Introduction to Tensor Networks as classical benchmark for quantum computation

Simone Notarnicola, University of Padova, INFN, Italy

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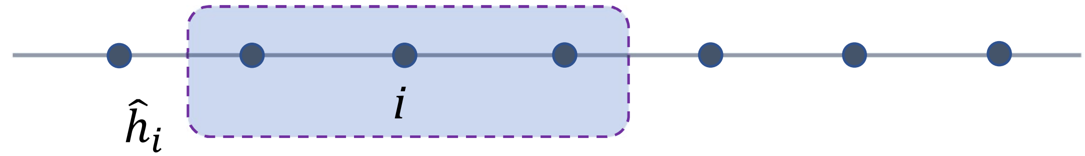
Quantum many-body systems

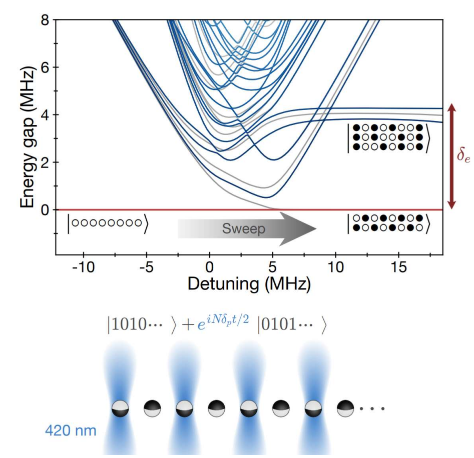
Lattice model, L sites, local dimension d $\rightarrow \dim \mathcal{H} = d^L$

1-D local Hamiltonian $\widehat{H} = \sum_i \widehat{h}_i$



$$|\Psi\rangle = \sum_{\alpha_1,\dots,\alpha_L} c_{\alpha_1,\dots,\alpha_L} |\alpha_1,\dots,\alpha_L\rangle$$

Experimental benchmarking



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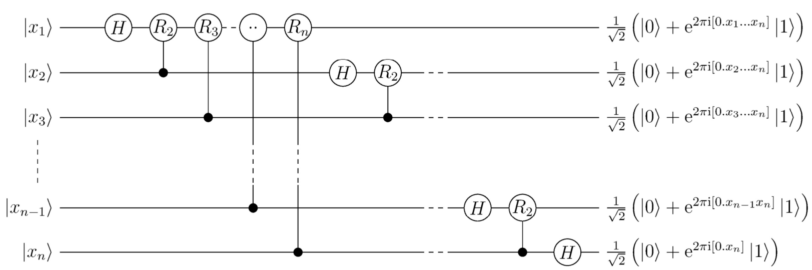
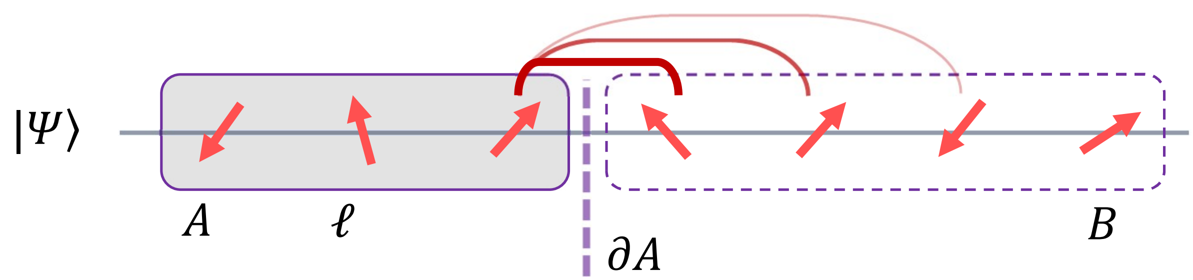
Quantum computing circuits



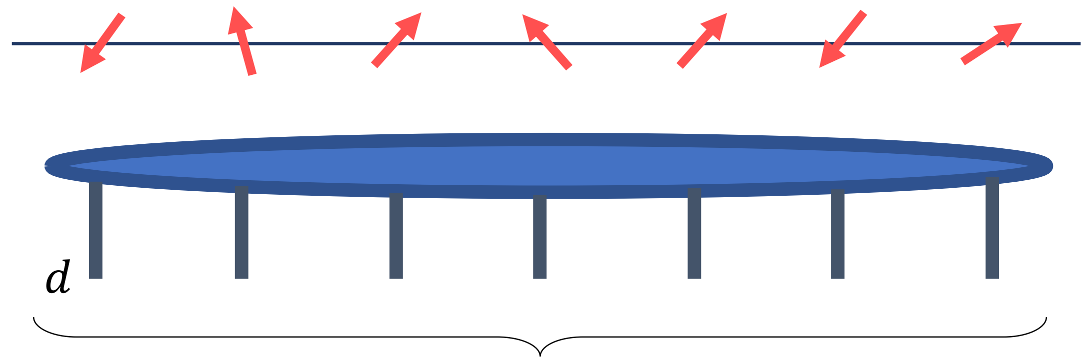
If correlations are local is not necessary knowing $|\Psi\rangle$ exactly



Tensor Networks and Matrix Product States

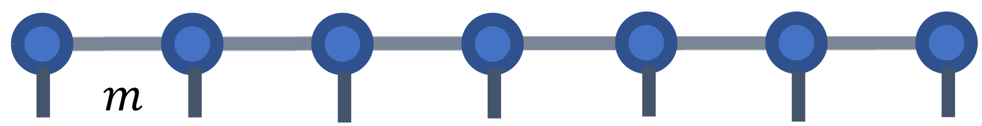
Efficient method to compress information

$$|\Psi\rangle = \sum_{\alpha_1,\dots,\alpha_L} c_{\alpha_1,\dots,\alpha_L} |\alpha_1,\dots,\alpha_L\rangle$$



Rank L tensor

Bond dimension m fixes the maximum amount of mutual information between subsystems

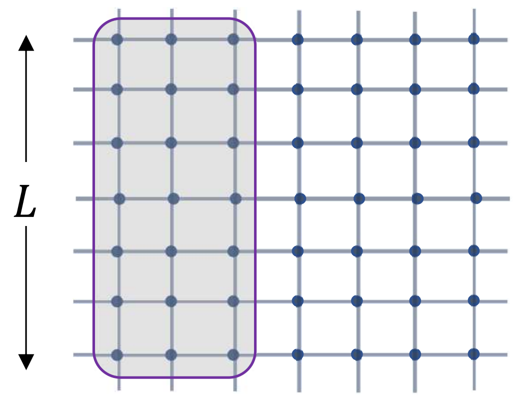


$$|\psi_{MPS}\rangle \sim L \cdot d \cdot m^2$$

Variational ansatz → $\min \langle \psi_{MPS} | \hat{H} | \psi_{MPS} \rangle$

$$S \sim \log m$$

Tree Tensor Networks



Which structure for 2D systems?

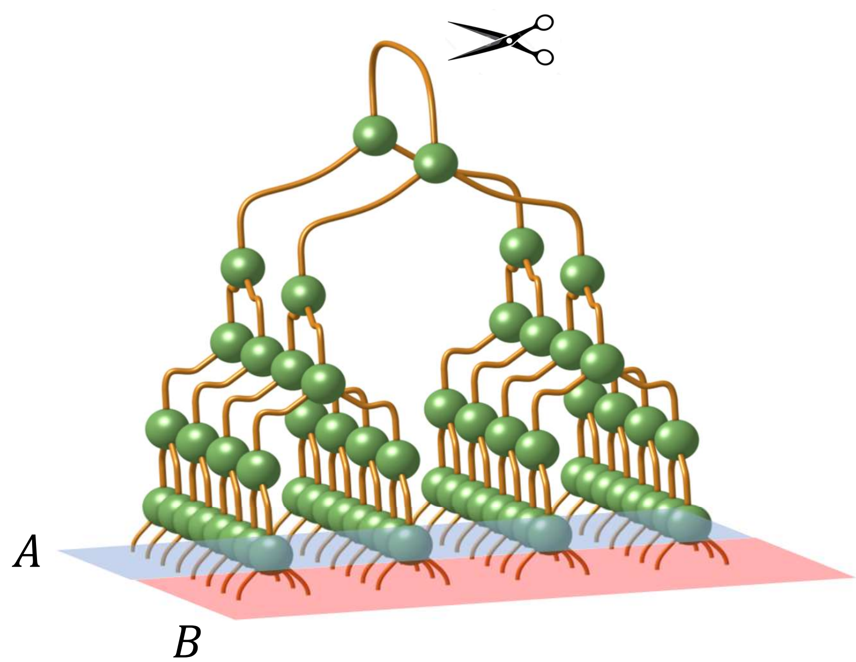
Binary Tree TN
Scaling invariant shape
Competitive computational complexity

Entropy computation?

$$S \sim \log m \sim L$$

↓

An exponentially large
bond dimension is needed



Augmented Tree Tensor Networks (T. Felser et al, arXiv 2011.08200)

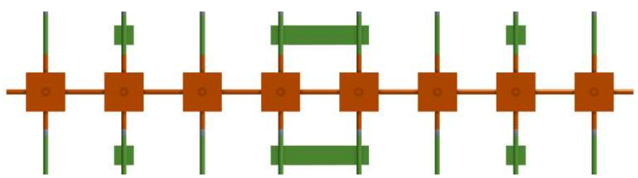
Starting point: TTN wave function $|\psi_{TTN}\rangle$

We add a layer made up of local unitary operators $\{u_k\}$

$$D(u) = \prod_k u_k$$

Placed along the boundaries individuated by the links

$\hat{H}_{eff}$



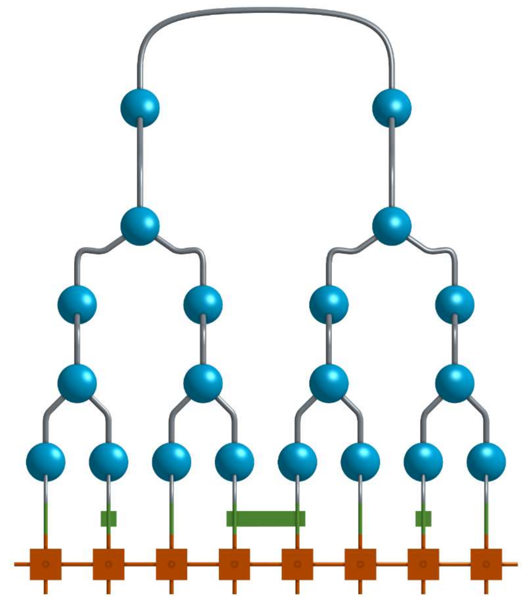
Ground state search:

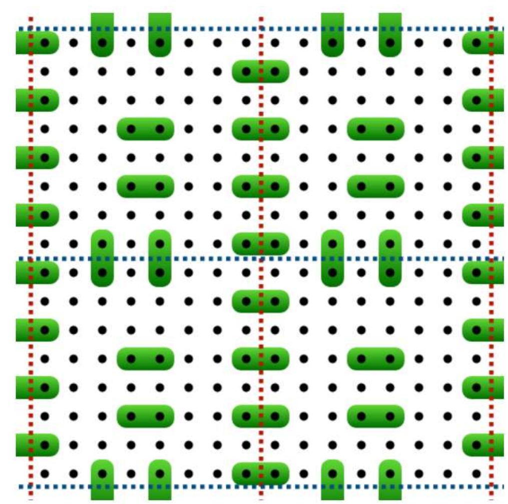
$$\begin{cases} \{\langle\psi_{TTN}|u_k^\dagger \hat{H} u_k|\psi_{TTN}\rangle\}_k \\ \langle\psi_{TTN}|\hat{H}_{eff}|\psi_{TTN}\rangle \end{cases}$$

$|\psi_{TTN}\rangle$

D

$\hat{H}$

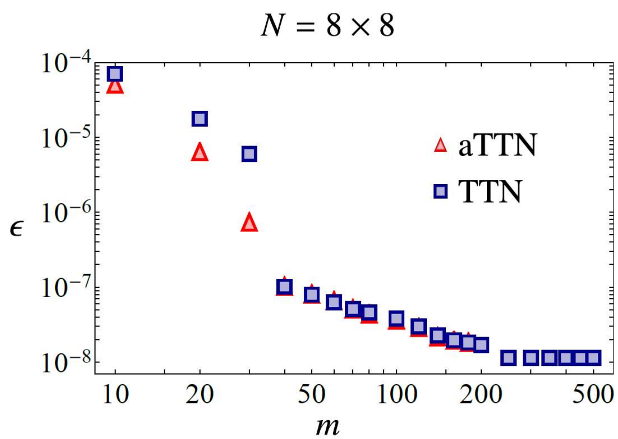


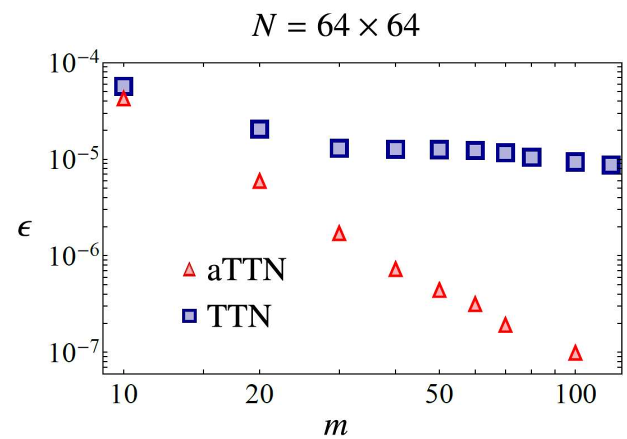


Augmented Tree Tensor Networks (T. Felser et al, arXiv 2011.08200)

Ising model

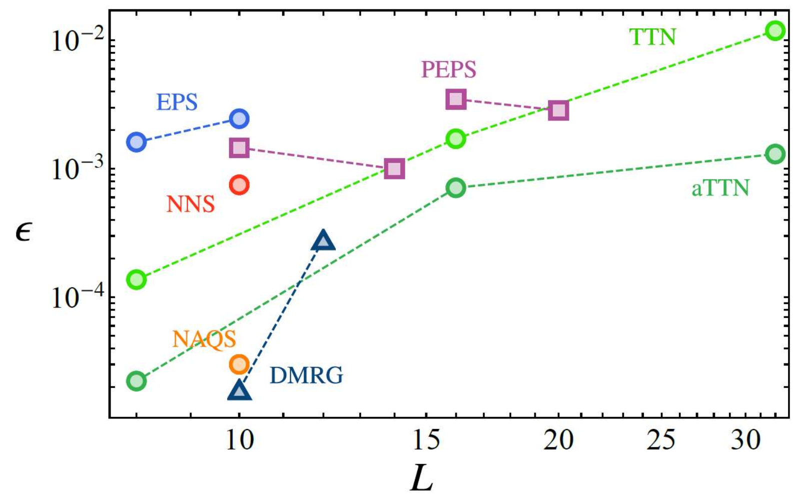
$$\hat{H} = \sum_{i,j=1}^{L} \sigma^x_{i,j}\sigma^x_{i,j+1} + \sigma^x_{i,j}\sigma^x_{i+1,j} + \sum_{i,j=1}^{L} \sigma^y_{i,j}\sigma^y_{i,j+1} + \sigma^y_{i,j}\sigma^y_{i+1,j}$$





Heisenberg model

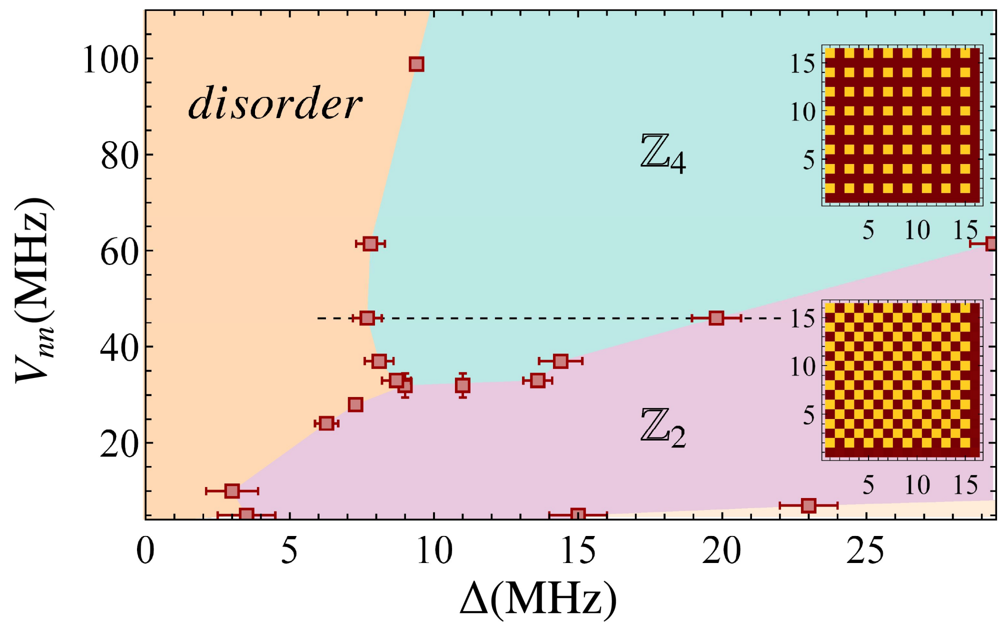
$$\hat{H} = \sum_{i,j=1,\gamma\in\{x,y,z\}}^{L} \sigma^\gamma_{i,j}\sigma^\gamma_{i,j+1} + \sigma^\gamma_{i,j}\sigma^\gamma_{i+1,j}$$



Augmented Tree Tensor Networks (T. Felser et al, arXiv 2011.08200)

2D Rydberg-atom lattice $$H_{ryd} = \sum_i \frac{\Omega}{2}\sigma_i^x + \sum_i (\Delta + \delta_i) n_i + \sum_{i<j} V_{ij} n_i n_j$$

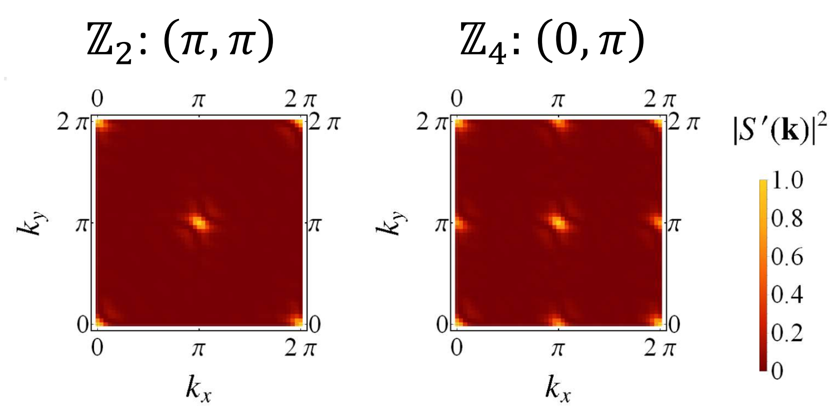
- $V_{ij} \equiv V(|i-j|) \sim \frac{c_6}{|i-j|^6}$
- Rydberg blockade radius r^* such that $V(r^*) = \Omega$



Different phases by changing V_{nn} and Δ

Static structure factor:

$$S(k) = \frac{1}{N^2}\sum_{r,s} e^{-ik\cdot(r-s)}\langle n_r n_s\rangle$$



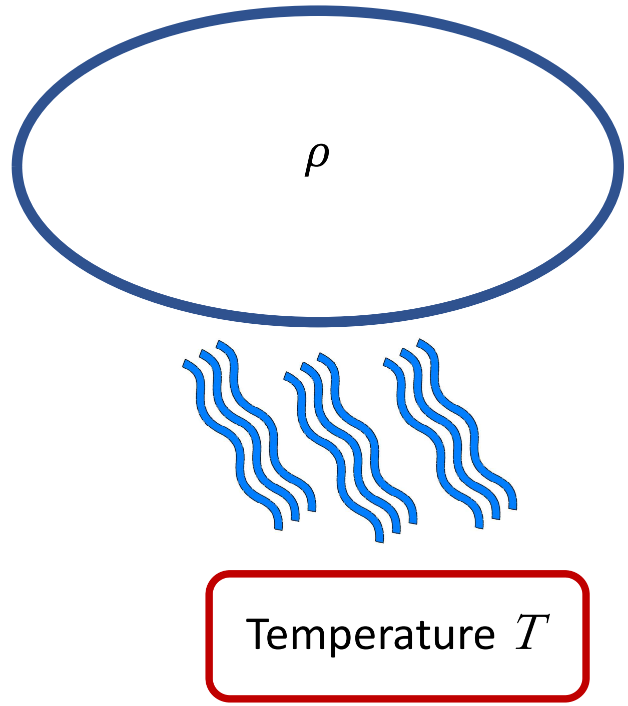
Tree Tensor operators (L. Arceci et al, arXiv 2011.10658)

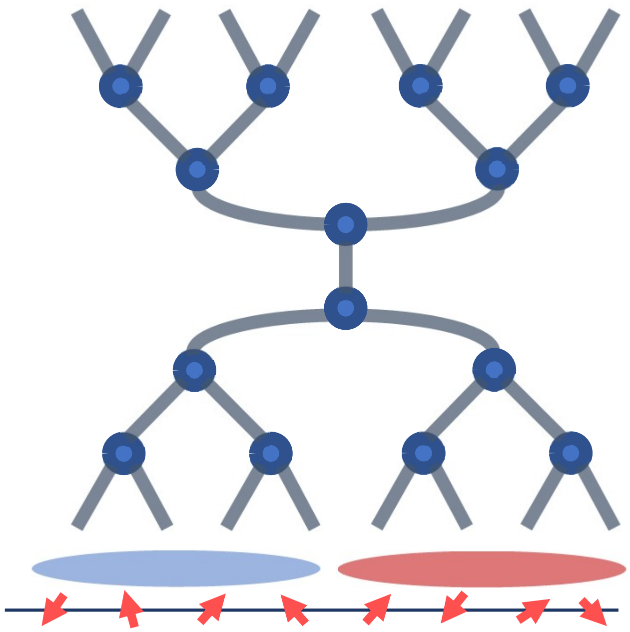
$$\rho = |\psi\rangle\langle\psi|$$

$$\downarrow$$

$$\rho = \sum_{j}^{K_0} p_j |\psi_j\rangle\langle\psi_j|$$

Classical mixture of pure states





$$\rho = X^\dagger X$$

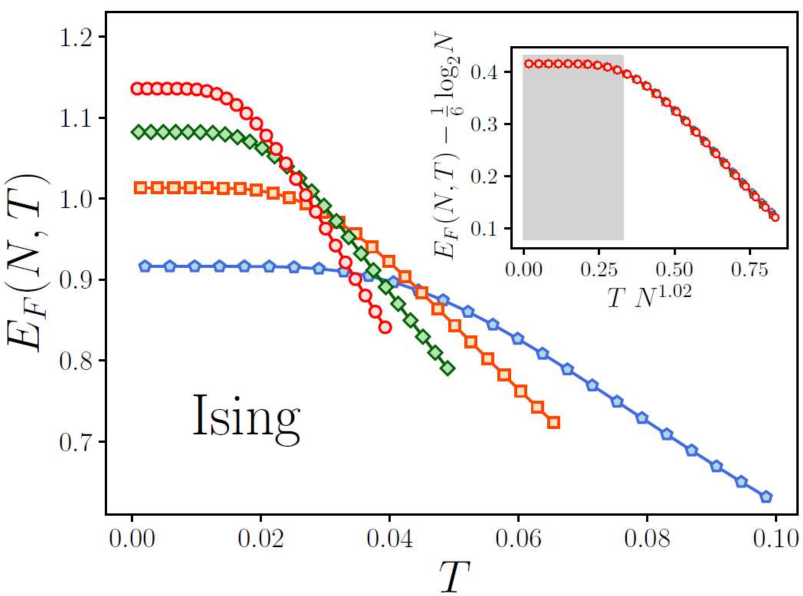
We can compute the entanglement of formation

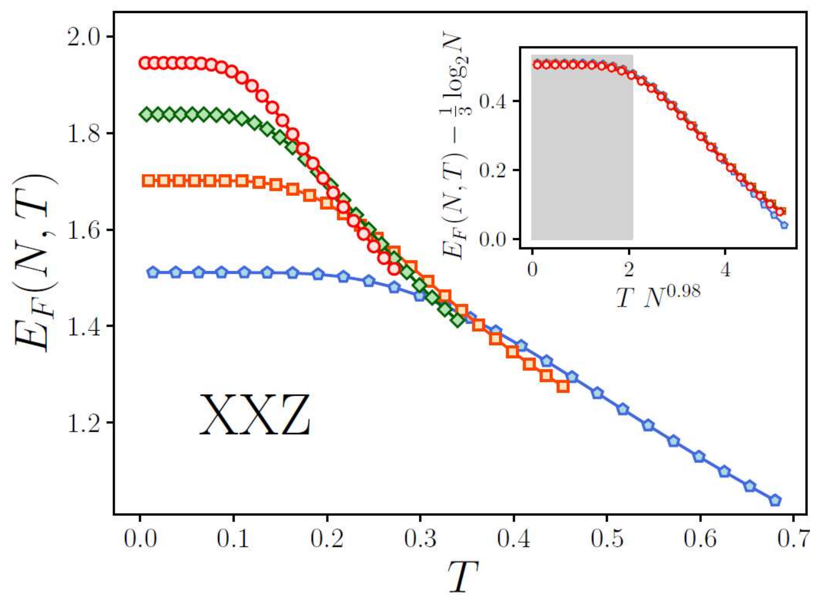
$$E_F(\rho) = \inf_{\{p_j,\psi_j\}} \left\{ \sum_j p_j \mathcal{S}(|\psi_j\rangle) : \rho = \sum_j p_j |\psi_j\rangle\langle\psi_j| \right\}$$

Tree Tensor operators

$$\hat{H}_{Ising} = J \sum_{i=1}^{N} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + h \hat{\sigma}_j^z \right) \qquad h = 1$$

$$\hat{H}_{XXZ} = J \sum_{j=1}^{N} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + \xi \, \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z \right) \qquad \xi = 0.5$$





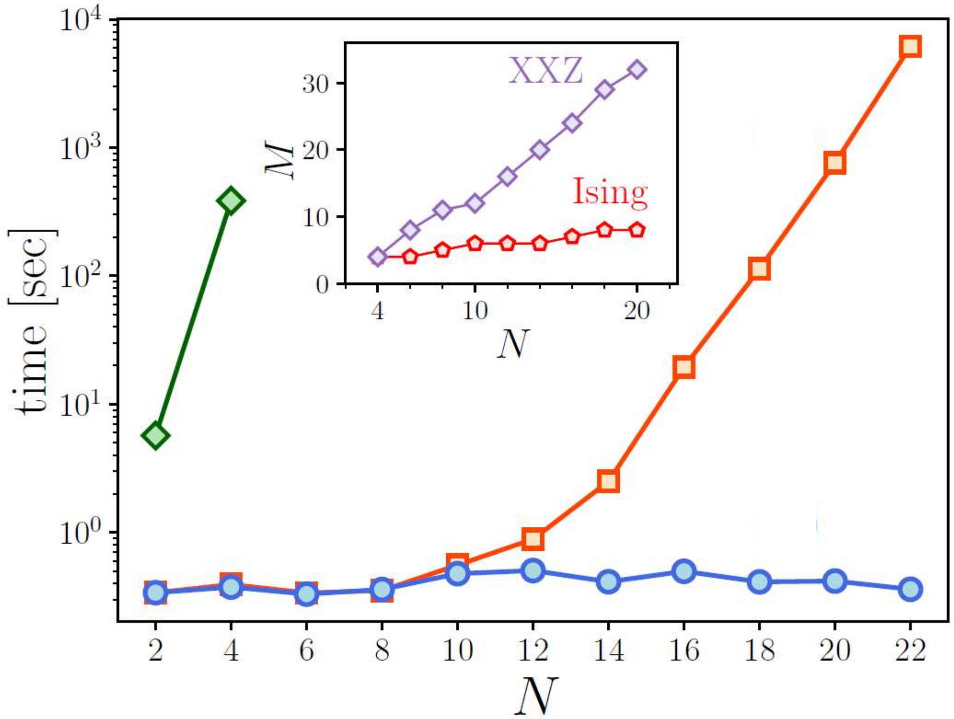
$T \leq 0.5\Delta$

$$E_F(T,N) = \frac{c}{3}\log N + g(TN^z)$$

Extension to finite temperature of conformal scaling of entanglement for critical systems

$\Delta \propto N^{-z}$

Tree Tensor operators



Full density matrix

Truncated K_0, uncompressed eigenstates

Truncated K_0 and TTO compression

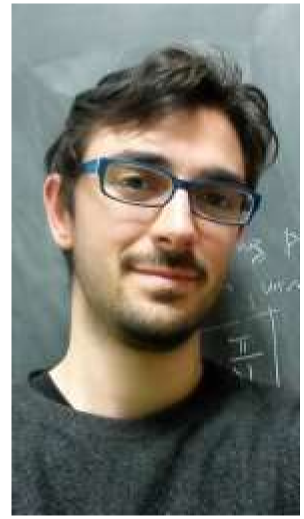
Conclusions

- Tensor Network techniques allow to tackle problems otherwise unaccessible
- Tree TNs guarantee a controlled computational complexity
- Augmented Tree TNs for studying high dimensional problems
- TTO for computing the Entanglment of Formation
- Next steps: dynamics implementation

S. Montangero

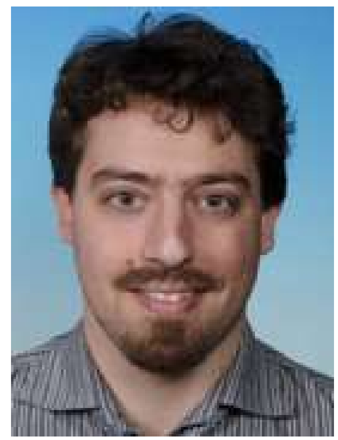
G. Magnifico

Thank you for your attention



L. Arceci

T. Felser



P. Silvi