Fighting qubit loss in topological QEC codes: theory and experiments

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1 - Brief introduction to Kitaev’s Toric Code

2 - Qubit Loss Error Correction: Theory...

3 - ... and Experiment
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3 - ... and Experiment
Main obstacle towards quantum computers: errors & losses

Coupling to the environment causes decoherence

Examples

1. Magnetic field fluctuations

\[ |\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle \quad \text{quantum state} \]

\[ \rho = |\alpha_0|^2 |0\rangle\langle 0| + |\alpha_1|^2 |1\rangle\langle 1| \quad \text{classical state} \]

2. Losses

Inaccessible qubits

This talk
Topological quantum error correction with the toric code
Kitaev's toric code


- Qubits on the links / bonds of a 2D square lattice
- 2 types of stabilisers

\[ S_x = XXXXX \]
\[ S_z = ZZZZZ \]

\[ |0\rangle |0\rangle \quad |0\rangle |1\rangle \quad |1\rangle |0\rangle \quad |1\rangle |1\rangle \]

logical states

Logical info

- X type error will anticommute with the Z-type stabilizers
- Z type error will anticommute with the X-type stabilizers

code space

\[ S_z |\psi_L\rangle = + |\psi_L\rangle \quad S_x |\psi_L\rangle = + |\psi_L\rangle \]
Logical operators must act non trivially within the code space.

Logical operators = strings that percolate through the lattice and change the logical state in the code space.
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Motivation:
Losses and leakage can damage the performance of (topological) QEC codes.

Challenges:
• Find protocols to deal with qubit loss
• Understand **robustness** of codes used
• Develop and experimentally test **in-situ leakage loss detection** and **correction** protocols

Theory
Grassl M, Beth T, Pellizari T PRA 56 (1997)

Photons
Redefine the plaquette/vertex and the logical operators

The loss affects
- two Z-stabilisers
- two X-stabilisers

Qubit losses in the toric code

The threshold for losses is given by the bond percolation threshold. Qubit losses in the toric code (square lattice) can be described by the percolation threshold $p_c = 1/2$. The diagram illustrates the concept with no percolating path leading to no logical operator.
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Qubit Loss Error Correction: Experiment

Goals

• Provide a toolbox for correcting losses in generic quantum codes
  Detect if the loss has happened
  Decide if correcting or not the code

• Devise the smallest example in a trapped ion setup

R Stricker, DV, A Erhard, L Postler, M Meth, M Ringbauer, P Schindler, T Monz, M Müller, R Blatt
Experimental deterministic correction of qubit loss, Nature 585, 207 (2020)
Qubit loss correction with four qubits

4 qubits

3 stabilisers \sim \text{toric code}

\begin{align*}
S_1^Z &= Z_1 Z_2 & S_2^Z &= Z_1 Z_3 \\
S_1^X &= X_1 X_2 X_3 X_4
\end{align*}

Logical operators

\begin{align*}
T^Z &= Z_1 Z_4 & T^X &= X_4
\end{align*}

Code space

\begin{align*}
S_1^Z |\psi_L\rangle &= + |\psi_L\rangle & S_2^Z |\psi_L\rangle &= + |\psi_L\rangle \\
S_1^X |\psi_L\rangle &= + |\psi_L\rangle
\end{align*}

3 qubits

\begin{align*}
\tilde{S}_1^Z &= S_1^Z S_2^Z = Z_2 Z_3 & \text{undetermined}
\end{align*}

2 stabilisers

\begin{align*}
\tilde{S}_1^Z &= S_1^Z S_2^Z = Z_2 Z_3 \\
\tilde{S}_1^X &= X_2 X_3 X_4 \\
\tilde{T}^Z &= T^Z S_1^Z = Z_2 Z_4 \\
\tilde{T}^X &= T^X = X_4
\end{align*}
Experimental qubit loss detection and correction: The whole picture

Encoding

Induce loss

QND loss detection

Case of no qubit loss

Case of qubit loss

Code reconstruction

Minimal example

4 physical qubits
Experimental qubit loss detection and correction: The whole picture

Encoding

QND loss detection

Code reconstruction

|0⟩

Induce loss

Minimal example

4 physical qubits
Qubit loss event

2 - Qubit loss event

Encoding

QND loss detection

Code reconstruction

Induce loss

Case of no qubit loss

Case of qubit loss

Expectation value

$D_{5/2}$

$S_{1/2}$

Tunable loss from $|0\rangle$

0-100 % loss probability

$p_L \sim \sin^2(\phi/2)$

Coherent rotation

$R(\phi)$

$^{40}\text{Ca}^+$
3 - QND qubit loss detection
3 - Qubit loss detection

Mølmer-Sørensen gate:
- Bichromatic laser field
- Two-photon resonant process

If code qubit is lost

If code qubit is not lost

Mølmer-Sørensen gate:
Qubit loss and correction - the entire cycle

\[ |\psi_L\rangle = \frac{1}{\sqrt{2}} (|0_L\rangle + i|1_L\rangle) \]

For the other logical states
R Stricker et al, Nature 585 207 (2020)
Conclusions

- Quantum error correcting codes can be realised in topological systems.
- Losses can affect quantum computers but can be cured with success.
- Experimental schemes for detecting and correcting losses can be developed.

Where this work has started…