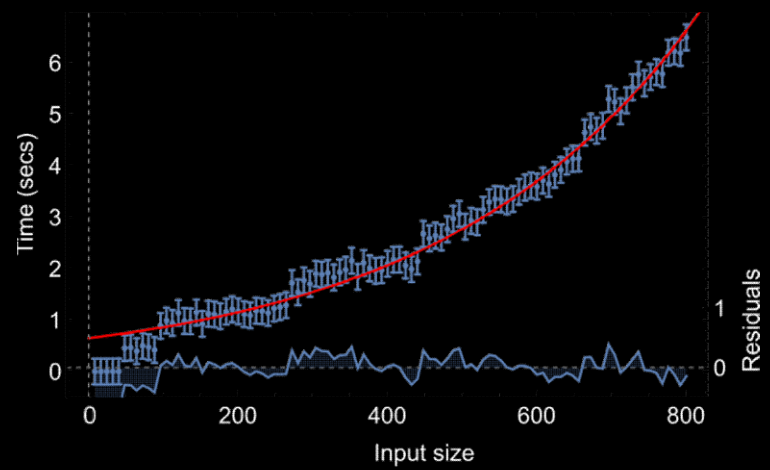
Example of QUBO matrix



Chimera graph

NPP – AN NP-HARD MODEL

3. Time complexity to solve NPP scales exponentially with input size



GOAL: Study quality of solutions given by D-Wave's $2000Q^{TM}$ Quantum Annealer

QUBO MODEL

- Quantum annealers minimize Ising Energy $H = \sum_i h_i S_i + \sum_{ij} J_{ij} S_i S_j$
- Changing variables, the problem is equivalently written as a QUBO:

$$\min E(x|Q) = \sum_{i \le j} x_i Q_{ij} x_j$$

Q | **Quadratic:**
the highest power of variables is x^2

U | **Unconstrained:**
no external constraints are applied

B | **Binary:**
variables are binary {0,1}

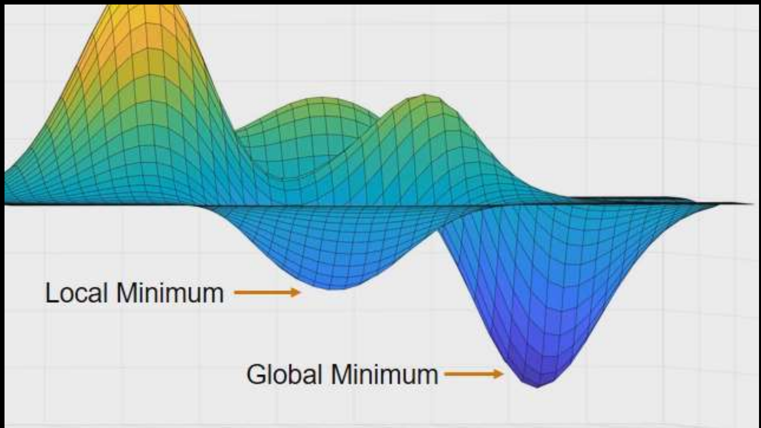
O | **Optimization:**
Minimization of an objective function (the energy)

NPP AS QUBO

- Recall: the NPP is the task of minimizing:

$$\left(\sum_{i\in A} a_i S_i\right)^2, \quad S_i \in \{-1, 1\} \forall i \in A$$

$$x_i = \frac{S_i + 1}{2}$$

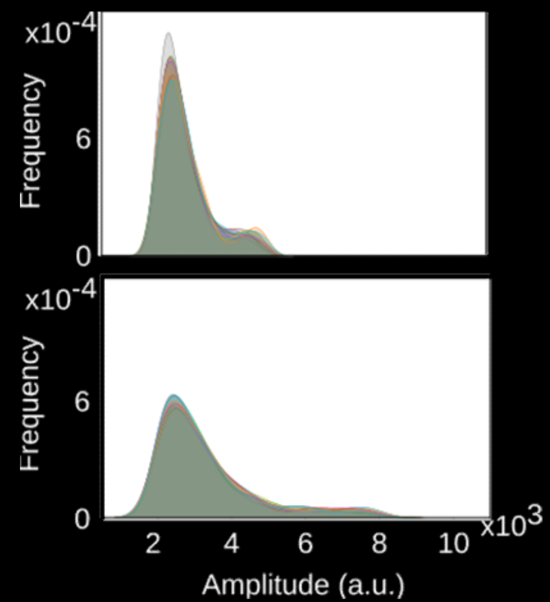


- We obtain a problem in the QUBO form:

$$\min x^T Q x, \quad x_i \in \{0,1\} \forall i \in A$$

PROBLEM DATA

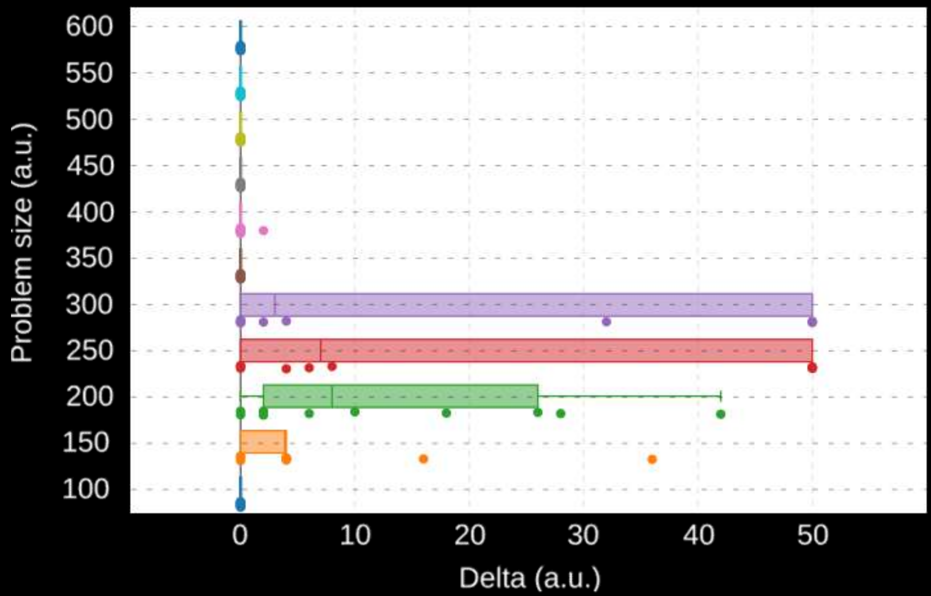
- Self-generated data with known ground state giving $E(x|Q) = 0$
- Problem sizes from 100 to 600 variables (positive integers)
- 10 instances for each problem size



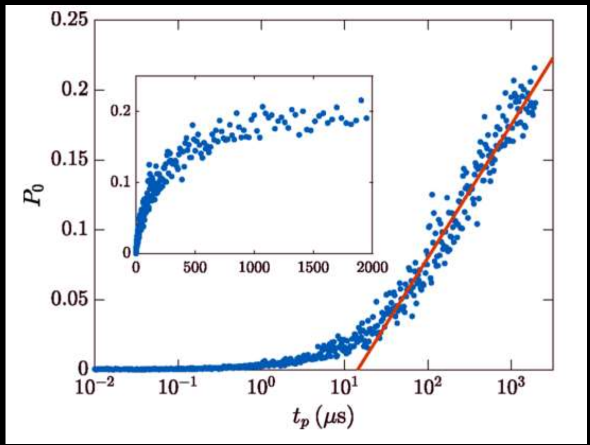
Data distributions for 200 and 500 variables

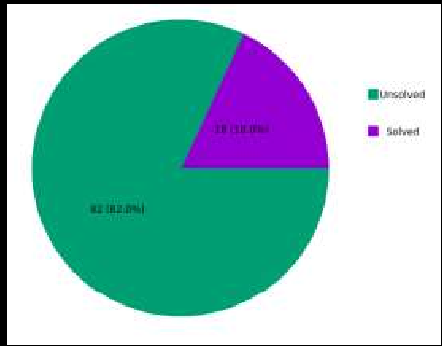
RESULTS

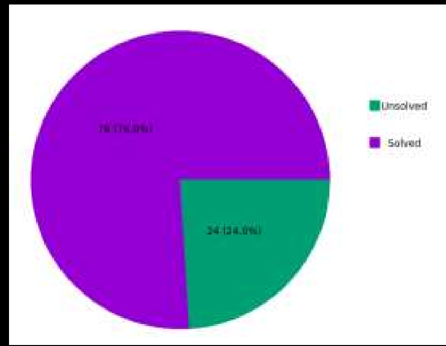
- (Truncated) deltas between $\sum_{i \in A_1} a_i$ and $\sum_{j \in A_2} a_j$ over 10 instances, for each problem size
- Optimal results for delta equal to 0



ANNEALING PAUSE





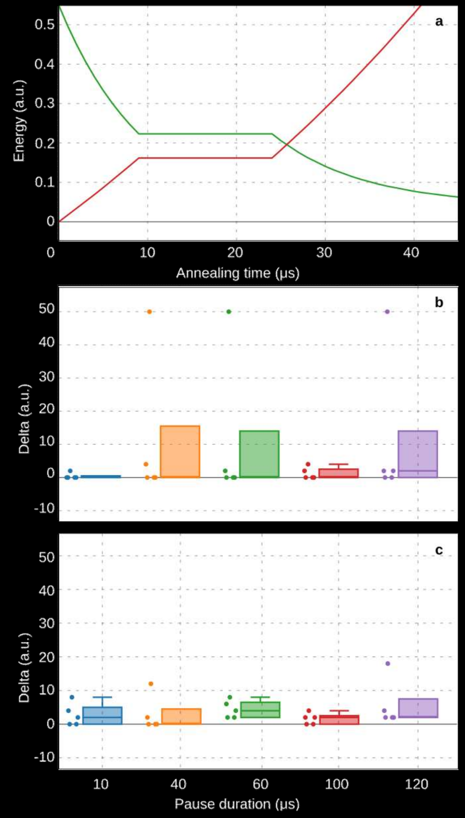


(J.Marshall, et al. **Power of Pausing: Advancing Understanding of Thermalization in Experimental Quantum Annealers**)

(D.Ottaviani, et al. **Low Rank Non-Negative Matrix Factorization with D-Wave 2000Q**)

PAUSE INVESTIGATION

- Goal: increase performances on problematic datasets
- How: allow for a pause in annealing cycle
- Collect statistics over 5 additional runs for problems with 300 (b) and 200 (c) variables



CONCLUSIONS

1. We described the general context of Number Partitioning Problem

2. We introduced the QUBO model for quadratic combinatorial optimization

3. We showed empirical results on the D-Wave $2000Q^{TM}$ Quantum Annealer solving a fully-connected problem, as input size scales up
   - For most complex tasks, we have recorded a positive contribute of the annealing pause in finding the ground state

THANK YOU FOR THE ATTENTION

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REPLY