Fighting qubit loss in topological QEC codes: theory and experiments

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1 - Brief introduction to Kitaev's Toric Code

2 - Qubit Loss Error Correction: Theory...

3 - ... and Experiment
1 - Brief introduction to Kitaev's Toric Code

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3 - ... and Experiment
Main obstacle towards quantum computers: errors & losses

Coupling to the environment causes decoherence

Examples

1. Magnetic field fluctuations

\[ |\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle \quad \text{quantum state} \]

\[ \rho = |\alpha_0|^2 |0\rangle\langle 0| + |\alpha_1|^2 |1\rangle\langle 1| \quad \text{classical state} \]

dephasing

2. Losses

Inaccessible qubits

This talk
Topological quantum error correction with the toric code
Kitaev's toric code

- Qubits on the links / bonds of a 2D square lattice
- 2 types of stabilisers

\[ S_z = ZZZZZZ \]
\[ S_x = XXXXX \]

\( |1\rangle \)
\( |0\rangle \)

they all commute and form a group

Errors
- X type error will anticommute with the Z-type stabilizers
- Z type error will anticommute with the X-type stabilizers

code space

\[ S_z |\psi_L\rangle = + |\psi_L\rangle \quad S_x |\psi_L\rangle = + |\psi_L\rangle \]

logical info

logical states

\(|\bar{0}\rangle |\bar{0}\rangle \quad |\bar{0}\rangle |\bar{1}\rangle \quad |\bar{1}\rangle |\bar{0}\rangle \quad |\bar{1}\rangle |\bar{1}\rangle \)
Logical operators must act non trivially within the code space.

Logical operators = strings that percolate through the lattice and change the logical state in the code space.

$$\bar{X}_1 = \sigma_1^x \sigma_2^x \ldots \sigma_n^x$$

Logical qubits

Qubit Losses

| 0 0 | 0 1 | 1 0 | 1 1 |
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Motivation:
Losses and leakage can damage the performance of (topological) QEC codes

Challenges:
• Find protocols to deal with qubit loss
• Understand robustness of codes used
• Develop and experimentally test in-situ leakage loss detection and correction protocols

Theory
Grassl M, Beth T, Pellizari T PRA 56 (1997)

Photons
Redefine the plaquette/vertex and the logical operators

The loss affects:
- two Z-stabilisers
- two X-stabilisers

Qubit losses in the toric code

The threshold for losses is given by the bond percolation threshold. Qubit losses in the toric code (square lattice) occur when there is no percolating path, which corresponds to no logical operator. The loss threshold is given by $p_c = 1/2$ (square lattice).
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Goals

• Provide a toolbox for correcting losses in generic quantum codes
  - Detect if the loss has happened
  - Decide if correcting or not the code

• Devise the smallest example in a trapped ion setup

R Stricker, DV, A Erhard, L Postler, M Meth, M Ringbauer, P Schindler, T Monz, M Müller, R Blatt
Experimental deterministic correction of qubit loss, Nature 585, 207 (2020)
Qubit loss correction with four qubits

4 qubits

3 stabilisers ~ toric code

\[ S_1^Z = Z_1 Z_2 \quad S_2^Z = Z_1 Z_3 \]
\[ S_1^X = X_1 X_2 X_3 X_4 \]

Logical operators

\[ T^Z = Z_1 Z_4 \quad T^X = X_4 \]

Code space

\[ S_1^Z |\psi_L\rangle = + |\psi_L\rangle \quad S_2^Z |\psi_L\rangle = + |\psi_L\rangle \]
\[ S_1^X |\psi_L\rangle = + |\psi_L\rangle \]

3 qubits

Loss

code-switching

2 stabilisers

\[ \tilde{S}_1^Z = S_1^Z S_2^Z = Z_2 Z_3 \quad \checkmark \]
\[ \tilde{S}_1^X = X_2 X_3 X_4 \quad \text{undetermined} \]

\[ \tilde{T}^Z = T^Z S_1^Z = Z_2 Z_4 \]
\[ \tilde{T}^X = T^X = X_4 \]
Experimental qubit loss detection and correction: The whole picture

1. Encoding
2. Induce loss
3. QND loss detection
4. Code reconstruction

Minimal example
4 physical qubits

Expectation value of $Z$ case

$S^Z_1$ $S^Z_2$
Experimental qubit loss detection and correction: The whole picture

1. Encoding
2. Induce loss
3. QND loss detection
4. Code reconstruction

Minimal example
4 physical qubits

\[ S_Z^1 \quad S_Z^2 \]
\[ S_X^1 \quad S_X^2 \]
2 - Qubit loss event

- Encoding
- Induce loss
- QND loss detection
- Code reconstruction

40Ca+

\[ D_{5/2} \]

\[ S_{1/2} \]

Coherent rotation \( R(\phi) \)

Tunable loss from \(|0\rangle\)

0-100% loss probability

\[ p_L \sim \sin^2(\phi/2) \]
3 - QND qubit loss detection

Encoding

QND loss detection

Code reconstruction

Induce loss

Case of no qubit loss

Case of qubit loss

Expectation value

Expectation value

Loss case

Ancilla

Code qubit

MS two-qubit gate

Bit-flip
### 3 - Qubit loss detection

**Mølmer-Sørensen gate:**
- Bichromatic laser field
- Two-photon resonant process

**QND loss detection**

![Diagram showing state transitions and gate operations](image)

**If code qubit is lost**

- \( D_{5/2} \)
- \( S_{1/2} \)
- Loss detected

**If code qubit is not lost**

- \( D_{5/2} \)
- \( S_{1/2} \)
- No loss detected

**Encoding**

- QND loss detection
- Code reconstruction

**Induce loss**

- If code qubit is lost
- If code qubit is not lost
Qubit loss and correction - the entire cycle

$$|\psi_L\rangle = \frac{1}{\sqrt{2}}(|0_L\rangle + i|1_L\rangle)$$

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For the other logical states
R Stricker et al, Nature 585 207 (2020)
Quantum error correcting codes can be realised in topological systems

Losses can affect quantum computers but can be cured with success

Experimental schemes for detecting and correcting losses can be developed

R Stricker, DV, A Erhard, L Postler, M Meth, M Ringbauer, P Schindler, T Monz, M Müller, R Blatt

*Experimental deterministic correction of qubit loss*, Nature 585, 207 (2020)
Where this work has started…